Math Modeling: The Epsilon School

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Problem Statement

To account for the considerable increase in school students for the upcoming school year, using the data from the school's current demographics, how should we properly distribute the seven new faculty hires so that class sizes remain fair?

We made the following assumptions:

1. Every student takes exactly 1 English class every year
2. Students are able to take multiple courses within each of the other subjects
3. Dropouts occur between years and are spread equally (2.5% of initial grade dropout between sophomore→junior, and 2.5% of initial grade dropout between junior→senior)
4. The proportion of sophomores, juniors, and seniors taking a certain class will remain across years
5. Existing foreign language teachers did not teach 2 languages (only 1 each)
6. A newly hired foreign language teacher can teach 2 languages
7. Current (2024) student to faculty ratios are ideal

Process

- Based off the assumption that “Current (2024) student to faculty ratios are ideal,” → determined that “fairness” means maintaining current student to faculty ratios as best as possible.
    - 2 options: raw increases in student to faculty ratios vs. percent increases
    - Decided on percent increases

Process:

- Determined the student to faculty ratio for each subject
- Constructed a new table considering the 2025 school population and course distributions
- Analyzed student to faculty ratio of the new population
- Compared the student to faculty ratio of this year to next year using percent increase
- Systematically added teachers to classes with highest percent increases in student to faculty ratio

Math 1

S_{2025} = # of sophomores taking a subject in 2025

S_{2024} = # of sophomores taking a subject in 2024

C_{S2024} = # of sophomores in 2024

C_{S2025} = # of sophomores in 2025

$S_{2025} = ((S_{2024}/C_{S2024})*C_{S2025})$

# of sophomores taking a subject in 2025 = ((# of sophomores taking the subject in 2024)/(# of sophomores in 2024))* (# sophomores in 2025)

Math 2

J_{2025} = # of juniors taking a subject in 2025

J_{2024} = # of juniors taking a subject in 2024

C_{J2024} = # of juniors in 2024

C_{J2025} = # of juniors in 2025

$J_{2025} = ((J_{2024}/C_{J2024})*C_{J2025})$

# of juniors taking a subject in 2025 = ((# of juniors taking the subject in 2024)/(# of juniors in 2024))* (# juniors in 2025)

Math 3

K_{2025} = # of seniors taking a subject in 2025

K_{2024} = # of seniors taking a subject in 2024

C_{K2024} = # of seniors in 2024

C_{K2025} = # of seniors in 2025

$K_{2025} = ((K_{2024}/C_{K2024})*C_{K2025})$

# of seniors taking a subject in 2025 = ((# of senior taking the subject in 2024)/(# of seniors in 2024))* (# seniors in 2025)

Calculation Example

S_{M2025} = # of sophomores taking math in 2025

S_{M2024} = 184 = # of sophomores taking math in 2024

C_{S2024} = 183 = # of sophomores in 2024

C_{S2025} = 292 = # of sophomores in 2025

$S_{M2025} = ((S_{M2024}/C_{S2024})*C_{S2025})$

$S_{M2025} = ((184/183)*292)$

S_{M2025} = 294 = # of sophomores taking math in 2025

Tables

Using the equations on slide 5, 6 and 7* we constructed class and subject population for the following year.

2024:

Subject	10th(183 ppl)	11th(155 ppl)	12th(152 ppl)	Total
Art	31	33	35	99
Biology	198	95	26	319
Chemistry	59	126	109	294
English	183	155	152	490
France	41	32	49	122
German	19	22	10	51
Spanish	51	26	33	110
Math	184	201	262	647
Music	50	56	49	155
physic	50	58	183	291
History	183	131	59	373

2025:

Subject	10th(292)	11th(183)	12th(155)	Total
Art	49	39	36	124
Biology	316	112	27	455
Chemistry	94	149	111	354
English	292	183	155	630
France	65	38	50	153
German	30	26	10	66
Spanish	81	31	34	146
Math	294	237	267	798
Music	80	66	50	196
physics	80	68	187	335
history	292	155	60	507

*# of <grade> taking a subject in 2025 = ((# of <grade> taking the subject in 2024) / (# of <grade> in 2024)) * (# of <grade> in 2025)

	10th(183 ppl)	11th(155 ppl)	12th(152 ppl)	Total
Art	31	33	35	99
Biology	198	95	26	319
Chemistry	59	126	109	294
English	183	155	152	490
French	41	32	49	122
German	19	22	10	51
Spanish	51	26	33	110
Math	184	201	262	647
Music	50	56	49	155
Physics	50	58	183	291
History	183	131	59	373

2024

	10th(292 ppl)	11th(183 ppl)	12th(155 ppl)	Total	Additional students from last year
Art	49	39	36	124	25
Biology	316	112	27	455	136
Chemistry	94	149	111	354	60
English	292	183	155	630	140
French	65	38	50	153	31
German	30	26	10	66	15
Spanish	81	31	34	146	36
Math	294	237	267	798	151
Music	80	66	50	196	41
Physics	80	68	187	335	44
History	292	155	60	507	134

2025

	10th(292)	11th(183*0.975)	12th(155*0.975)	Total	Additional students from last year
Art	49	38	35	122	23
Biology	316	109	26	451	132
Chemistry	94	145	108	348	54
English	292	178	151	622	132
French	65	37	49	151	29
German	30	25	10	66	15
Spanish	81	30	33	144	34
Math	294	231	260	785	138
Music	80	64	49	193	38
Physics	80	67	182	328	37
History	292	151	59	501	128

2025

With drop-out rates considered (along with subject and class population)

Our Solution - Calculating Student/Faculty Ratios

R_{202X} = overall student to faculty ratio for a subject (in 202X)

T_{202X} = total # of students taking a subject (in 202X)

F_{202X} = total # of faculty teaching a subject (in 202X)

$R_{202X} = T_{202X} / F_{202X}$

Example: 2025, Math

T_{2025} = 785 = total # of students taking math (in 2025)

F_{2025} = 6 = total # of faculty teaching math (in 2025)

R_{2025} = 785 / 6 = ~130.9107234 = overall student to faculty ratio for math (in 2025)

Student to Faculty Ratio Tables

Constructed using process from previous slide applied to each course in each year

2024:

Course	Total Students	Faculty	Student/Faculty Ratio
Art	99	1	99
Biology	319	4	79.75
Chemistry	294	3	98
English	490	5	98
French	122	1	122
German	51	1	51
Spanish	110	1	110
Math	647	6	107.8333333
Music	155	1	155
Physics	291	3	97
History	373	5	74.6

2025:

Course	Total Students	Faculty	Student/Faculty Ratio
Art	122	1	122.2502587
Biology	451	4	112.7855033
Chemistry	348	3	115.8522892
English	622	5	124.31
French	151	1	150.9748217
German	66	1	65.58421281
Spanish	144	1	144.1164369
Math	785	6	130.9107234
Music	193	1	192.9625742
Physics	328	3	109.4978169
History	501	5	100.291653

Our Solution - Calculating % Increase For Student/Faculty Ratios

$R_{\%}$ = % increase in student to faculty ratio in a subject between 2024-2025

$R_{\%} = (R_{2025} - R_{2024}) / R_{2024} * 100\%$ = change in student to faculty ratio over initial student faculty ratio for a subject

Example: Math

R_{2025} = 130.9107234 = student to faculty ratio for math (in 2025)

R_{2024} = 107.8333333 = student to faculty ratio for math (in 2024)

$R_{\%}$ = (130.9107234 - 107.8333333) / 107.8333333 * 100%

= ~21.400980006% = % increase in student to faculty ratio in math between 2024-2025

Student to Faculty Ratio % Increase Table

Constructed using process from previous slide applied to each course

Course	% Increase in Student/Faculty Ratio
Art	23.49%
Biology	41.42%
Chemistry	18.22%
English	26.85%
French	23.75%
German	28.60%
Spanish	31.01%
Math	21.40%
Music	24.49%
Physics	12.88%
History	34.44%

Our Solution - Deciding Faculty Allocation

1. Identify subject with highest % increase in student/faculty ratio
2. Allocate an additional faculty to the subject
3. Recalculate % increase in student/faculty ratio for the subject

4. Repeat steps 1-3 six more times to allocate all 7 new faculty

Our Solution - Deciding Faculty Allocation cont.

Before Adding Faculty:

Recalculated % Increase After:

+ 1 Biology Faculty
(Total Added: 1)

Course	Faculty	Student/Faculty Ratio	% Increase in Student/Faculty Ratio
Art	1	122.2502587	23.49%
Biology	4	112.7855033	41.42%
Chemistry	3	115.8522892	18.22%
English	5	124.31	26.85%
French	1	150.9748217	23.75%
German	1	65.58421281	28.60%
Spanish	1	144.1164369	31.01%
Math	6	130.9107234	21.40%
Music	1	192.9625742	24.49%
Physics	3	109.4978169	12.88%
History	5	100.291653	34.44%

Course	Faculty	Student/Faculty Ratio	% Increase in Student/Faculty Ratio
Art	1	122.2502587	23.49%
Biology	5	90.22840265	13.14%
Chemistry	3	115.8522892	18.22%
English	5	124.31	26.85%
French	1	150.9748217	23.75%
German	1	65.58421281	28.60%
Spanish	1	144.1164369	31.01%
Math	6	130.9107234	21.40%
Music	1	192.9625742	24.49%
Physics	3	109.4978169	12.88%
History	5	100.291653	34.44%

+ 1 History Faculty
(Total Added: 2)

Course	Faculty	Student/Faculty Ratio	% Increase in Student/Faculty Ratio
Art	1	122.2502587	23.49%
Biology	5	90.22840265	13.14%
Chemistry	3	115.8522892	18.22%
English	5	124.31	26.85%
French	1	150.9748217	23.75%
German	1	65.58421281	28.60%
Spanish	1	144.1164369	31.01%
Math	6	130.9107234	21.40%
Music	1	192.9625742	24.49%
Physics	3	109.4978169	12.88%
History	5	100.291653	34.44%

Course	Faculty	Student/Faculty Ratio	% Increase in Student/Faculty Ratio
Art	1	122.2502587	23.49%
Biology	5	90.22840265	13.14%
Chemistry	3	115.8522892	18.22%
English	5	124.31	26.85%
French	1	150.9748217	23.75%
German	1	65.58421281	28.60%
Spanish	1	144.1164369	31.01%
Math	6	130.9107234	21.40%
Music	1	192.9625742	24.49%
Physics	3	109.4978169	12.88%
History	6	83.57637751	12.03%

Our Solution - Deciding Faculty Allocation cont.

+ 1 Language Faculty that can teach German & Spanish
(Total Added: 3)

+ 1 English Faculty
(Total Added: 4)

Before Adding Faculty:

Course	Faculty	Student/Faculty Ratio	% Increase in Student/Faculty Ratio
Art	1	122.2502587	23.49%
Biology	5	90.22840265	13.14%
Chemistry	3	115.8522892	18.22%
English	5	124.31	26.85%
French	1	150.9748217	23.75%
German	1	65.58421281	28.60%
Spanish	1	144.1164369	31.01%
Math	6	130.9107234	21.40%
Music	1	192.9625742	24.49%
Physics	3	109.4978169	12.88%
History	6	83.57637751	12.03%

Course	Faculty	Student/Faculty Ratio	% Increase in Student/Faculty Ratio
Art	1	122.2502587	23.49%
Biology	5	90.22840265	13.14%
Chemistry	3	115.8522892	18.22%
English	5	124.31	26.85%
French	1	150.9748217	23.75%
German	2	32.79210641	-35.70%
Spanish	2	72.05821846	-34.49%
Math	6	130.9107234	21.40%
Music	1	192.9625742	24.49%
Physics	3	109.4978169	12.88%
History	6	83.57637751	12.03%

Recalculated % Increase After:

Course	Faculty	Student/Faculty Ratio	% Increase in Student/Faculty Ratio
Art	1	122.2502587	23.49%
Biology	5	90.22840265	13.14%
Chemistry	3	115.8522892	18.22%
English	5	124.31	26.85%
French	1	150.9748217	23.75%
German	2	32.79210641	-35.70%
Spanish	2	72.05821846	-34.49%
Math	6	130.9107234	21.40%
Music	1	192.9625742	24.49%
Physics	3	109.4978169	12.88%
History	6	83.57637751	12.03%

Course	Faculty	Student/Faculty Ratio	% Increase in Student/Faculty Ratio
Art	1	122.2502587	23.49%
Biology	5	90.22840265	13.14%
Chemistry	3	115.8522892	18.22%
English	6	103.5916667	5.71%
French	1	150.9748217	23.75%
German	2	32.79210641	-35.70%
Spanish	2	72.05821846	-34.49%
Math	6	130.9107234	21.40%
Music	1	192.9625742	24.49%
Physics	3	109.4978169	12.88%
History	6	83.57637751	12.03%

Our Solution - Deciding Faculty Allocation cont.

Before Adding Faculty:

Course	Faculty	Student/Faculty Ratio	% Increase in Student/Faculty Ratio
Art	1	122.2502587	23.49%
Biology	5	90.22840265	13.14%
Chemistry	3	115.8522892	18.22%
English	6	103.5916667	5.71%
French	1	150.9748217	23.75%
German	2	32.79210641	-35.70%
Spanish	2	72.05821846	-34.49%
Math	6	130.9107234	21.40%
Music	1	192.9625742	24.49%
Physics	3	109.4978169	12.88%
History	6	83.57637751	12.03%

Recalculated % Increase After:

Course	Faculty	Student/Faculty Ratio	% Increase in Student/Faculty Ratio
Art	1	122.2502587	23.49%
Biology	5	90.22840265	13.14%
Chemistry	3	115.8522892	18.22%
English	6	103.5916667	5.71%
French	1	150.9748217	23.75%
German	2	32.79210641	-35.70%
Spanish	2	72.05821846	-34.49%
Math	6	130.9107234	21.40%
Music	2	96.4812871	-37.75%
Physics	3	109.4978169	12.88%
History	6	83.57637751	12.03%

+ 1 Music Faculty
(Total Added: 5)

Course	Faculty	Student/Faculty Ratio	% Increase in Student/Faculty Ratio
Art	1	122.2502587	23.49%
Biology	5	90.22840265	13.14%
Chemistry	3	115.8522892	18.22%
English	6	103.5916667	5.71%
French	1	150.9748217	23.75%
German	2	32.79210641	-35.70%
Spanish	2	72.05821846	-34.49%
Math	6	130.9107234	21.40%
Music	2	96.4812871	-37.75%
Physics	3	109.4978169	12.88%
History	6	83.57637751	12.03%

+ 1 Language
(French) Faculty
(Total Added: 6)

Course	Faculty	Student/Faculty Ratio	% Increase in Student/Faculty Ratio
Art	1	122.2502587	23.49%
Biology	5	90.22840265	13.14%
Chemistry	3	115.8522892	18.22%
English	6	103.5916667	5.71%
French	2	75.48741085	-38.13%
German	2	32.79210641	-35.70%
Spanish	2	72.05821846	-34.49%
Math	6	130.9107234	21.40%
Music	2	96.4812871	-37.75%
Physics	3	109.4978169	12.88%
History	6	83.57637751	12.03%

Our Solution - Deciding Faculty Allocation cont.

+ 1 Art Faculty
(Total Added: 7)

Before Adding Faculty:

Course	Faculty	Student/Faculty Ratio	% Increase in Student/Faculty Ratio
Art	1	122.2502587	23.49%
Biology	5	90.22840265	13.14%
Chemistry	3	115.8522892	18.22%
English	6	103.5916667	5.71%
French	2	75.48741085	-38.13%
German	2	32.79210641	-35.70%
Spanish	2	72.05821846	-34.49%
Math	6	130.9107234	21.40%
Music	2	96.4812871	-37.75%
Physics	3	109.4978169	12.88%
History	6	83.57637751	12.03%

Recalculated % Increase After:

Course	Faculty	Student/Faculty Ratio	% Increase in Student/Faculty Ratio
Art	2	61.12512934	-38.26%
Biology	5	90.22840265	13.14%
Chemistry	3	115.8522892	18.22%
English	6	103.5916667	5.71%
French	2	75.48741085	-38.13%
German	2	32.79210641	-35.70%
Spanish	2	72.05821846	-34.49%
Math	6	130.9107234	21.40%
Music	2	96.4812871	-37.75%
Physics	3	109.4978169	12.88%
History	6	83.57637751	12.03%

Final Allocation Results:

1 Biology, 1 History, 1 German+Spanish, 1 English, 1 Music, 1 French, 1 Art

Strengths

- Method for factoring in drop-out rates
  - More common that students will leave between grades
- Student-to-faculty ratios are an accommodating indicator of whether or not a department needs another teacher

Weaknesses

- We rounded the calculated number of students per class, data may have been slightly affected
- The difference in previous and current student-to-faculty ratio varied by ~55% among some classes, which is not ideal

Future Work

Although we aimed to find a solution that would maximize fairness and precision, there are some aspects of the model that could be further developed in the future.

- Consider teachers' student capacity levels - is there a limit to how many students they will teach per class that we should respect?
- Consider class intensity - would harder vs. easier classes of the same subject affect the number of students enrolled?
- Create a mathematical model that can continue our work for an expected increase in incoming students every year

Acknowledgements/Questions