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SPECIAL ISSUE PAPER

Achieving capacity fairness for wireless mesh networks

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ABSTRACT

This paper addresses a joint problem of power control and channel assignment within a wireless mesh network. A wireless mesh network is made up of two kinds of nodes: mesh routers (MRs) and user nodes (UNs). The MRs form a backbone network, while UNs receive data from the backbone network by connecting to the MRs *via* one hop. This paper aims to find the optimal joint solution of power control and channel assignment of the wireless mesh networks such that the minimum capacity of all links is maximized. We develop an upper bound for the objective by relaxing the integer variables and linearization. Subsequently, we put forward a heuristic approach to approximate the optimal solution, which tries to increase the minimal capacity of all links *via* setting tighter constraint and solving a binary integer programming problem. Simulation results show that solutions obtained by this algorithm are very close to the upper bounds obtained *via* relaxation, thus suggesting that the solution produced by the algorithm is near-optimal. Copyright © 2009 John Wiley & Sons, Ltd.

KEYWORDS
max–min fariness; power control; channel assignment; wireless mesh network

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1. INTRODUCTION

In this paper, we consider a two-tiered wireless mesh network that consists of a number of user nodes (UNs) and multiple mesh routers (MRs). UNs are deployed at strategic positions, transmitting and receiving information to/from the nearby MR *via* a single hop (see Figure 1). Such a two-tiered architecture has been used in many wireless communication systems. In disaster area, the transceivers carried by the first responders play the role of UNs while the vehicles with powerful antennas near those UNs work as MRs to facilitate communication between first responders and the command centre. As another example, powerful receivers are set near humidity sensors on farms to gather information for controlling irrigation systems. As can be seen, both examples share a similar scenario: UNs are placed at fixed positions in the sensing field and sending information to nearby MRs. In our proposed network scenarios, UNs are distributed on various spots to collect or receive information, thus their positions cannot be altered in the sensing field. MRs, a higher-level nodes forming the backbone network, are responsible for communicating with UNs.

Recently fairness of network throughput of channels has been placed with great importance. The reason is that the traditional goal of maximizing the summation of throughput on all links could result in unbalanced use of network resources. In this paper, we aim to enhance the fairness of network throughput by maximizing the minimal capacity of all UNs. In our proposed two-tiered wireless network model, one main issue is to deal with an increasing number of UNs while using orthogonal frequency-division multiple access (OFDMA) as a multiple access mechanism. Obviously, the fairness of network throughput is highly dependent on the channel assignment as well as power control of each MR. Transmissions on the same channel could seriously depreciate each other's capacity due to interference. Therefore, in this paper, we aim to exploit spatial diversity of UNs and channel reuse to maximize the minimal capacity of all assigned links.

Speaking of the implementation of OFDM, the orthogonality allows for efficient modulator and demodulator implementation using the Fast Fourier Transform (FFT) algorithm on the receiver side, and inverse FFT on the sender side. The compute intensive and time critical functions that

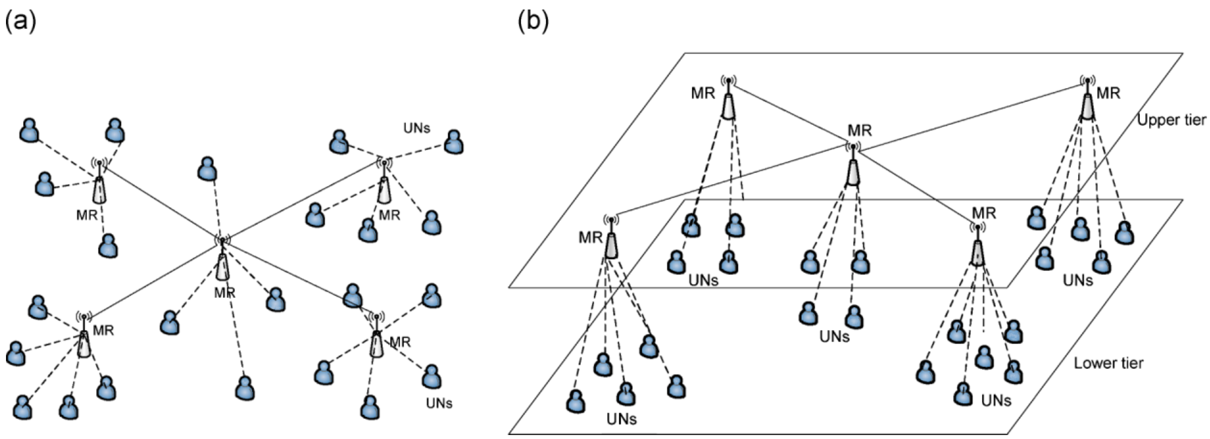

Figure 1. Reference architecture for a two-tiered wireless mesh network. (a) Physical topology. (b) Hierarchical view. The dashed line represents the link connection between network UNs and MRs. The solid line represents the links between MRs.

were traditionally implemented in hardware are nowadays being implemented in software running on those low-cost digital signal processers that can efficiently calculate the FFT.

Jointly studying power control on physical layer and channel assignment on MAC layer falls into cross-layer design problems. It would be natural to consider the power control parameter as a continuous variable and the channel assignment parameter as a binary integer variable. As a result, the fairness problem formulation falls into mixed integer nonlinear programming (MINLP), which is NP-hard in general. Thus we resort to approximating the problem by setting the power control parameter as a discrete variable, e.g. a finite number of equally spaced power levels. Subsequently, we put forward a heuristic approach: Binary Integer Programming-based Algorithm with fairness constraint (BIPA) to achieve suboptimal result when the power of MRs can only be set as discrete variables. In particular, the channel assignment and power control results are updated *via* solving a binary integer programming problem at each iteration when the fairness constraints are set tighter. In order to measure the quality of the suboptimal results obtained by BIPA, we develop an upper bound of the fairness objective by relaxing the integer variables and linearization. Simulation results show that the results obtained by BIPA are very close to the upper bound, thus suggest that (1) the upper bound is very tight; and (2) the solution obtained by BIPA is near-optimal.

The rest of this paper is organized as follows. In Section 2, we review the related work about channel assignment, power control and fairness issues in wireless networks. In Section 3, we describe the network and interference model. In Section 4, we formulate the channel assignment and power control problem to maximize the minimal capacity of all assigned links. In Section 5, we propose BIPA as the heuristic approach followed by the method to obtain the upper bound solution presented in Section 6. In Section 7, simulation results are presented to compare the solutions obtained by BIPA and the upper bound. Section 8 concludes this paper.

2. RELATED WORK

Many research efforts have been done in maximizing the network throughput. The strategies adopted include power control, channel assignment, using multi-tired architecture, minimizing interference and so on. In this section we will briefly review related work on multi-tiered wireless networks, power control, channel assignment and max–min fairness problems.

2.1. Multi-tiered wireless networks

Many researchers have proposed their network models with a multi-tiered architecture, most of which set the limit of tiers as 2. The second tier of network nodes are introduced due to various reasons. For instance, In [1], additional relay nodes, which are identical to a regular node but without sensor unit, are disseminated into a dense wireless sensor network. Joint optimization of relay deployment and power control is performed for lifetime elongation. A heuristic algorithm is presented in [2] for energy provisioning and relay node placement in wireless sensor networks to elongate the network lifetime. In [3], relay points with access to power supply are strategically placed to improve the throughput of wireless LAN. An iterative procedure is developed to compute the best placement of a fixed number of relay points. Simulation results for the 802.11-like system model show significant performance gain through optimal placement. For our work, MRs can connect to each other without distance and interference constraints. At least one of the MRs can access to the backbone network.

2.2. Power control

Power control schemes has been extensively studied to enhance wireless network performance. Sun *et al*. [4] propose that increasing nodal transmit power can lessen the interference caused by hidden node problem, and thus the network throughput could be increased. Behzad and Rubin [5] analyse and investigate the effect of nodal transmit power on the maximum level of the source-destination throughput. It adopts a different system model: the network nodes are accessing the channel based on time-division-multiple-access (TDMA) scheme to transmit packets. Tang *et al*. [6] study joint link scheduling and power control in a TDMA-based multihop wireless network with the objective of maximizing network throughput. The successful transmission is guaranteed only when the SINR is above a certain threshold value. Narayanaswamy *et al*. propose that within a heterogeneous *ad hoc* network, all the network nodes choose identical transmit power can maximize traffic carrying throughput, extend battery life as well as reduce contention at MAC layer. The so-called COMPOW protocol selects a common minimum transmit power for all nodes such that network connectivity is preserved [7]. Kawadia and Kumar [8] consider the power control problem when nodes are non-homogeneously dispersed, and proposes three solutions, CLUSTERPOW, tunnelled CLUSTERPOW and MINPOW for joint clustering and power control problem. All above-mentioned works are assuming that network nodes are sharing a common transmit channel, thus are different from our network model. In [9], Qiao *et al*. analyse the relationship among different radio ranges and transmit power's effects on the interference in 802.11a/h systems, and propose several frame-based intelligent power control mechanisms, which employ the best combination of the physical layer mode and transmit power level. The objective is different from ours: it pursues the minimization of communication energy consumption in 802.11 systems. In [10], Ho and Liew point out that the minimum transmit power scheme [7] can create hidden node problems, thus decrease network performance. However, most of the power control schemes are implemented in a scenario with one common shared channel.

2.3. Channel assignment

A number of channel assignment schemes have been proposed in recent years. In [11], Chin *et al*. address the problem of dynamically assigning channels in *ad hoc* wireless networks *via* power control in order to satisfy their minimum QoS requirements. The objective then is to maximize the number of co-channel links subject to some stability conditions. In [12], a cluster-based multipath topology control and channel assignment scheme is proposed, which explicitly creates a separation between the channel assignment and topology control functions, thus minimizes flow disruptions. In [13], Raniwala *et al*. propose a greedy load-aware channel assignment scheme when network nodes are with multiple radios. The goal of channel assignment is to bind each network interface to a radio channel such that the available bandwidth on each link is proportional to its expected load. In [14], Alicherry *et al*. mathematically formulate the joint channel assignment and routing problem, taking into account the interference constraints, the number of channels in the network and the number of radios available at each MR. A centralized algorithm is developed to solve the problem to yield the optimized network throughput. The channel assignment algorithm is used to adjust the flow on the flow graph to keep the increase of interference for each channel to a minimum. In [15], Ramachandran *et al*. propose an interference-aware channel assignment algorithm and protocol for multi-radio wireless mesh networks. The proposed solution intelligently assigns channels to radios to minimize interference and thus enhance network performance. Few research efforts have addressed the problem of utilizing power control mechanism to influence the interference as well as channel gains between network nodes and further determine the optimal channel assignment.

2.4. Max–min fairness

In the area of Operations Research, the max–min problem has been extensively studied. In [16], Tang develops a nonsimplex-based algorithm that finds an optimal solution to a max–min allocation problem with nonnegative integer variables. Fairness has been well studied in both network layer and MAC layer. Recently, in [17], Hou *et al*. develop an elegant polynomial time algorithm to calculate the rate allocation under a network lifetime constraint with respect to a two-tiered wireless sensor network. In [6], a Linear Programming (LP) formulation is provided to solve the max–min guaranteed maximum throughput bandwidth allocation problem. In addition, the Lexicographical Max–Min Bandwidth Allocation (LMMBA) problem is solved by a polynomial time optimal algorithm. In [18], Liang *et al*. investigate resource allocation for fading relay channels under separate power constraints, which falls into max–min problems. However, it is studied within a context of three-terminal networks. Few research efforts truly address the max–min fairness problem of link throughput *via* channel assignment and power control.

3. NETWORK MODELING

In this section, we present an example of cross-layer optimization problem for a wireless mesh network. We first address some technical aspects of this mesh network in terms of network architecture, path loss model as well as interference model. Then we set out to formulate the cross-layer optimization problem. We list the notation in this paper in Table I.

Table I. Notation.

Symbol	Definitions
d_0	An amount of distance
d_1	An amount of distance
P_0	Signal power measured at d_0 metres from transmitter
P_1	Signal power measured at d_1 metres from transmitter
P_t	General transmit power
G_t	Transmitter antenna gain
G_r	Receiver antenna gain
λ	Wavelength of the transmitted signal
c	Velocity of radio-wave propagation in free space
P_r	General received power
P_{noise}	Noise Power
A	General Capacity of a channel
W	Bandwidth of the channel
d	Distance between the transmitter and receiver
N	Number of UNs
M	Number of MRs
$\mathcal{U}$	Set of UNs
$\mathcal{R}$	Set of MRs
u_i	The ith UN
r_i	The ith MR
$P_{\max}$	Maximum transmit power
Q	Number of power levels
l	Index of a power level
Pr_{ij}^{lk}	Received power at u_j when the transmitter power of r_i is set as $\frac{l}{Q}P_{\max}$ on channel k
t_I	Interference threshold
t_R	Receiving threshold
$\mathcal{A}$	Channel capacity matrix A
Pr_{ij}	Power of the received signal at u_j from r_i
ρ_{ij}^{lk}	Binary assignment variable indicating the assignment of the kth channel of r_i to u_j at the transmit power of $\frac{l}{Q}P_{\max}(1 \le l \le Q)$
ζ	Minimum capacity of all assigned links
τ	Obtained second smallest capacity of all assigned links
$\mathcal{X}$	A general set of continuous variables
x_i	A general variable within the set $\mathcal{X}$
σ	Optimal max–min value of $\mathcal{X}$
η	Summation of all variables $\mathcal{X}$
n	Number of variables in $\mathcal{X}$
μ	Value of variables in $\mathcal{X}$, when $x_1 = x_2 = \cdots = x_n$

3.1. Network architecture

We focus on a two-tiered wireless mesh networks. There are two types of nodes in a mesh network: UNs and MRs. All nodes are placed at fixed locations. Each UN can connect to only one MR by establishing a channel with an adjustable transmit power. Since the mesh network uses OFDM for multiple access, we assume each MR can support the same limited number of channels. As for typical mesh network, the downlink data traffic is much more than uplink data traffic, thus we only consider the downlink case. It is also assumed that MRs can connect to each other without distance and interference constraints. At least one of the MRs can access to the backbone network.

3.2. Path loss model

The network throughput depends heavily on the packet reception rate, which can be modeled using path loss channel model in physical layer. In this paper, the following path loss model (1) is being used.

$$P_0 d_0^{\alpha} = P_1 d_1^{\alpha} \tag{1}$$

P_0 and P_1 are the signal power measured at d_0 and d_1 metres away from the transmitter, respectively. α denotes the path loss exponent. If we set d_0 to be 1 m, thus P_0 is the reference signal power measured at 1 m away from the transmitter. Then Equation (1) is simplified as $P_1 = \frac{P_0}{d_1^{\alpha}}$. P_0 can be calculated using the free space propagation model [19] as Equation (2).

$$P_0 = P_t G_t G_r \left(\frac{\lambda}{4\pi}\right)^2 \tag{2}$$

P_t is the transmit power. G_t and G_r are the transmitter and receiver antenna gains, respectively. $\lambda = c/f$ is the wavelength of the transmitted signal. c is the velocity of radio-wave propagation in free space, which is equal to the speed of light. Then the received power P_r at a distance d metres away from the transmitter can be calculated as Equation (3).

$$P_r = \frac{P_t G_t G_r \left(\frac{\lambda}{4\pi}\right)^2}{d^{\alpha}} \tag{3}$$

Let P_{noise} denote the noise power, then the signal-to-noise ratio (SNR) at the receiver end can be calculated as Equation (4).

$$\text{SNR} = \frac{P_r}{P_{\text{noise}}} \tag{4}$$

Based on Shannon formula [20], the capacity of the channel of a link can be expressed as Equation (5).

$$A = W \times \log_2\left(1 + \frac{P_t G_t G_r (\lambda/4\pi)^2}{d^{\alpha} P_{\text{noise}}}\right) \tag{5}$$

where A denotes the capacity of the channel, W denotes the bandwidth of the channel and d denotes the distance between the transmitter and receiver of the link.

4. PROBLEM FORMULATION

First of all, we need to provide some notation. Let the number of UNs be N, the number of MRs be M and the number of OFDM channels be C. Denote the set of UNs as $\mathcal{U} = \{u_1, u_2, \ldots, u_N\}$, the set of MRs as $\mathcal{R} = \{r_1, r_2, \ldots, r_M\}$. Denote the maximum transmit power as $P_{\max}$. We then introduce an integer parameter Q that represents the total number of power levels to which a transmitter can be adjusted, i.e. $\frac{1}{Q}P_{\max}, \frac{2}{Q}P_{\max}, \ldots, P_{\max}$. If the transmitter power of r_i is set as $\frac{l}{Q}P_{\max}(1 \le l \le Q)$ on channel k, the received power at u_j is denoted as Pr_{ij}^{lk}. Secondly, receiving constraint is considered regarding a successful transmission. Suppose there is a transmission from r_i to u_j, then the received power at u_j should be no less than the receiving constraint, denoted as t_R. Then we can define a channel capacity matrix $\mathcal{A}$ as Equation (6).

$$\mathcal{A}_{ij}^{lk} = \begin{cases} W \times \log_2\left(1 + \frac{Pr_{ij}^{lk}}{P_{\text{noise}}}\right) & : \ if \quad Pr_{ij}^{lk} \ge t_R; \\ 0 & : \ \text{otherwise} \end{cases} \tag{6}$$

Let us denote ρ_{ij}^{lk} as the binary assignment variable with $\rho_{ij}^{lk} = 1$ indicating the assignment of the kth channel of r_i to u_j at the transmit power of $\frac{l}{Q}P_{\max}$; $\rho_{ij}^{lk} = 0$ means the kth channel at power level l is not assigned between r_i and u_j. Then we consider constraint on interference. Suppose there is a transmission from r_i to u_j on the kth channel, then there is a limitation on the transmission powers of all other concurrent transmissions on the same channel. Specifically, we consider the interference power on u_j due to all other concurrent transmissions on the same channel is negligible if the overall received interference power is less than a threshold t_I $(t_I \le t_R)$, as shown by Equation (7).

$$\sum_{a=1}^{M}\sum_{b=1}^{N}\sum_{l=1}^{Q} \rho_{ab}^{lk} \times Pr_{aj}^{lk} \le t_I \tag{7}$$

Note that Equation (7) only holds when u_j receive signals on the kth channel. Therefore, it is necessary to develop Equation (7) into a more general form as shown in Equation (8).

$$\sum_{a=1}^{M}\sum_{b=1}^{N}\sum_{l=1}^{Q} \rho_{ab}^{lk} \times Pr_{aj}^{lk} + (\epsilon - t_I) \times \sum_{i=1}^{M}\sum_{l=1}^{Q} \rho_{ij}^{lk} \le \epsilon \tag{8}$$

where ϵ is a large value such that $\forall\, j\,(1 \le j \le N)$, $\sum_{a=1}^{M}\sum_{b=1}^{N}\sum_{l=1}^{Q}\rho_{ab}^{lk} \times Pr_{aj}^{lk} \le \epsilon$ always holds. In Equation (8), the term $\sum_{i=1}^{M}\sum_{l=1}^{Q}\rho_{ij}^{lk}$ accurately indicates if u_j is working on the kth channel.

Using the channel capacity matrix (6) and the interference constraint as Equation (8), our fairness problem can be formulated as Equation (9).
Fairness Problem: (Optimal channel assignment and power control problem)

$$\begin{aligned} \text{maximize} \quad & \zeta \\ \text{s.t.} \quad & \rho_{ij}^{lk} \in \{0, 1\} \\ & \sum_{i=1}^{M}\sum_{k=1}^{C}\sum_{l=1}^{Q} \rho_{ij}^{lk} = 1 \\ & \sum_{a=1}^{M}\sum_{b=1}^{N}\sum_{l=1}^{Q} \rho_{ab}^{lk} \times Pr_{aj}^{lk} + (\epsilon - t_I) \\ & \times \sum_{i=1}^{M}\sum_{l=1}^{Q} \rho_{ij}^{lk} \le \epsilon \\ & \sum_{i=1}^{M}\sum_{k=1}^{C}\sum_{l=1}^{Q} \rho_{ij}^{lk} \times \mathcal{A}_{ij}^{lk} \ge \zeta \\ & \forall\, r_i, u_j, 1 \le k \le C, 1 \le l \le Q \end{aligned} \tag{9}$$

The objective of our optimization problem is to maximize the minimal capacity of all assigned links, which is denoted as ζ in Equation (9). The second constraint is to guarantee that each UN can only be assigned to one channel linked to an MR. The third constraint is to ensure each transmission is interference free from other transmissions on the same channel, as the interference constraint (8).

5. BINARY INTEGER PROGRAMMING BASED ALGORITHM WITH FAIRNESS CONSTRAINT (BIPA)

We now take a closer look at the fairness problem as formulated in Equation (9) in Section 4. Observe that the objective is not a linear function of the set of variables ρ_{ij}^{lk}, which obstructs many classic algorithms from being applicable. In other words, the key obstacle in solving this fairness problem lies in the transformation of the objective function, while still maintaining the motive of the *max–min* problem: the minimal capacity of all assigned links are maximized. To this end, it comes down very naturally that the sum of the capacity of all assigned links could be recognized as the simplest linear function. Based on the foregoing discussion, the newly generated problem is described as Equation (10).

Suboptimal Fairness Problem:

$$
\begin{aligned}
\max \quad & \sum_{j=1}^{N}\sum_{i=1}^{M}\sum_{k=1}^{C}\sum_{l=1}^{Q} \rho_{ij}^{lk} \times \mathcal{A}_{ij}^{lk} \\
\text{s.t.} \quad & \rho_{ij}^{lk} \in \{0, 1\} \\
& \sum_{i=1}^{M}\sum_{k=1}^{C}\sum_{l=1}^{Q} \rho_{ij}^{lk} = 1 \\
& \sum_{a=1}^{M}\sum_{b=1}^{N}\sum_{l=1}^{Q} \rho_{ab}^{lk} \times Pr_{aj}^{lk} + (\epsilon - t_I) \\
& \times \sum_{i=1}^{M}\sum_{l=1}^{Q} \rho_{ij}^{lk} \leq \epsilon \\
& \sum_{i=1}^{M}\sum_{k=1}^{C}\sum_{l=1}^{Q} \rho_{ij}^{lk} \times \mathcal{A}_{ij}^{lk} \geq \zeta \\
& \forall r_i, u_j, 1 \leq k \leq C, 1 \leq l \leq Q
\end{aligned} \tag{10}
$$

In Equation (10), the second constraint is to guarantee that each UN can only be assigned to one channel with one MR. The third constraint is to ensure each transmission is interference free from other transmissions on the same channel, as the interference constraint (8). We notice that the fourth constraint demands that each of the assigned links has a capacity larger than ζ. It is obvious that if ζ is set the same as the maximized value of the minimal capacity of all assigned links, the optimal solution of the fairness problem can be yielded. Naturally, the key to obtain the optimal result to the fairness problem resides in adjusting the value of ζ. The value of ζ is very subtle to the extent that the suboptimal fairness problem would be infeasible with very high value of ζ while we would end up obtaining a solution that is far from the optimal when ζ is set too low. Then the intuition would be to reach the maximal value of ζ such that the suboptimal fairness problem is feasible. It would seem quite straightforward to update the value of ζ after each iteration with the minimal value of all assigned links. However, a closer examination would repel this idea due to the fact that the updated constraint would not alter the results in following iterations since the current solution is already optimized. But since possibilities still remain that the minimal capacity could be increased if the constraint is set tighter, we choose to take the risk that the suboptimal fairness problem be infeasible by updating ζ with the second smallest capacity of all assigned links yielded in the previous iteration. Since transmit power on all assigned links has been maximized (otherwise there is still space for the summation of all assigned links to increase, which violates the objective of the suboptimal fairness problem), it would also be justifiable to point out that setting ζ as the second smallest capacity could break the current assignment: eliminating the link with smallest capacity and assigning that corresponding UN a 'better' link. Since the cases that the two smallest capacities are equal are very rare due to different distances among all UN–MR pairs, this iterative procedure terminates as soon as it cannot reach a feasible problem.

We also need to justify the validity of the transformation of the objective function from maximizing the minimal capacity to maximizing the summation of the capacity of all assigned links. Given a value of ζ, the objective of the suboptimal fairness problem should help produce solutions with higher minimal capacity of all assigned links. Based on this consideration, the objective of maximizing the summation of capacity on all assigned links has the following two merits: (1) it is the most simplified form of linear functions; (2) it pushes the minimal capacity of all assigned links higher.

To solve the suboptimal fairness problem, we use a LP-based branch-and-bound algorithm [21]. The algorithm creates a search tree by repeatedly adding constraints to the problem, called Branching. At a branching step, the algorithm chooses a variable x_j whose current value is not an integer and adds the constraint $x_j = 0$ to form one branch and the constraint $x_j = 1$ to form the other branch. This process can be represented by a binary tree, in which the nodes represent the added constraints. At each node, the algorithm solves the LP-relaxation problem using the constraints at that node and decides whether to branch or to move to another node depending on the outcome. There are three possibilities: (1) If the LP relaxation problem at the current node is infeasible or its optimal value is greater than that of the best integer point, the algorithm removes the node from the tree, after which it does not search any branches below that node. The algorithm then moves to a new node according to the pre-specified method. (2) If the algorithm finds a new feasible integer point with lower objective value than that of the best integer point, it updates the current best integer point and moves to the next node. (3) If the LP-relaxation problem's optimal value is not an integer and the optimal objective value of the LP relaxation problem is less than the best integer point, the algorithm branches below this node.

The algorithm could potentially search all 2^n binary integer vectors, where n is the number of variables. However, it has been proved that such a binary integer programming problem could be transformed into a linear optimal distribution problem [22] by generating a directed graph, to reduce the computation complexity to only $O(n^3)$.

From the foregoing, we propose BIPA algorithm as the following:

- Step 1: Set $\zeta = 0$;
- Step 2: Solve the suboptimal fairness problem as Equation (10);
- Step 3: Obtain the second smallest value τ of all assigned links based on the channel assignment solution obtained from Step 2;
- Step 4: Update ζ with τ, repeat Steps 2 and 3 until no feasible solution exists.

6. AN UPPER BOUND FOR THE OBJECTIVE FUNCTION

In Section 5, we propose a heuristic approach to aggressively approximate the maximized minimal value by iteratively tightening the constraint that the capacity of each link is larger than a threshold value ζ. In this section, we develop an upper bound for the objective function. First of all, we claim Theorem 1 and give the proof subsequently.

Theorem 1 (Equivalent Conditions for Max–min Objective). *Given a set of continuous variables* $\mathcal{X} = \{x_1, x_2, \ldots, x_n\}$*, max–min* $\mathcal{X}$ *is achieved iff*

- $\sum \mathcal{X}$ *is maximized;*
- $x_1 = x_2 = \cdots = x_n$.

Proof. Suppose $x_i = \sigma$ denotes the optimal max–min value. Since $x_j \geq x_i$ ($\forall j \neq i$), then we can reduce the value of x_j ($j \neq i$) to the point that $x_1 = x_2 = \cdots = x_n$. Because max–min value σ cannot be increased with respect to the set $\mathcal{X}$, then $\sum \mathcal{X} = n \times \sigma$ is maximized. Thus the necessary condition is validated.

We will validate the sufficient condition based on contradiction. Assume $\sum \mathcal{X} = \eta$ is maximized, $x_1 = x_2 = \cdots = x_n$, and the optimal max–min value $\mu \geq \frac{\sum \mathcal{X}}{n}$. Then we can set $x_1 = x_2 = \cdots = x_n = \mu$, obviously $n \times \mu \geq \eta$, which violates the assumption that $\sum \mathcal{X}$ is maximized. Therefore the sufficient condition is proved to be true. ■

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{C}\sum_{l=1}^{Q} \rho_{ij}^{lk} \times \mathcal{A}_{ij}^{lk} \\
\text{s.t.} \quad & \rho_{ij}^{lk} \in [0, 1] \\
& \sum_{i=1}^{M}\sum_{k=1}^{C}\sum_{l=1}^{Q} \rho_{ij}^{lk} = 1 \\
& \sum_{a=1}^{M}\sum_{b=1}^{N}\sum_{l=1}^{Q} \rho_{ab}^{lk} \times Pr_{aj}^{lk} + (\epsilon - t_I) \\
& \times \textstyle\sum_{i=1}^{M} \sum_{l=1}^{Q} \rho_{ij}^{lk} \leq \epsilon \\
& \sum_{i=1}^{M}\sum_{k=1}^{C}\sum_{l=1}^{Q} \rho_{i1}^{lk} \times \mathcal{A}_{i1}^{lk} \\
& = \textstyle\sum_{i=1}^{M} \sum_{k=1}^{C} \sum_{l=1}^{Q} \rho_{i2}^{lk} \times \mathcal{A}_{i2}^{lk} \\
& = \ldots \\
& = \textstyle\sum_{i=1}^{M} \sum_{k=1}^{C} \sum_{l=1}^{Q} \rho_{iN}^{lk} \times \mathcal{A}_{iN}^{lk} \\
& \forall r_i, u_j, 1 \leq k \leq C, 1 \leq l \leq Q
\end{aligned}
\tag{11}
$$

Inspired by Theorem 1, we figure out that the optimal max–min capacity of all assigned links can be achieved iff the overall network capacity is maximized and all assigned links have the same capacity. Now we reexamine the fairness problem in Equation (9). Since the capacity values are discrete due to a finite number of power levels, it is hardly possible to reach a point that the same capacity is achieved on all links with different distances of MR–UN pairs. To render the capacity values continuous, we can relax the binary integer requirement on ρ_{ij}^{lk} by setting $0 \leq \rho_{ij}^{lk} \leq 1$. In this way, we can pursue an upper bound for the objective when the throughput of each assigned link is equal to each other. Such relaxation enables us to formulate the *max–min* problem as Equation (11).

In Equation (11), the second constraint is to guarantee that each UN can only be assigned to one channel with one MR. The third constraint is to ensure each transmission is interference free from other transmissions on the same channel, as the interference constraint (8). The fourth constraint demands the capacity values of all links are the same. This new (relaxed) formulation falls into a standard LP problem. We can obtain the solution in polynomial time. Due to the relaxation, the solution to this LP problem corresponds to an upper bound to the objective of the original problem in Equation (9). There may not exist a feasible solution to achieve this upper bound. Nevertheless, this upper bound offers a benchmark to measure the quality of the feasible solution obtained from BIPA proposed in Section 5.

7. SIMULATION RESULTS

In this section, we present simulation results on max–min capacity values yielded by BIPA and the corresponding upper bound. In particular, we compare the fairness performance yielded by BIPA and the upper bound by proposing a new evaluation metric named *performance ratio*. We produce performance ratios under different parameter sets to investigate the impact of different factors on the fairness performance, such as the number of channels, the number of power levels and the interference threshold.

7.1. Simulation setup

A set of UNs are randomly distributed in a square region ($\{(x, y)|0 \leq x \leq 100, 0 \leq y \leq 100\}$, (x, y) denotes the Cartesian coordinate of a point). Totally four MRs are placed within the area at coordinates (25, 25), (25, 75), (75, 75), (75, 25), respectively. The bandwidth of each channel is set as 1 MHz. The transmitter and receiver antenna gains are both set as 100. The path loss exponent $\alpha = 2$. The wavelength λ is set as $\frac{3\times10^8}{2.4\times10^9}$ m, which corresponds to the centre frequency of 2.4GHz. The noise power is set as 1×10^{-6} watt. For each set of system parameters, we generate 20 instances of the network scenarios with randomly distributed UNs to obtain the average performance.

Intuitively, the following parameters may have significant impact on system performance: the number of users (N), the total number of channels (C), the number of power levels (Q) and the interference threshold (t_I). To obtain extensive results, we vary the values of three parameters: Q, C and t_I. To construct different network topologies, we randomly place 10 UNs in the given region at the first trial and 15 UNs at the second trial. Figures 2 and 3 show the performance ratios when we change the values of C, Q and t_I for 20 random instances of 10-UN scenarios and 15-UN scenarios,

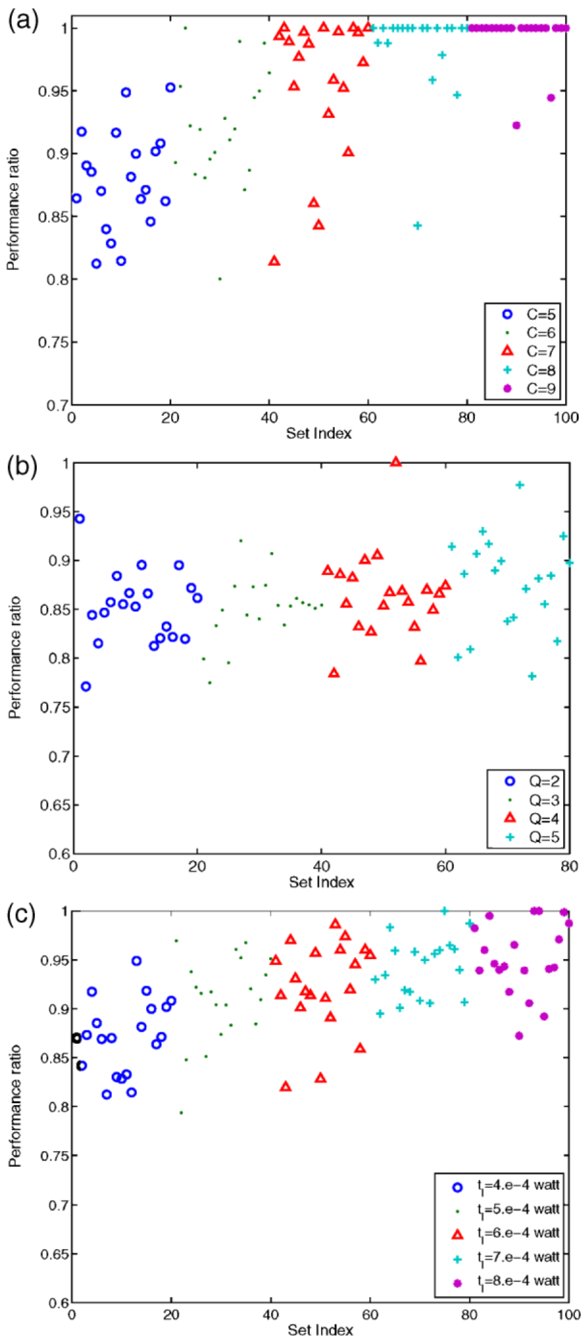

Figure 2. Performance ratios for 10-UN scenarios. 20 random-generated network topologies are tested for each parameter set. (a) Performance ratios when C changes for $P_{\max} = 5\,\text{W}$, $t_R = 10^{-3}\,\text{W}$, $Q = 10$ and $t_I = 4 \times 10^{-4}\,\text{W}$. (b) Performance ratios when Q changes for $P_{\max} = 5\,\text{W}$, $t_R = 10^{-3}\,\text{W}$, $C = 5$ and $t_I = 4 \times 10^{-4}\,\text{W}$. (c) Performance ratios when t_I changes for $P_{\max} = 5\,\text{W}$, $t_R = 10^{-3}\,\text{W}$, $C = 5$ and $Q = 10$.

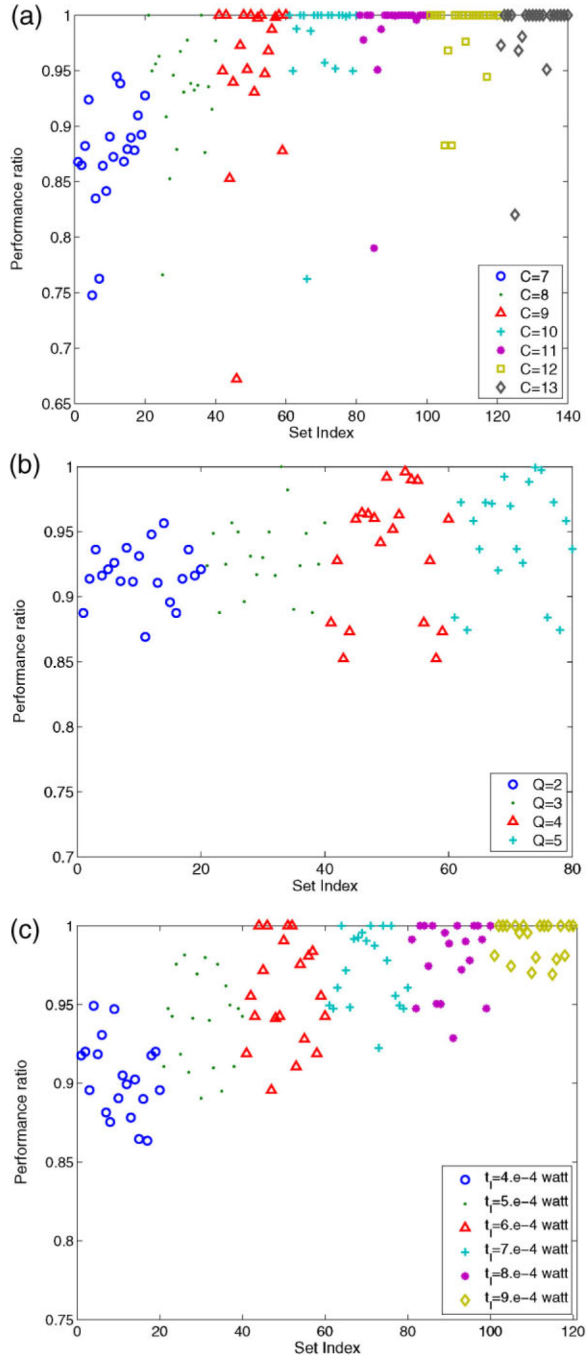

Figure 3. Performance ratios for 15-UN scenarios. 20 random-generated network topologies are tested for each parameter set. (a) Performance ratios when C changes for $P_{\max} = 2.5\,\text{W}$, $t_R = 10^{-3}\,\text{W}$, $Q = 3$ and $t_I = 4 \times 10^{-4}\,\text{W}$. (b) Performance ratios when Q changes for $P_{\max} = 2.5\,\text{W}$, $t_R = 10^{-3}\,\text{W}$, $C = 7$ and $t_I = 4 \times 10^{-4}\,\text{W}$. (c) Performance ratios when t_I changes for $P_{\max} = 2.5\,\text{W}$, $t_R = 10^{-3}\,\text{W}$, $C = 7$ and $Q = 3$.

respectively. Figures 4 and 5 show the average value of each set of 20 performance ratios corresponding with the same parameter set with respect to 10-UN scenarios and 15-UN scenarios, respectively.

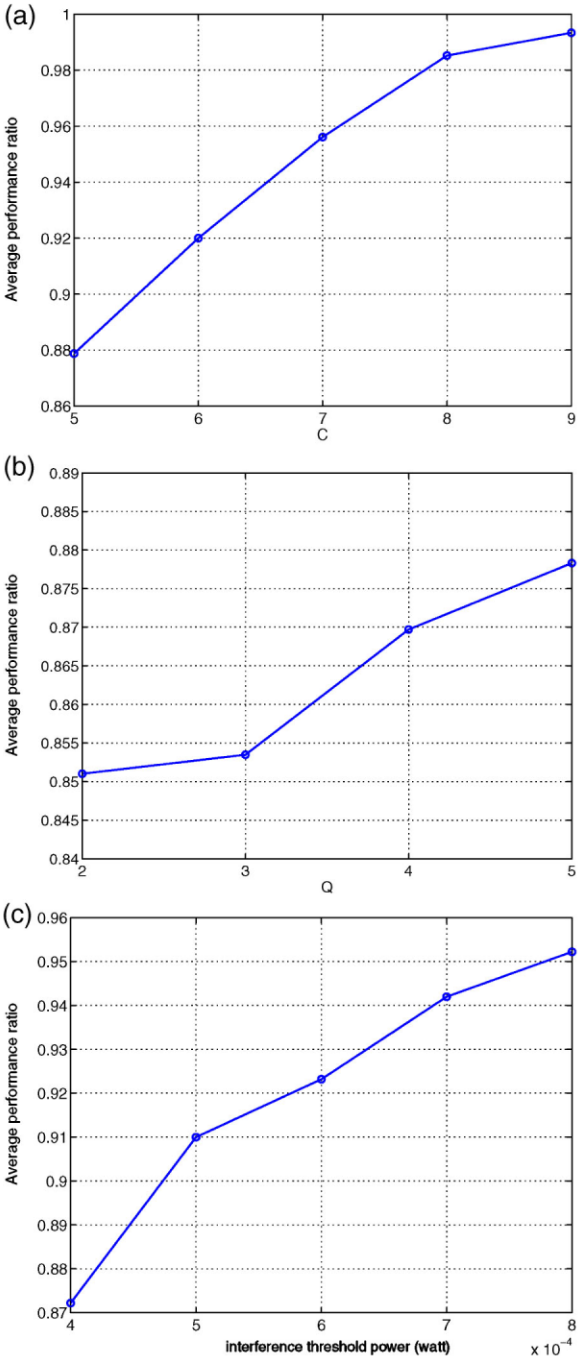

Figure 4. Average performance ratios for 10-UN scenarios. (a) Average performance ratios when C changes for $P_{\max} = 5\,\text{W}$, $t_R = 10^{-3}\,\text{W}$, $Q = 10$ and $t_I = 4 \times 10^{-4}\,\text{W}$. (b) Average performance ratios when Q changes for $P_{\max} = 5\,\text{W}$, $t_R = 10^{-3}\,\text{W}$, $C = 5$ and $t_I = 4 \times 10^{-4}\,\text{W}$. (c) Average performance ratios when t_I changes for $P_{\max} = 5\,\text{W}$, $t_R = 10^{-3}\,\text{W}$, $C = 5$ and $Q = 10$.

7.2. Evaluation metric

To evaluate the performance of BIPA, we propose a metric named as performance ratio, as the main evaluation

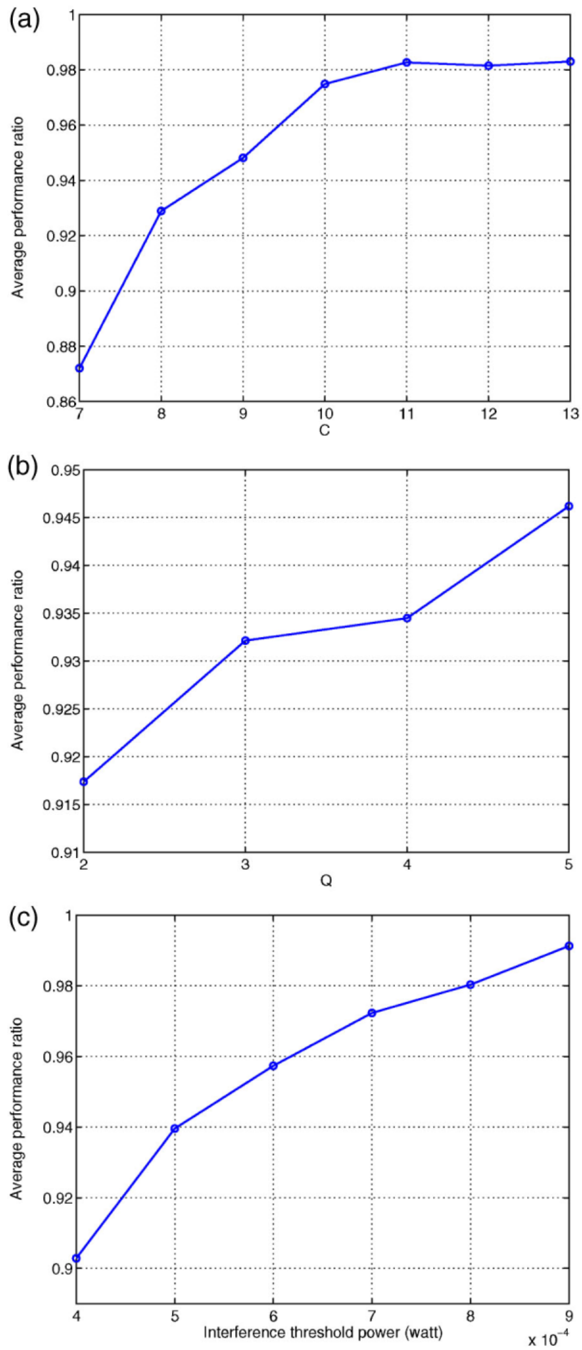

Figure 5. Average performance ratios for 15-UN scenarios. (a) Average performance ratios when C changes for $P_{\max} = 2.5\,\text{W}$, $t_R = 10^{-3}\,\text{W}$, $Q = 3$ and $t_I = 4 \times 10^{-4}\,\text{W}$. (b) Average performance ratios when Q changes for $P_{\max} = 2.5\,\text{W}$, $t_R = 10^{-3}\,\text{W}$, $C = 7$ and $t_I = 4 \times 10^{-4}\,\text{W}$. (c) Average performance ratios when t_I changes for $P_{\max} = 2.5\,\text{W}$, $t_R = 10^{-3}\,\text{W}$, $C = 7$ and $Q = 3$.

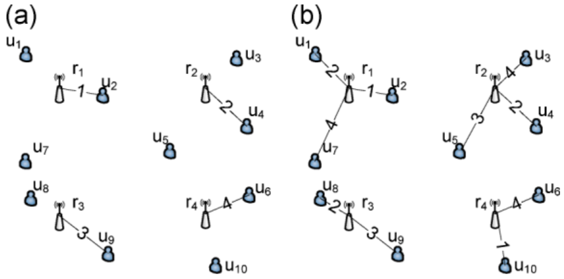

Figure 6. An example showing channel assignment with respect to our simulation scenario. The number on the line between a UN–MR pair shows the index of channel assigned for the link. (a) No power control ($Q = 1$). (b) With power control ($Q = 10$).

measure for the simulation results. The performance ratio is defined as the ratio of the suboptimal solution yielded by BIPA to the upper bound. As the suboptimal solution is always less than the upper bound, BIPA yields better solutions when performance ratios are closer to 1.

7.3. Impact of power control

Power control exerts a pivotal influence on results of channel assignment for a wireless mesh network. In Figure 6, we show an example of channel assignment with respect to our simulation area. From Figure 6(a), it can be seen that to avoid interference, only four UNs can be assigned channels. Any other assigned channel would be seriously interfered by one of the four channels as all the MRs are transmitting signal at their maximum power. In contrast, Figure 6(b) shows that all UNs can be assigned channels when power control can be realized at MRs. This is due to the fact each MR can automatically adjust its transmit power to avoid causing interference to other transmissions, and thus channels can be assigned at each MR to accommodate a lot more UNs.

7.4. Impact of number of channels

In this subsection, we evaluate the impact of number of channels on the performance ratios in 10-UN and 15-UN network scenarios. The performance ratios are calculated using the minimal capacity yielded by BIPA and the upper bound computed from Equation (11). First of all, it can be seen from Figures 2(a) and 3(a) that most performance ratios are very close to 1, which shows that the gap between solutions yielded by BIPA and the upper bound is very narrow. In addition, since the unknown optimal solution is between the solution obtained by BIPA algorithm and the upper bound, the upper bound is very tight, and BIPA yields near-optimal solutions. Secondly, from Figure 2(a), it is obvious that as we enlarge C from 5 to 9 regarding the same 20 10-UN scenarios, the points of performance ratios are positioned higher and the majority of the points are equal to 1 when $C = 9$. The reason is that with more channels, the interference constraint would be less tight and some MRs can increase its transmit power on certain channels. Thus the UN associated with the minimal capacity link would have the chance to be assigned a link with higher capacity. Accordingly, the same phenomenon is witnessed in Figure 4(a) where each point represents the average value of the performance ratios of the 20 10-UN scenarios corresponding with the same parameter set. In addition, for 15-UN network scenarios, similar trend can be witnessed as C is increased from 7 to 13 in Figures 3(a) and 5(a).

7.5. Impact of number of power levels

In this subsection, we evaluate the impact of number of power levels on the performance ratios in 10-UN and 15-UN network scenarios. The performance ratios are calculated using the minimal capacity yielded by BIPA and the upper bound computed from Equation (11). First of all, it can be seen from Figures 2(b) and 3(b) that most performance ratios are near 1, which shows that solutions yielded by BIPA is close to the upper bound. Therefore BIPA yields near-optimal solutions. Secondly, from Figure 2(b), the points of performance ratios do not get obvious higher positions as Q is increased from 2 to 5. However, as we can see from Figure 4(b), the average value of each set of 20 performance ratios does increase when Q is larger. This is because the larger number of power levels increases the tunability of power control, and as a result, the fairness performance gets closer to the optimal. Again, for 15-UN network scenarios, although obvious performance enhancement cannot be easily observed in Figure 3(b), the change in average performance ratios in Figure 5(b) does suggest that the increase of power levels works in the fairness performance's favour. However, it should also be noted that the fairness performance cannot be increased dramatically by more power levels. Due to the exponentially increased complexity induced by larger Q, it would not be recommended to pursue better fairness performance by rendering Q a large number.

7.6. Impact of interference threshold

In this subsection, we evaluate the impact of interference threshold on the performance ratios in 10-UN and 15-UN network scenarios. The performance ratios are calculated using the minimal capacity yielded by BIPA and the upper bound computed from Equation (11). It can be seen from Figures 2(c) and 3(c) that most performance ratios are very close to 1, which shows that the solutions yielded by BIPA is very close to the upper bound, and thus near the unknown optimal solution. It can also be implied that the upper bound is very tight. As can be seen from Figures 2(c) and 4(c), the max–min performance is enhanced as t_I grows from 4×10^{-4} to 8×10^{-4} W. The rationale behind is that when the value of t_I is larger, MRs can transmit at higher power levels without violating the interference constraints, causing the general link capacities to increase. Therefore the minimal capacity is increased accordingly. Similar trend is also witnessed in Figures 3(c) and 5(c) for 15-UN scenarios.

8. CONCLUSION

This paper addresses fairness problem on the throughput of all links for a two-tiered wireless mesh network. The fairness problem is formulated with cross-layer behaviours and constraints, i.e. channel assignment on MAC layer and power control on physical layer. We successfully transform the max–min objective to more solvable linear objective with additional constraints in compromise of optimality. In particular, we propose a heuristic approach BIPA to maximize the minimal capacity of all links by optimally assigning channels as well as setting transmit powers for each link. To measure the quality of solutions yielded by BIPA, we develop an upper bound to estimate the objective function subsequently. Simulation results show that solutions yielded by BIPA are very close to the upper bound, which suggests that they are near-optimal.

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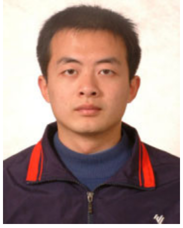

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