Generalized Jacobi functions of fractional degree and optimal polynomial approximations in fractional Sobolev-type spaces

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We define the generalised Jacobi functions of fractional degree (GJF-Fs) by allowing the (integer) degree of the Jacobi polynomials (JPs) defined by the hypergeometric functions to be real. The significance of this family of GJF-Fs resides in that they enjoy some fractional integral/derivative formulas, leading to analytical tools for both algorithm development and analysis of fractional PDEs. Indeed, the Jacobi polyfractonomials [1] and generalised Jacobi functions/polynomials [2, 3] are special cases of GJF-Fs. In this talk, we highlight their impactful role in spectral approximation theory. In a nutshell, using the fractional integration by parts, the Jacobi polynomial expansion coefficients for functions with fractional Sobolev regularity can be naturally, precisely expressed in terms of GJF-Fs. We are then able to optimally estimate the decay rate of the expansion coefficients, so do the errors of the orthogonal projections, polynomial interpolation and quadratures. In particular, we can significantly improve the L^∞-estimate of the Chebyshev expansion in [4] for functions with limited regularity. For example, we can recover the best possible order by polynomial approximation of functions with singularity, e.g., $|x-a|^\beta$ with real $\beta > 0$ and $a \in [-1, 1]$.

In addition to this topic, we shall report an efficient Hermite spectral method for nonlocal problems [5] in an unbounded domain.

KEY WORDS: Generalized Jacobi functions, fractional Sobolev-type spaces, orthogonal polynomial expansions, error estimates

REFERENCES

1. M. Zayernouri and G.E. Karniadakis. Fractional Sturm-Liouville eigen-problems: Theory and numerical approximation. *J. Comput. Phys.*, 252:495–517, 2013.
2. B.Y. Guo, J. Shen, and L.L. Wang. Generalized Jacobi polynomials/functions and their applications. *Appl. Numer. Math.*, 59(5):1011–1028, 2009.
3. S. Chen, J. Shen, and L.L. Wang. Generalized Jacobi functions and their applications to fractional differential equations. *Math. Comp.*, 85 (300), 1603-1638, 2016.
4. L.N. Trefethen. Is Gauss quadrature better than Clenshaw-Curtis? *SIAM Review*, 51(1):67–87, 2008.
5. Q. Du, M. Gunzburger, R. Lehoucq, and K. Zhou. Analysis and approximation of nonlocal diffusion problems with volume constraints. *SIAM Review*, 54(4), 667-696, 2012.