Modeling-Based Technique for 3D Permittivity Profile Reconstruction

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The paper presents a microwave imaging technique for determination of arbitrary 3D profiles of complex permittivity in dielectric samples in closed multi-port systems. The technique relies on preliminary modeling of the system and measurement of its S-parameters. A numerical inversion is performed with the use of an artificial neural network procedure. To improve the accuracy, a central composite orthogonal design is applied to characterize the permittivity profile. Illustrative results of reconstruction of two test distributions are shown for a rectangular block in a 6-port waveguide structure.

Keywords: Artificial neural network, electromagnetic modeling, microwave imaging, permittivity profile.

INTRODUCTION

Determination of permittivity profiles of dielectric samples is a microwave (MW) imaging task essential in the area of non-destructive evaluation and testing (NDE/NDT), and has many important applications [1]. Our interest in this area is conditioned by demand from the technologies associated with high temperature MW processing and sintering. Two key applications of MW imaging in this field may be in testing materials before and after processing. At a more advanced stage of development, MW imaging may be thought of as a technique of monitoring the internal structure of the samples in the course of heating.

While most techniques of MW imaging are developed for open scenarios, in [2] we proposed an approach relevant to MW power engineering and applicable to closed systems (such as waveguides or resonators). The technique is backed by data on the system obtained from modeling (i.e., 3D conformal finite-difference time-domain (FDTD) analysis) and measurements of *S*-parameters, and is based on an inversion procedure driven by artificial neural networks (ANN). The technique [2], however, is limited to reconstructing 2D profiles described by pre-defined functions and works only when the samples are rotated.

In this paper, we present an upgrade of the technique in [2] that overcomes these limitations. This FDTD-backed, ANN-based technique using elementary measurements of *S*-parameters is capable of reconstructing arbitrary 3D permittivity profiles. Instead of a waveguide (2-port) structure [2], this technique employs a multi-port closed system, which allows for getting more input information about the scenario without requiring the

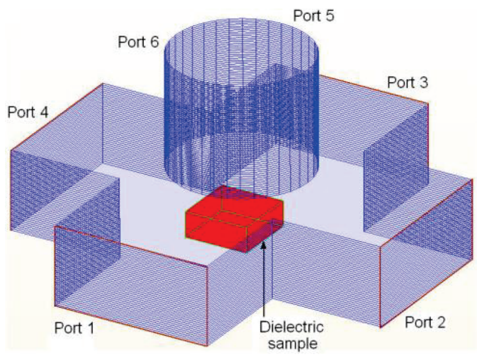


Figure 1. A multiport waveguide system with material for testing.

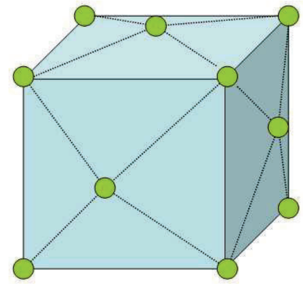


Figure 2. Base points around a block of the sample.

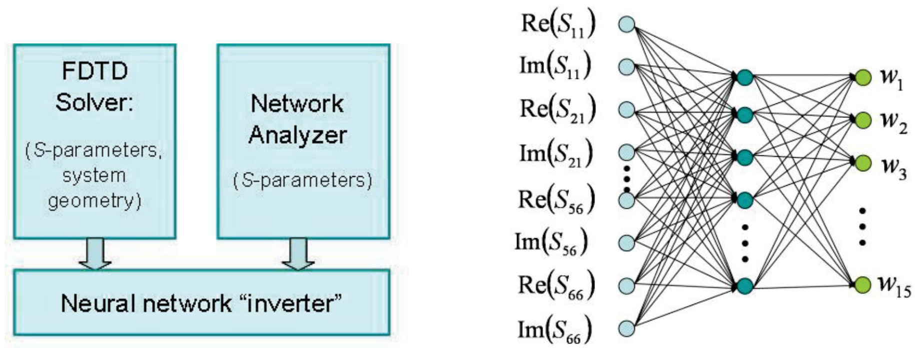
sample to be rotated. The inversion technique driven by an optimal experimental design methodology [3] is used to account for variations of complex permittivity in all three dimensions with acceptable accuracy.

TECHNIQUES AND MATERIALS

Our approach to determining 3D complex permittivity profiles is based on measurements of *S*-parameters and an ANN inversion procedure supported by massive FDTD-generated data on a multi-port closed system. In the present implementation of the technique, this system is a 6-port turnstile junction (Fig. 1) with four ports in the rectangular waveguides and two ports corresponding to two orthogonal modes in the circular waveguide. The dielectric sample is located on the bottom wall of the structure. The measured parameters are the complex reflection and transmission coefficients (i.e., the full *S*-matrix of the junction). The structure in Fig. 1 is chosen in order to provide information sufficient for reconstructing 3D permittivity profiles of the tested sample; in general, there may be other multiport systems whose *S*-parameters are sensitive enough to 3D variations of permittivity of the sample to provide accurate reconstructions.

The ANN procedure of numerical inversion employs the idea of combining an ANN with a regression model. In the first step, the 3D permittivity profile in the sample is described with some mechanism providing the mapping between the spatial coordinates and the values of complex permittivity at the specified points. In the mapping mechanism, we apply a quadratic polynomial function. The next step is application of an ANN with global cubic radial basis functions as a numerical inverter that sets the mapping between the measurable values (i.e., *S*-parameters) and the coefficients of the function chosen in the first step. This ANN, after appropriate training, is used for reconstruction of the permittivity profiles corresponding to the measured *S*-matrix of the system.

Our technique aims to reconstruct permittivity profiles of practical materials that may have an arbitrary distribution of complex permittivity inside the sample. At the same time, we apply a model function, namely, a quadratic polynomial, to describe these profiles as precisely as possible. In other words, we apply a regression technique with three independent variables (spatial coordinates x, y, z), to match the experimental profile



(a) (b)

Fig. 3. Functional scheme (a) and an ANN architecture (b) of the inversion procedure.

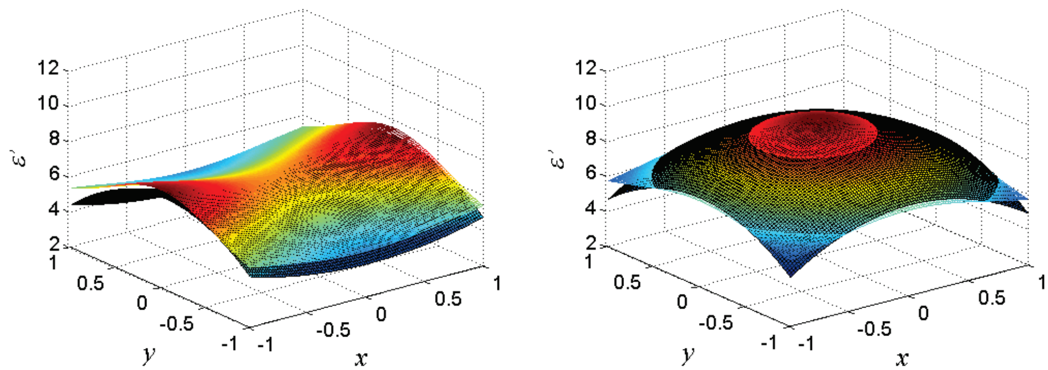
with a quadratic polynomial. Following the optimal experimental design theory, to build the regression we apply central composite orthogonal design [3] involving function values taken at 15 points inside and outside the sample. Assuming that the sample has the form of a rectangular parallelepiped, we use 8 points at the corners, 6 points outside the sample (opposite to the centers of the sides), and 1 point at the center (Fig. 2). Permittivity values at the base points are used for reconstruction of a quadratic surface. The reconstruction is performed by an ANN with global cubic radial basis functions.

Furthermore, our technique is based on application of an ANN as a numerical inverter (Fig. 3a). Here, we combine the assumption of a certain sophisticated function governing permittivity profiles with a numerical model representing the experimental system. In our procedure, *S*-parameters come to the inverter from both an FDTD solver (used in a training stage) and a vector network analyzer.

An ANN used for reconstruction of permittivity profiles and shown in Fig. 3b is a radial basis function (RBF) network with a cubic basis function. The inputs of the ANN are complex *S*-parameters of the electromagnetic system (the real and imaginary parts of 36 complex coefficients of the *S*-matrix) whereas the outputs are the parameters governing the permittivity profile inside the sample (i.e., 15 coefficients of the regression quadratic equation). The network is trained by numerical data obtained with the use of the 3D conformal FDTD simulator *QuickWave-3D* [4]. The training points are generated by the multiple runs of the model as random combinations of the function coefficients w_1, …, w_{15} uniformly distributed in the specified intervals. Mathematical treatment of the ANN training is given in detail in [2].

RESULTS

Here we illustrate the performance of the technique by two examples using a sample with dimensions 100 × 100 × 80 mm. Reconstructions are performed in the turnstile junction that has rectangular waveguides of cross-section 248 x 124 mm and a circular waveguide with radius 107.6 mm. The operating frequency is 915 MHz. The sample is centered on the bottom wall. The medium of the sample is assumed lossless, and the dielectric constant ε' is assumed to vary smoothly in all three dimensions in the interval from 2 to 12. Two actual profiles of ε' in the sample are given as functions of all three coordinates:



(a) (b)

Fig. 4. Actual (transparent) and reconstructed (solid) profiles for functions (1) (a) and (2) (b).

$$\varepsilon' = (1/2 - x^2 + y^2)\exp(1 - x^2 - y^2/2 - z) + 5 \tag{1}$$

$$\varepsilon' = a(1 + b\exp(-((x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2)/r_0^2)) \tag{2}$$

with the parameters a, = 5, b, = 1, $r_0 = 1$, and $x_0 = y_0 = z_0 = 0$. The reconstructed surfaces are shown in Fig. 4 along with functions (1) and (2). In both cases, the results are shown for the horizontal plane through the center of the sample. The errors of reconstruction are 9.2% and 5.1%, respectively. More details on the technique and its performance can be found in [5].

CONCLUSION

A new modelling-based technique of 3D MW imaging in closed systems has been outlined. The tested sample with an arbitrary 3D profile of complex permittivity should be placed in a multi-port structure. Accuracy of reconstruction depends on the system and adequacy of the model, but only elementary measurements of *S*-parameters are required.

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