

**MA 1024:**  
***Calculus IV***  
***(Multivariable Calculus)***

**Essential Class Notes & Graphics**

**D'17 \* Sections D03 & D04**

**2016-2017**

# GRAPHS OF FUNCTIONS OF TWO VARIABLES

## Two Main Types of Graphs:

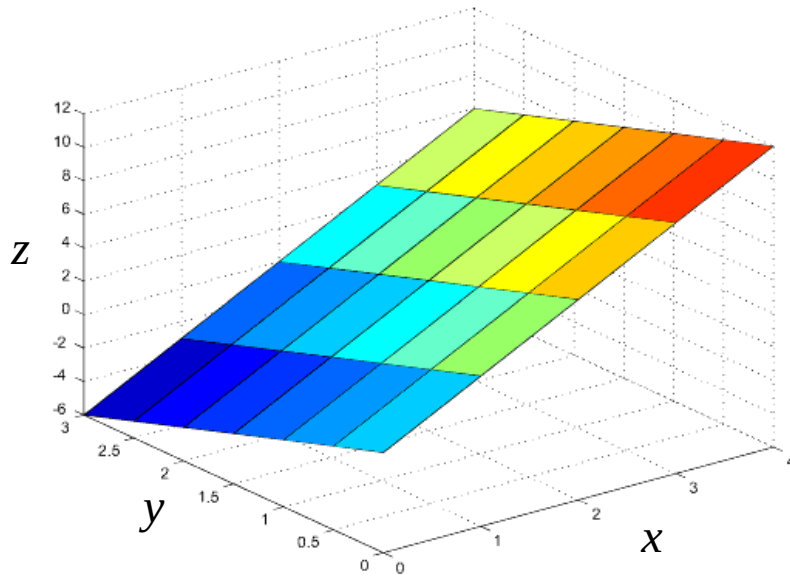
- Contour lines, or level curves
- Surfaces

## Technical Means of Generation:

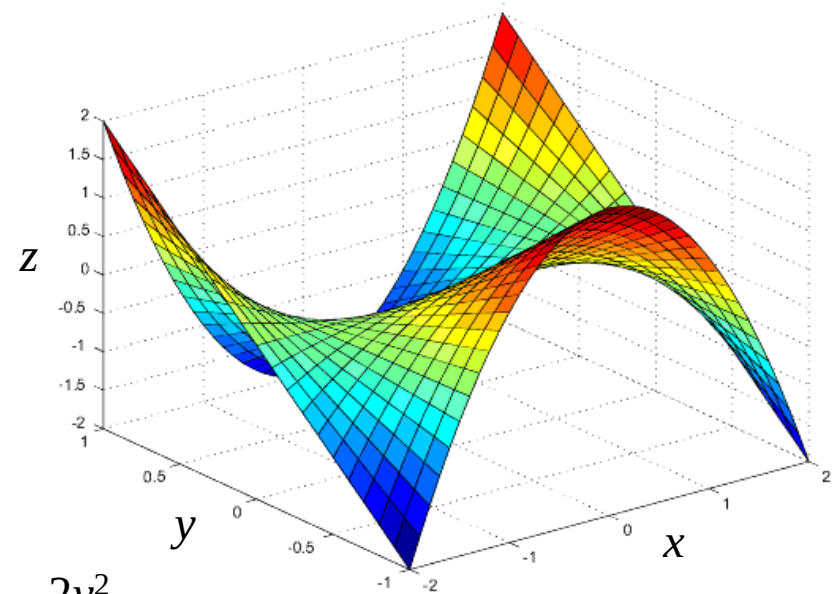
- Manual sketches
- Precise images by graphical calculators & Computer Algebra Systems (Maple, MATLAB, Mathematica, etc.)

# Examples of Surfaces (1)

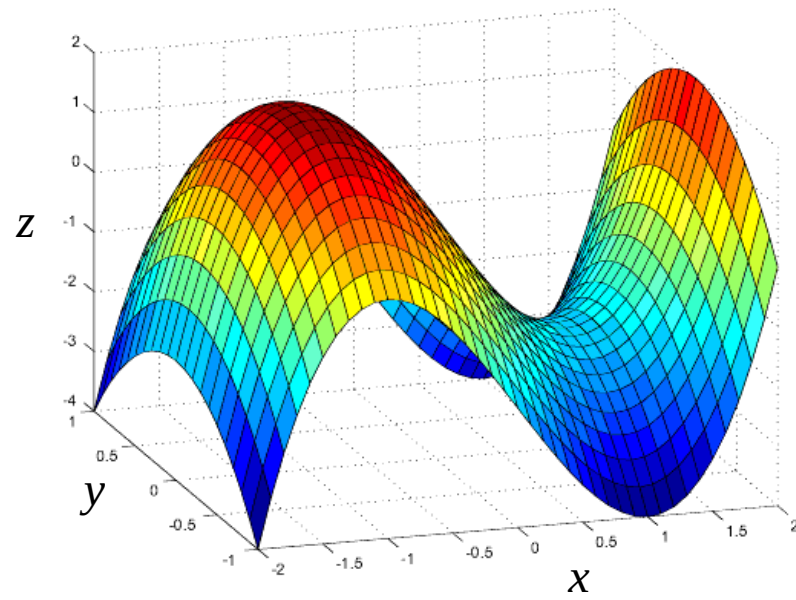
$$z = 3x - 2y$$



$$z = x^2y - 2y$$

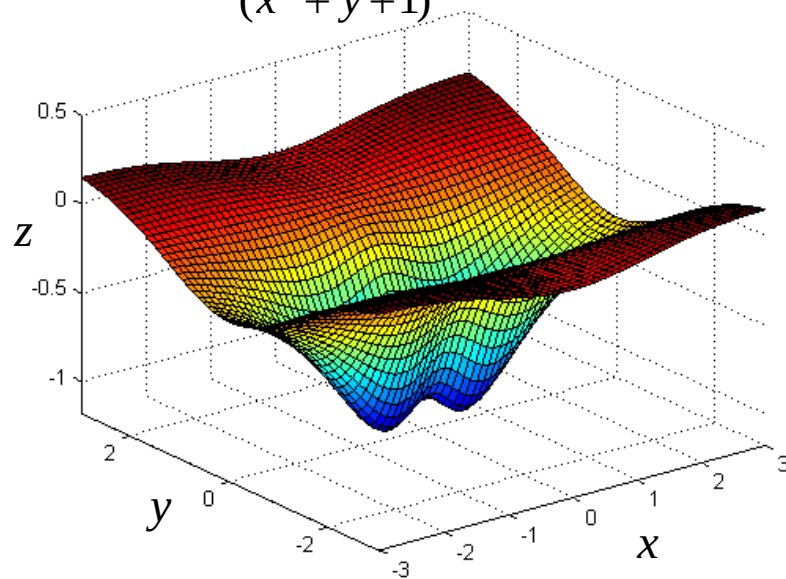


$$z = x^3 - 3y - 2y^2$$

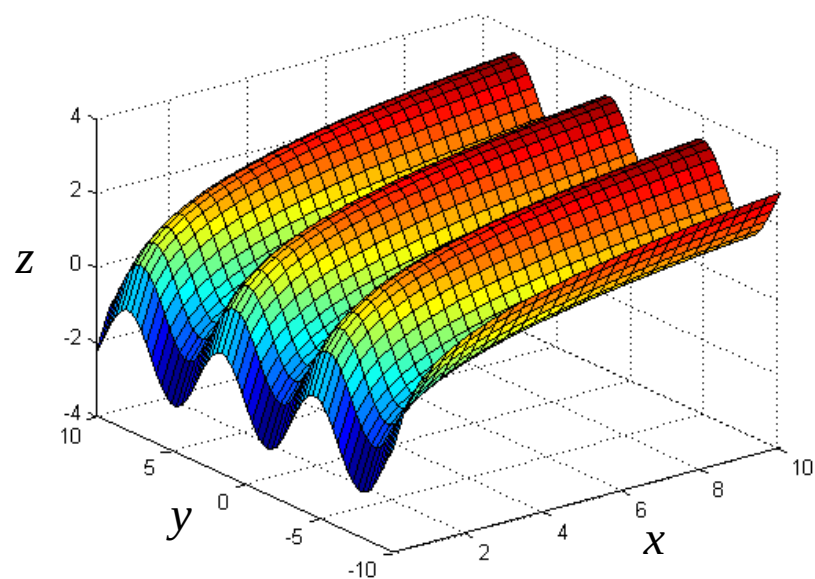


# Examples of Surfaces (2)

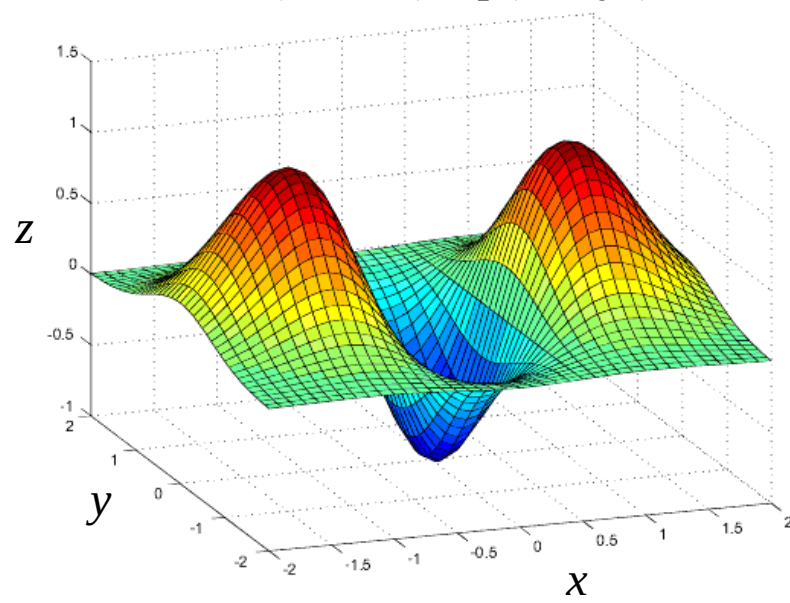
$$z = \frac{(x^2 - 1)(y^2 - 4) + x^2 + y^2 - 5}{(x^2 + y + 1)^2}$$



$$z = \ln x + \sin y$$

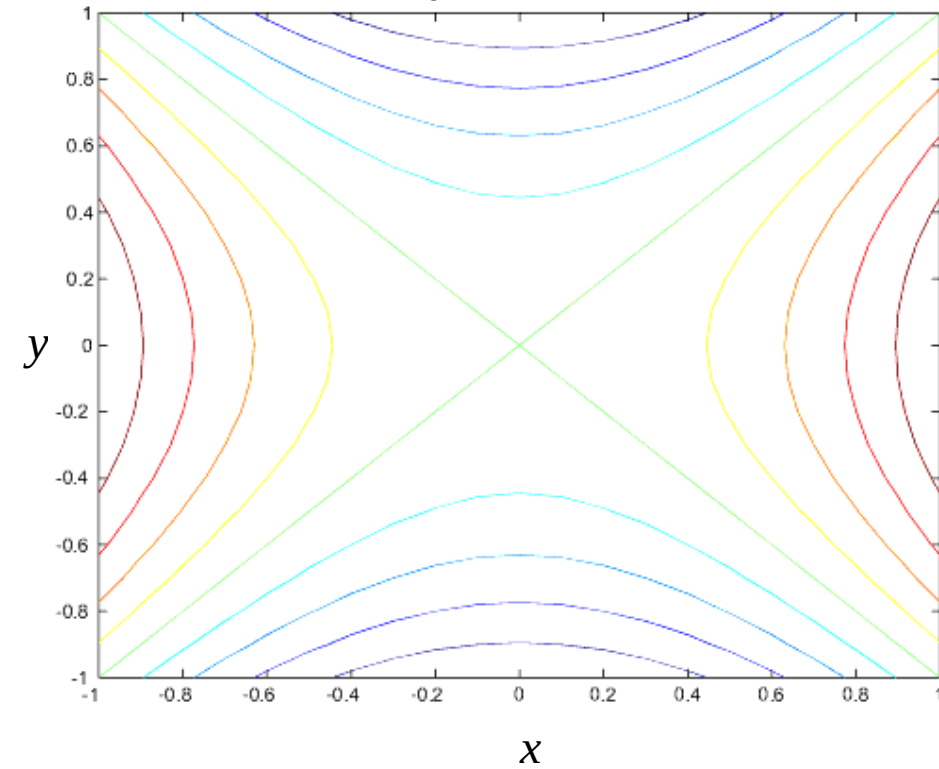


$$z = (4x^2 - 1)\exp(-x^2 - y^2)$$

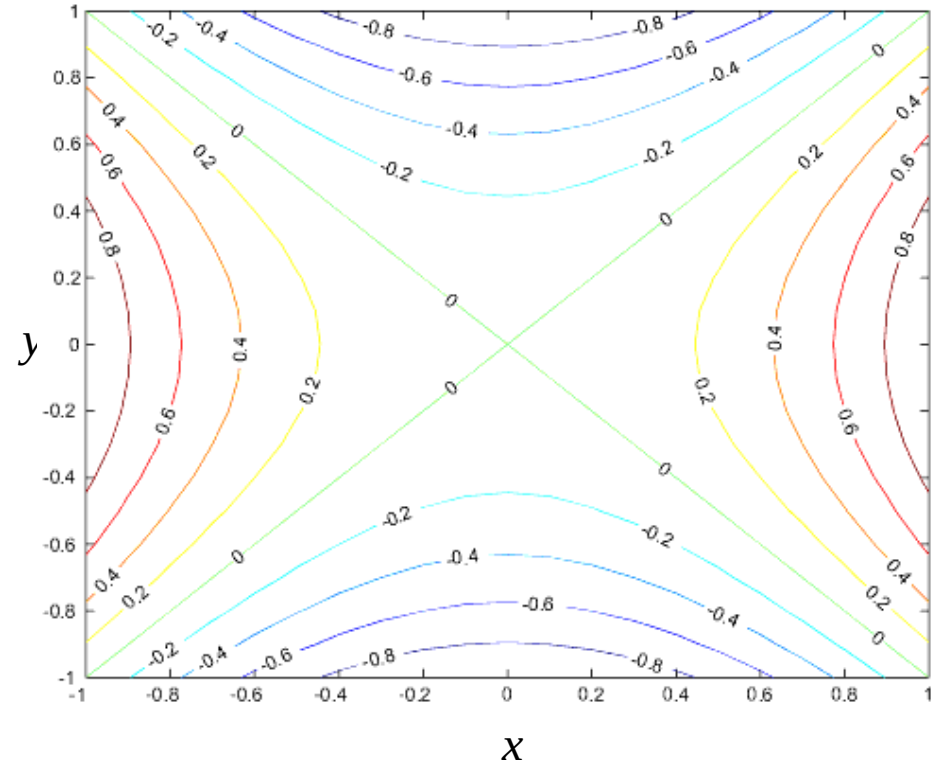


# Examples of Level Curves

$$z = x^2 - y^2 \quad (\text{no labels})$$

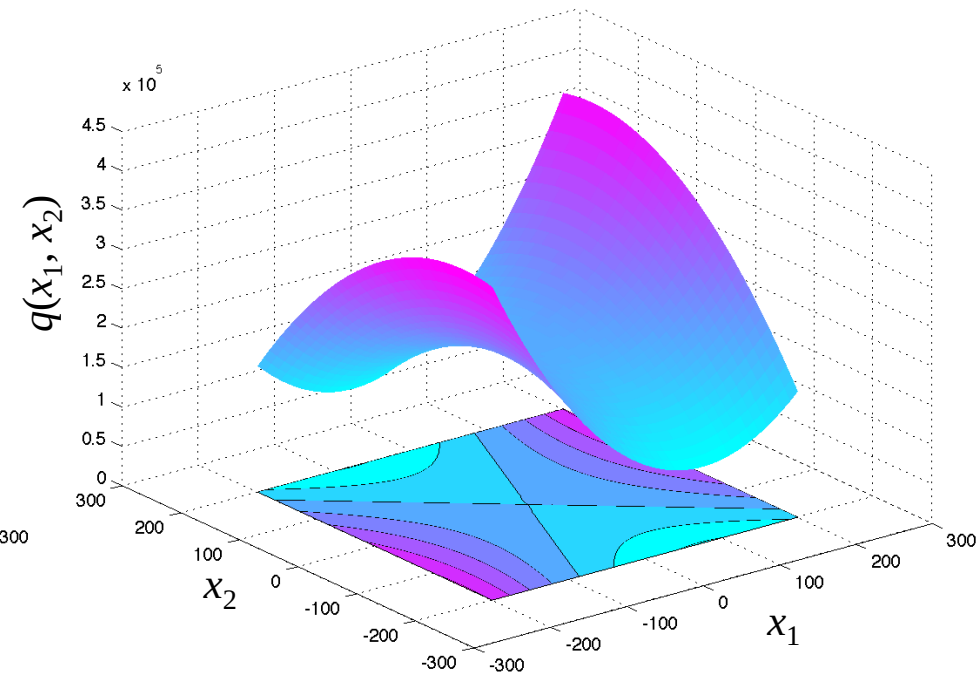
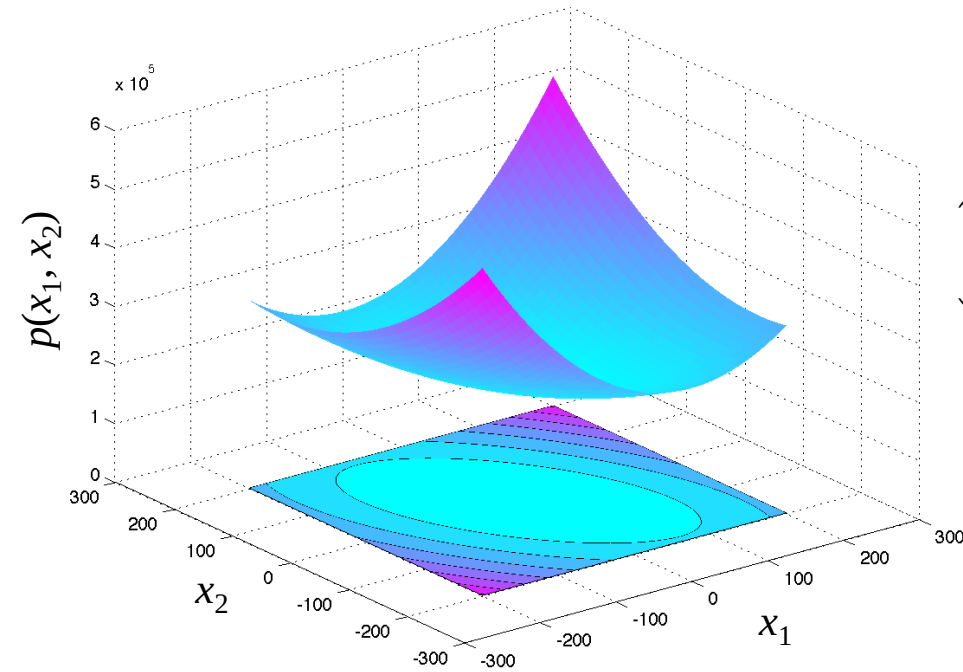


$$z = x^2 - y^2 \quad (\text{with labels})$$



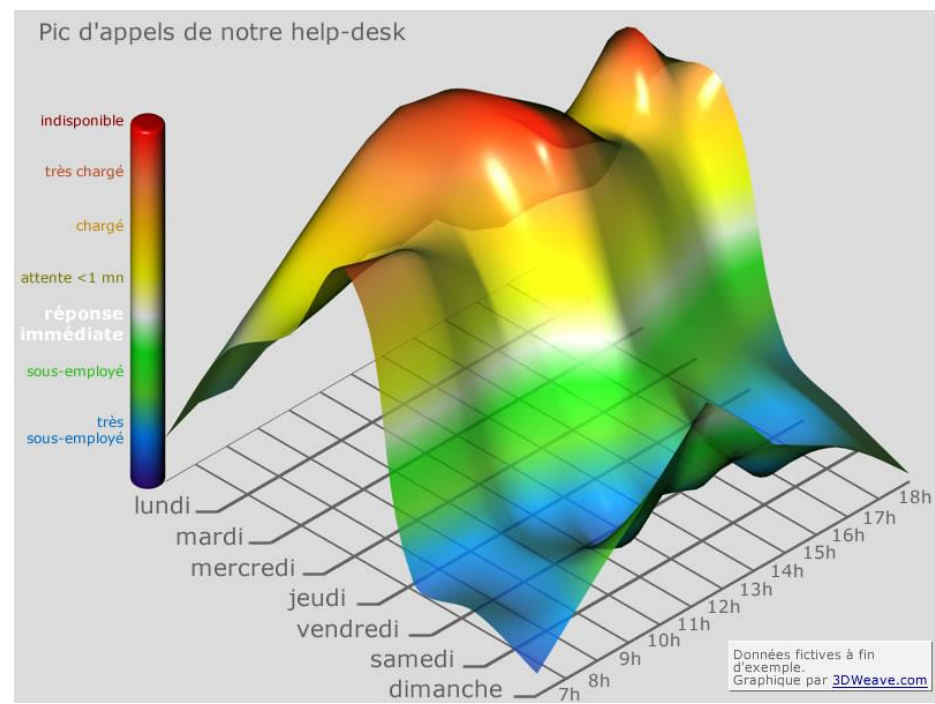
# Surfaces & Level Curves

$$\begin{aligned} p(x) &= 4x_1^2 + 2x_2^2 + 3x_1x_2 + 4x_1 + 5x_2 + 2 \times 10^5 \\ q(x) &= 4x_1^2 - 2x_2^2 + 3x_1x_2 + 4x_1 + 5x_2 + 2 \times 10^5 \end{aligned}$$

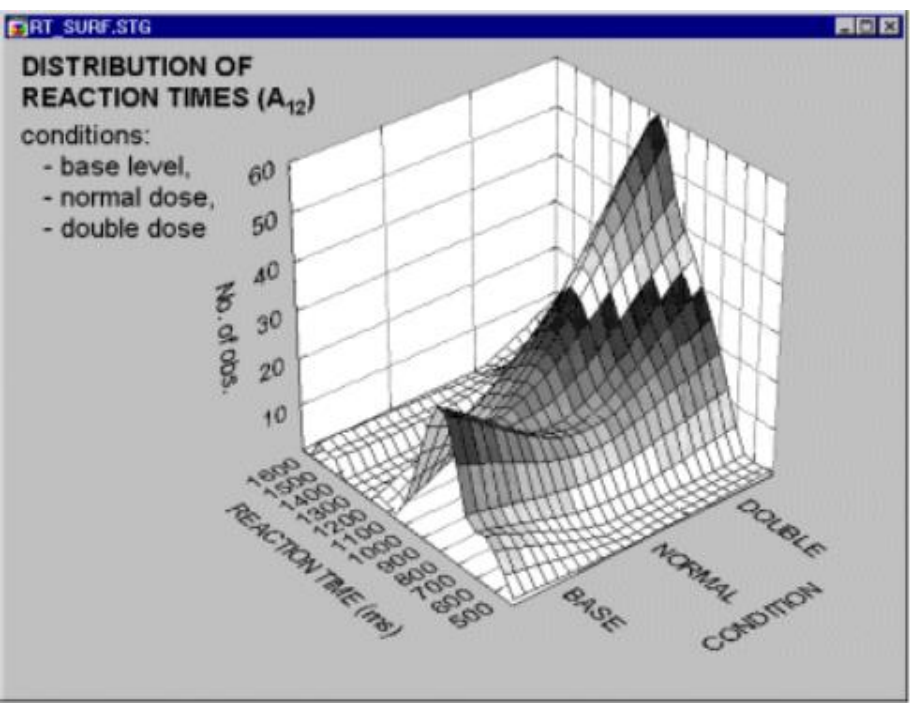


# Surfaces in Applications

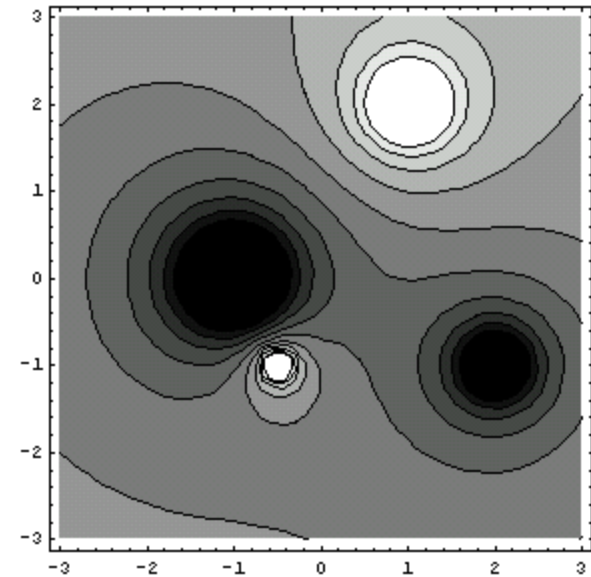
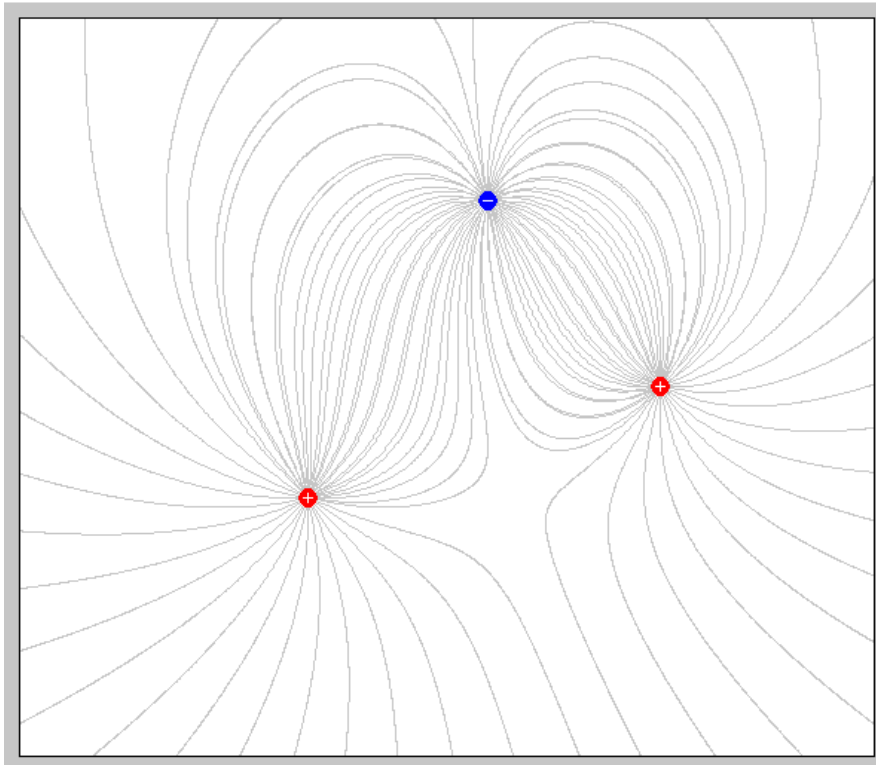
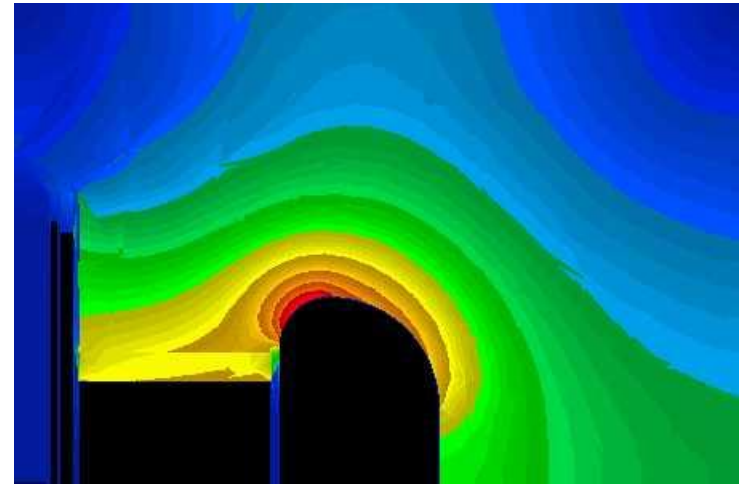
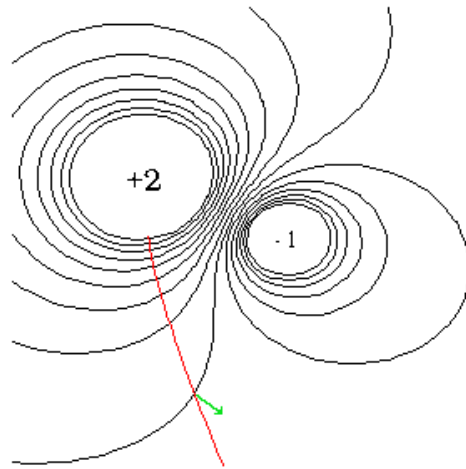
## Operation of a Company's *Help Desk*



## Characterization of a Chemical Reaction

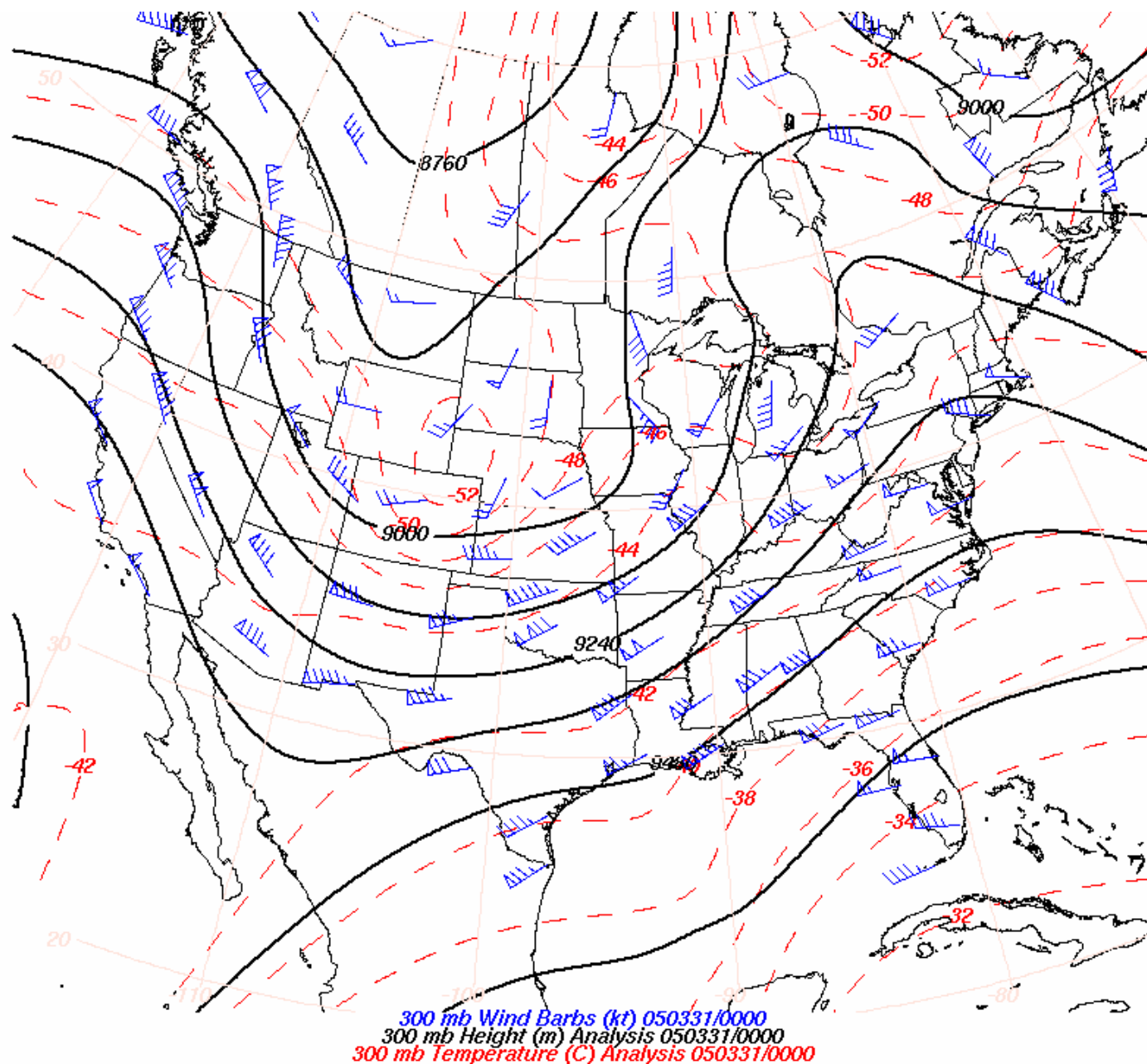


# Level Curves: Electromagnetic Field

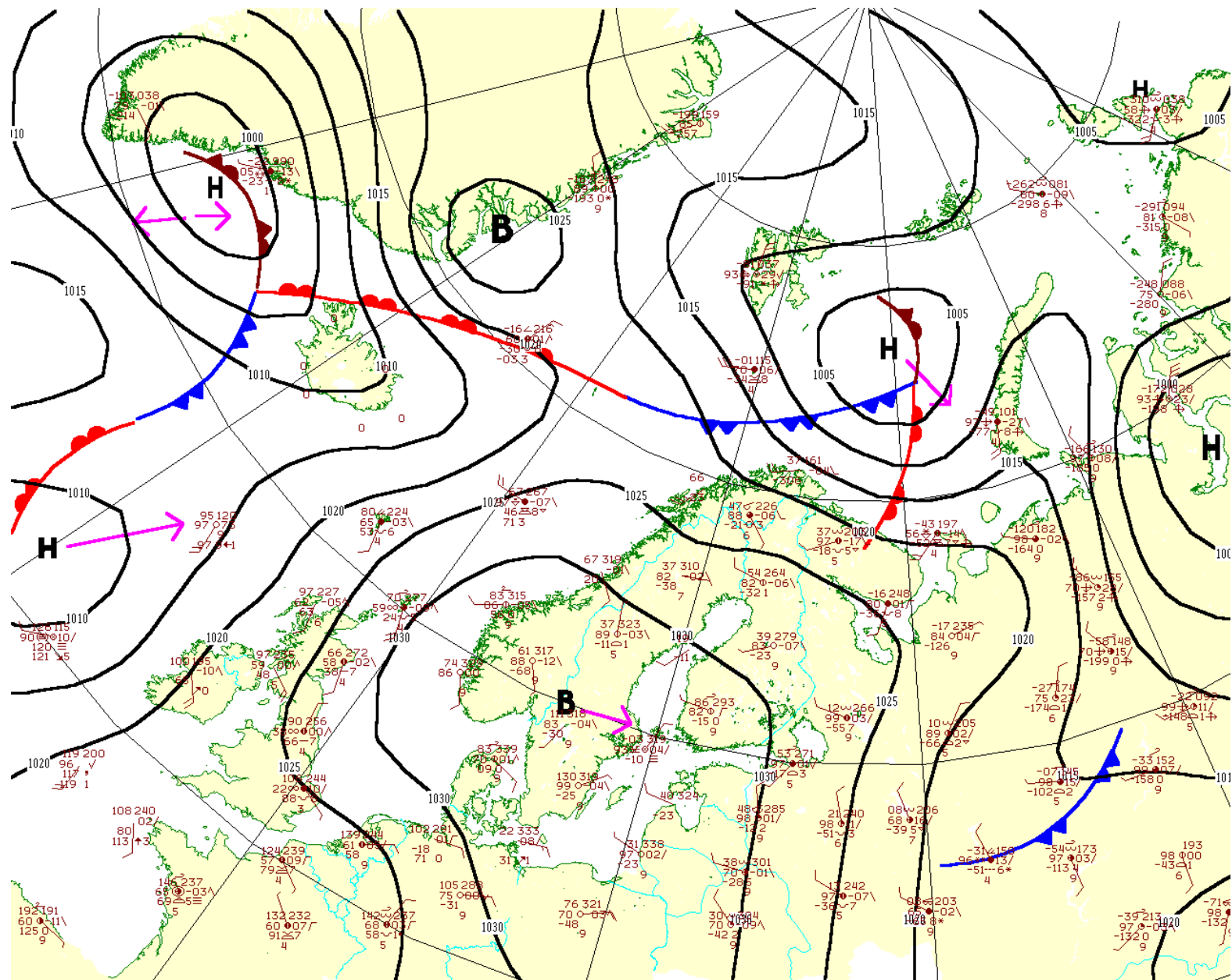




# Level Curves: Weather Maps (1)

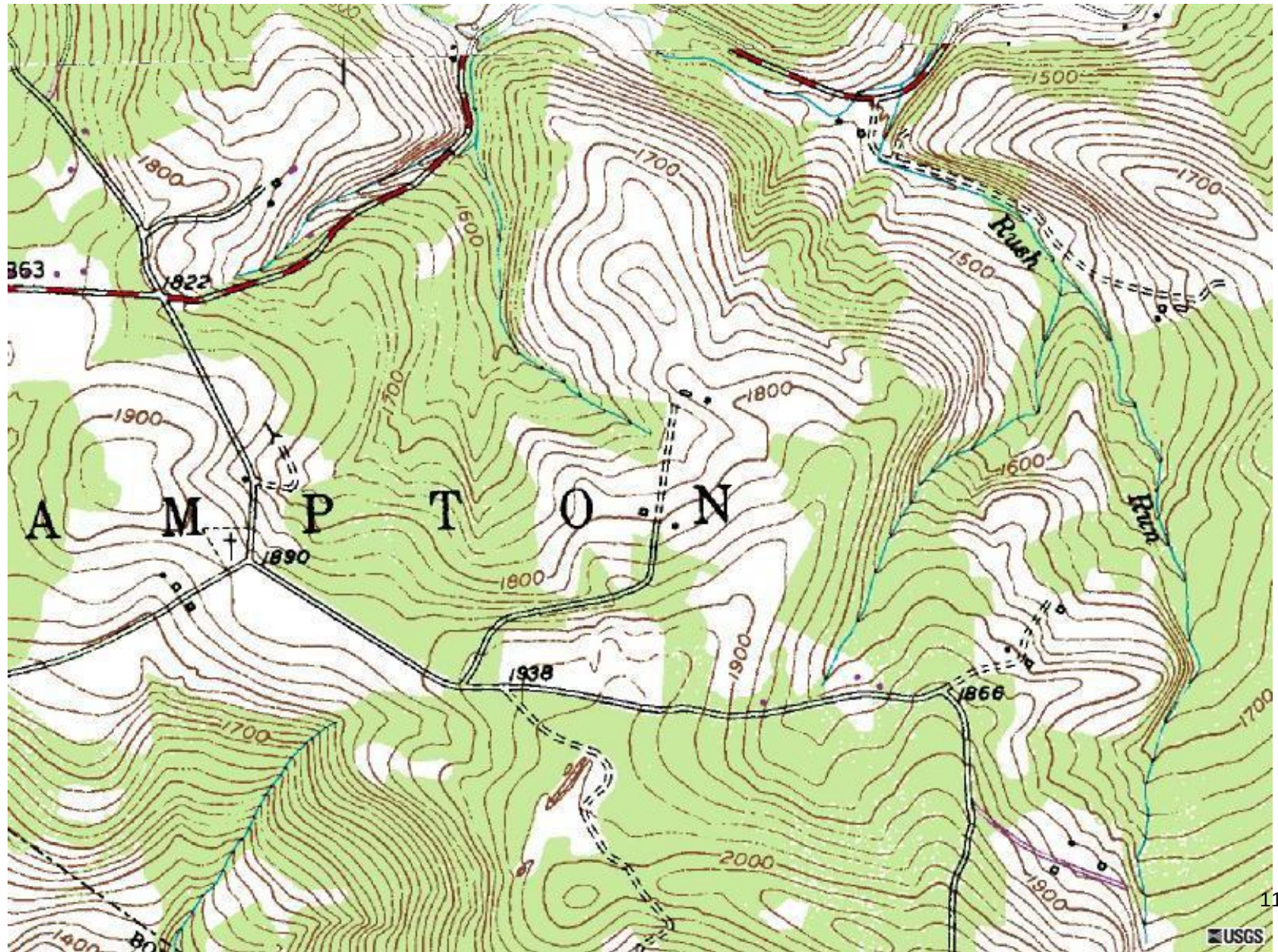


# Level Curves: Weather Maps (2)



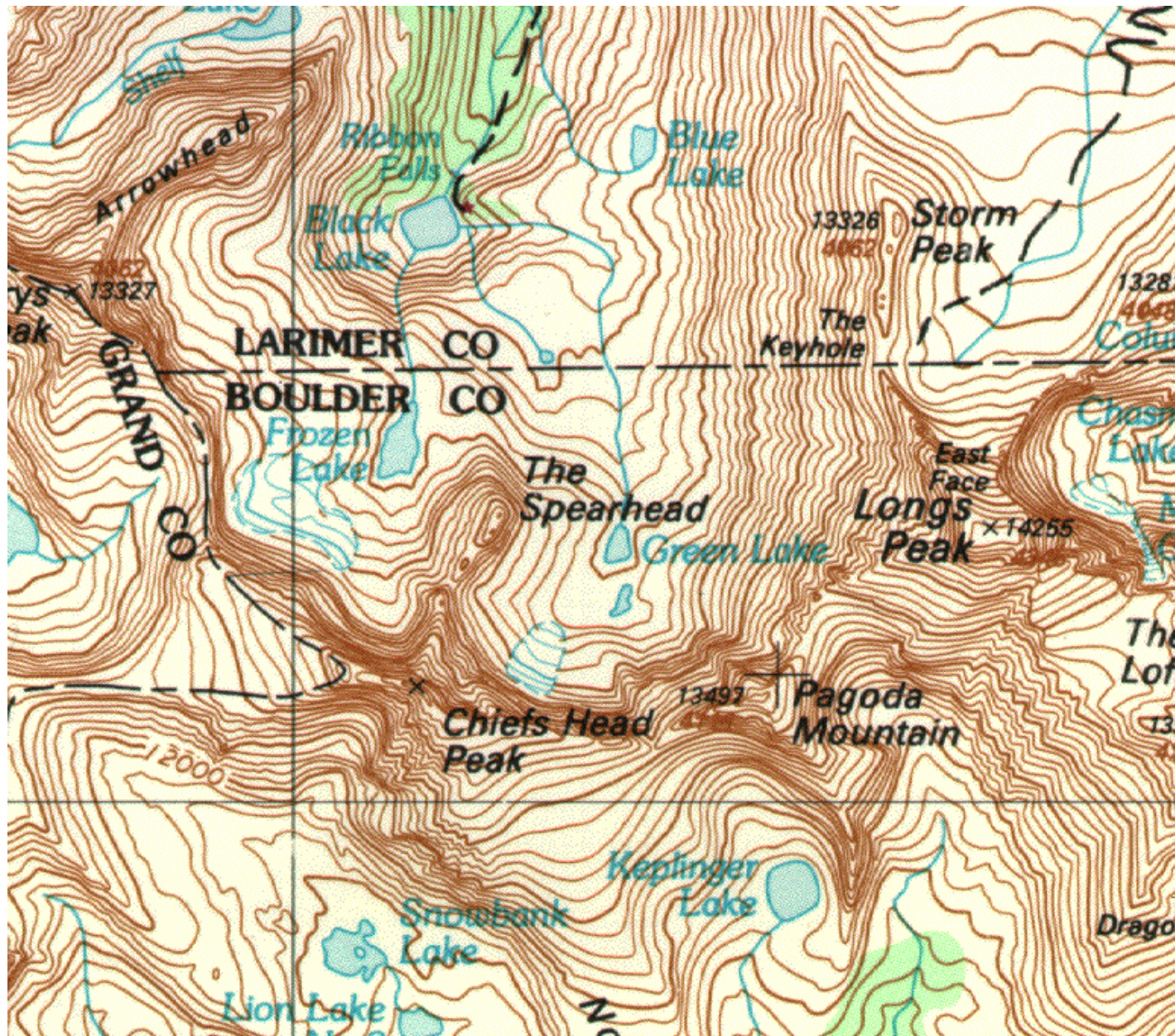


# Level Curves: Topographic Maps (1)

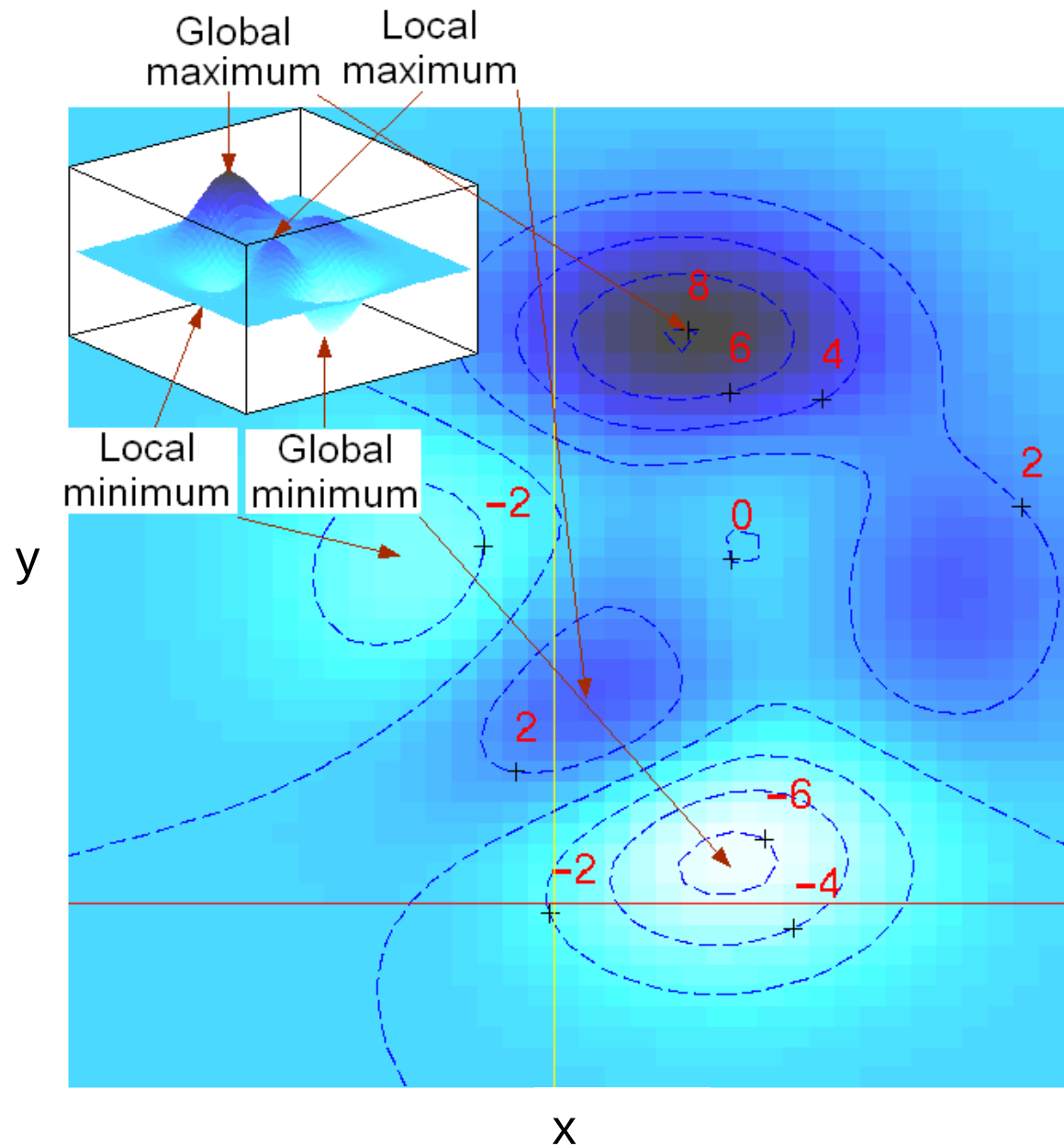




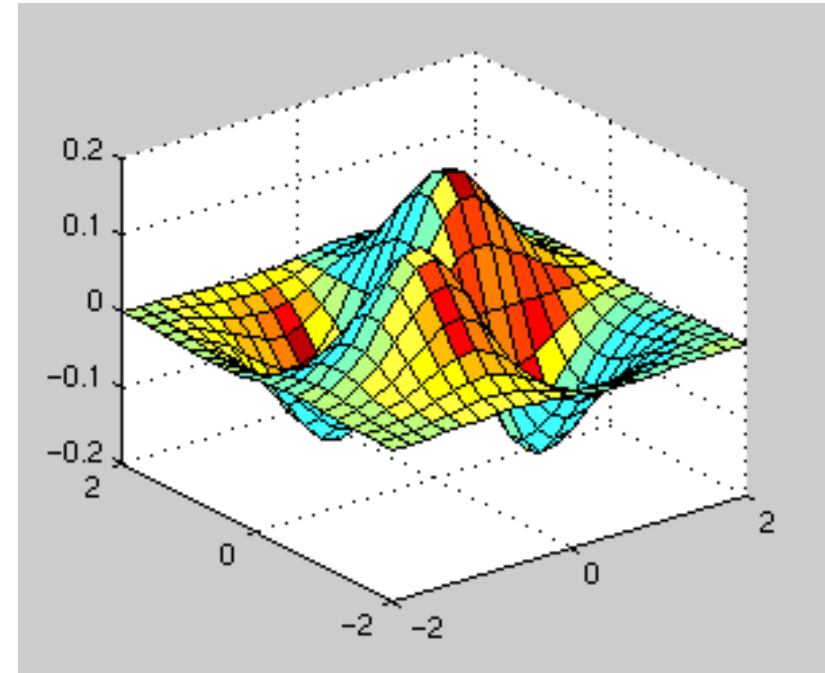
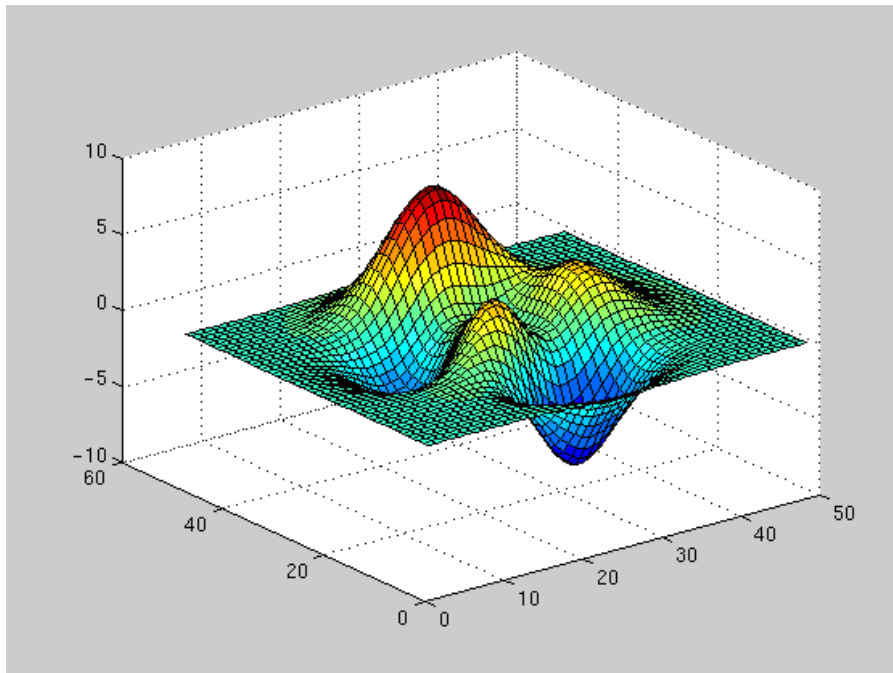
# Level Curves: Topographic Maps (2)



# GLOBAL & LOCAL MAX/MIN VALUES

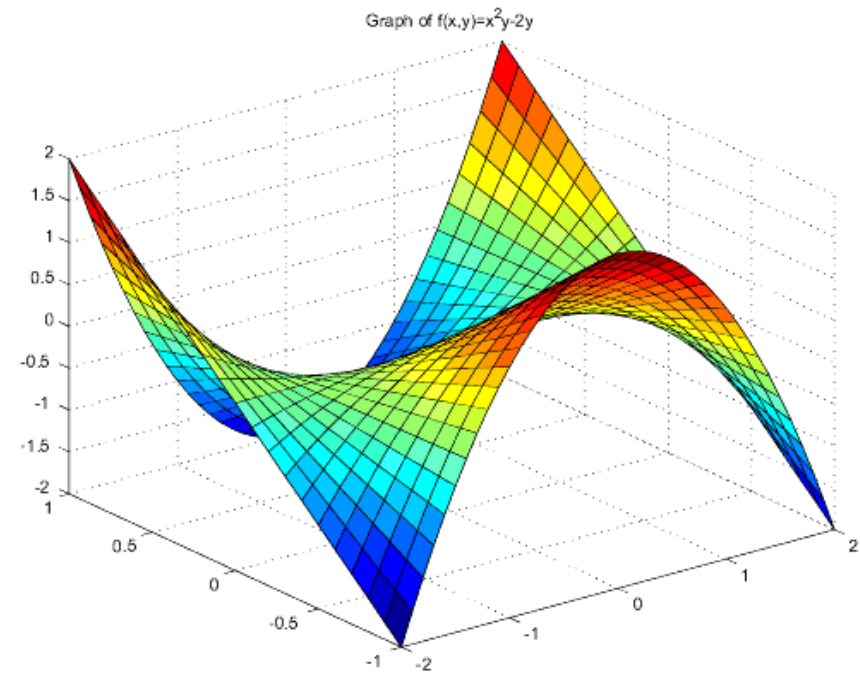
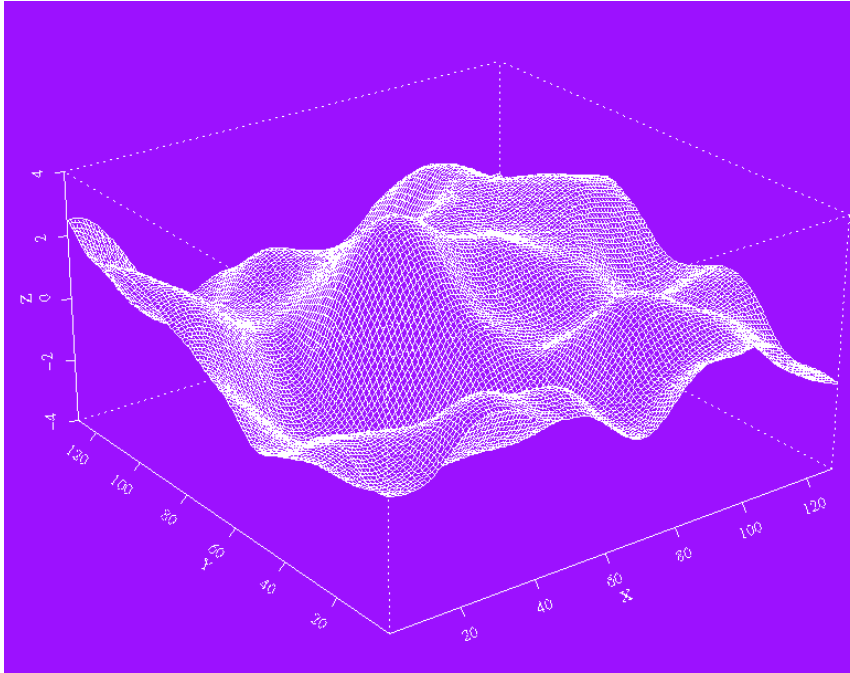


# Surfaces with Multiple Extreme Points (1)

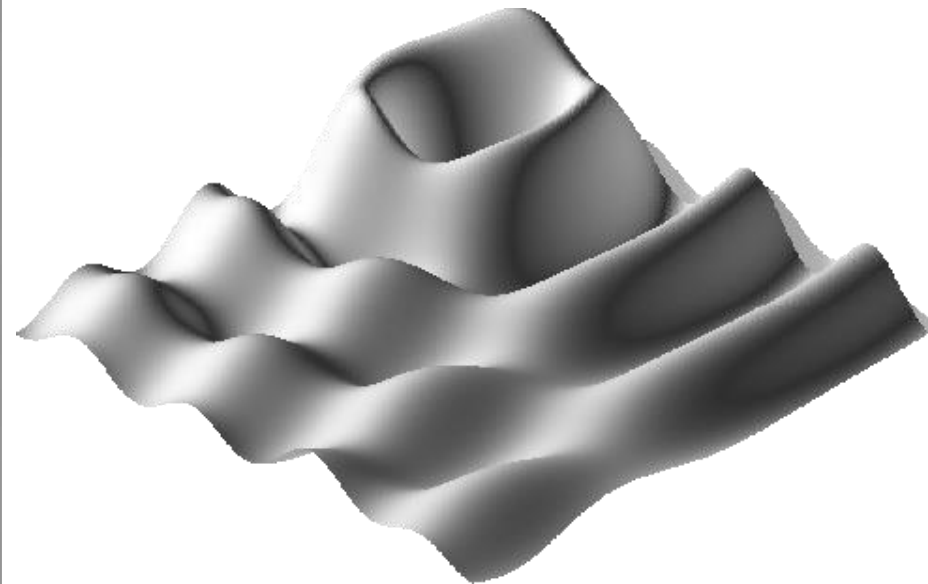
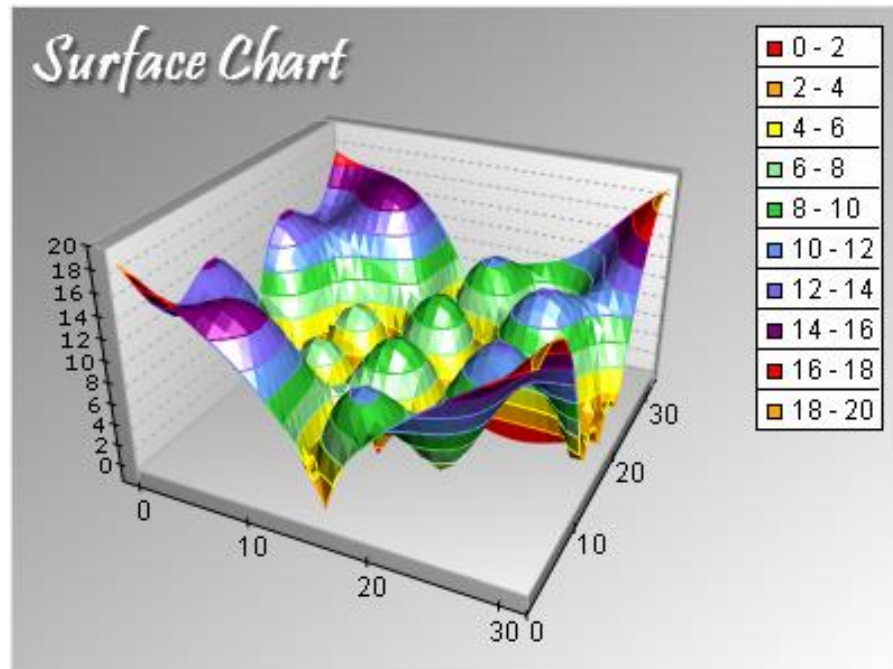




# Surfaces with Multiple Extreme Points (2)



# Surfaces with Multiple Extreme Points (3)





# TABLE OF ELEMENTARY INTEGRALS

$$\int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1 \quad (1)$$

$$\int \frac{1}{u} du = \ln |u| + C \quad (2)$$

$$\int e^u du = e^u + C \quad (3)$$

$$\int a^u du = \frac{a^u}{\ln a} + C \quad (4)$$

$$\int \cos u du = \sin u + C \quad (5)$$

$$\int \sin u du = -\cos u + C \quad (6)$$

$$\int \sec^2 u du = \tan u + C \quad (7)$$

$$\int \csc^2 u du = -\cot u + C \quad (8)$$

$$\int \sec u \tan u du = \sec u + C \quad (9)$$

$$\int \csc u \cot u du = -\csc u + C \quad (10)$$

# Elementary Integrals (*cont'd*)

$$\int \tan u \, du = \ln|\sec u| + C \quad (11)$$

$$\int \cot u \, du = \ln|\sin u| + C \quad (12)$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C \quad (13)$$

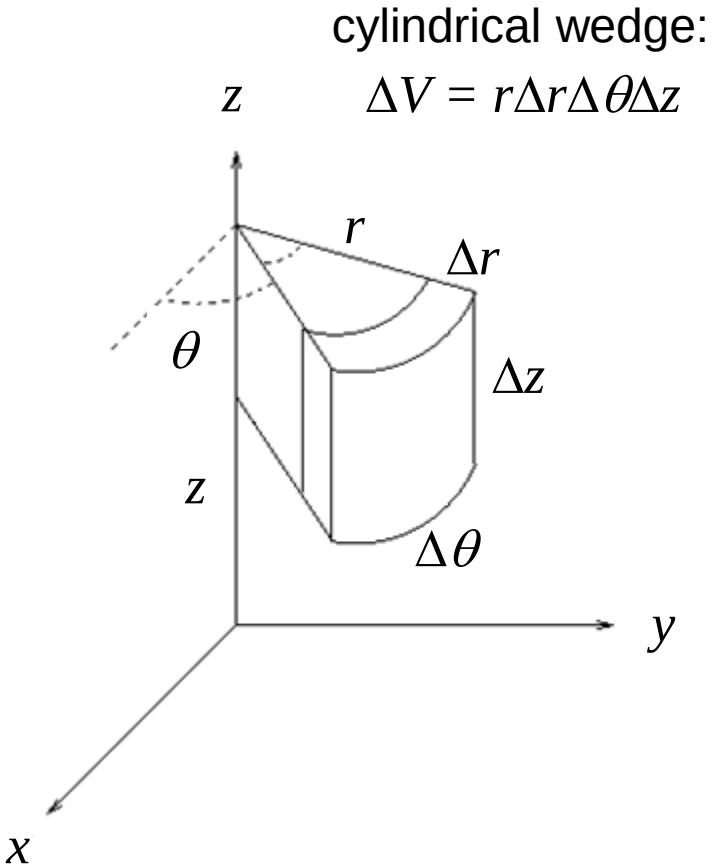
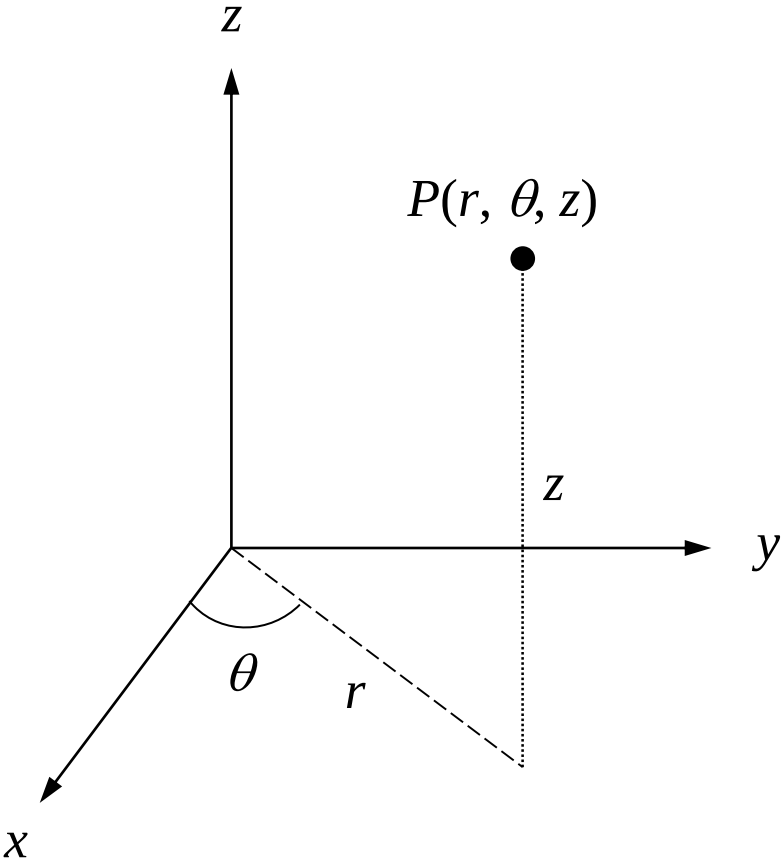
$$\int \csc u \, du = -\ln|\csc u + \cot u| + C \quad (14)$$

$$\int \frac{1}{\sqrt{a^2 - u^2}} \, du = \sin^{-1} \frac{u}{a} + C \quad (15)$$

$$\int \frac{1}{a^2 + u^2} \, du = \frac{1}{a} \tan^{-1} \frac{u}{a} + C \quad (16)$$

$$\int \frac{1}{u\sqrt{u^2 - a^2}} \, du = \frac{1}{a} \sec^{-1} \left| \frac{u}{a} \right| + C \quad (17)$$

# Cylindrical Coordinates



$\theta$

# Spherical Coordinates

