

MA1023:
Calculus III

Supplementary Class Notes

C'20
2019-2020

Calc III as a Part of *Calculus*

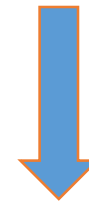
**MA1023
(Calc III)**

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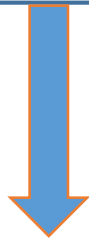
**Series,
Approximations,
Polar
Coordinates,
and Vectors**

MA1021
MA1022
MA1023
MA1024

Calculus



The course on fundamentals of mathematics, i.e., about some of crucial elements of basic human knowledge and essential components of the universal world culture





Key Introductory Points

Major Topics of the Course:

- **Indeterminate Forms & Improper Integrals**
- **Infinite Sequences and Infinite Series**
- **Parametric Curves & Polar Coordinates**
- **Vectors, Curves, Surfaces (in the Plane and in Space)**

Important Advice – Important to Follow:

- ❑ *Be here - come to class - don't miss lecture meetings!*
- ❑ *Read the textbook!*
- ❑ *Do the homework!*

Structure of the Tests:

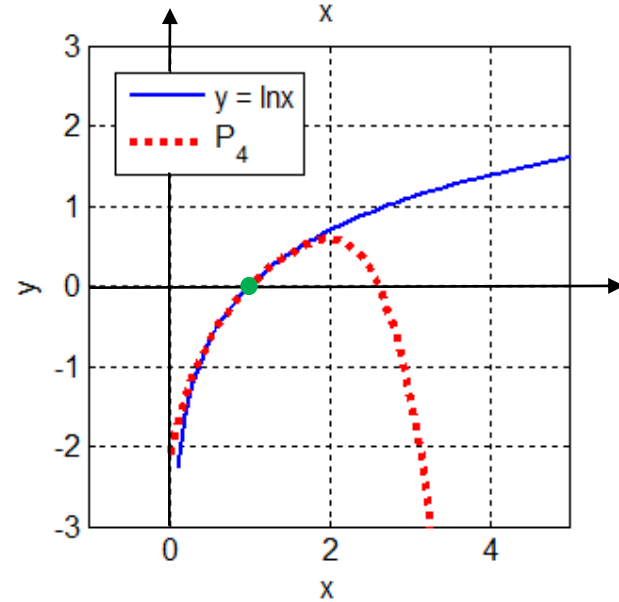
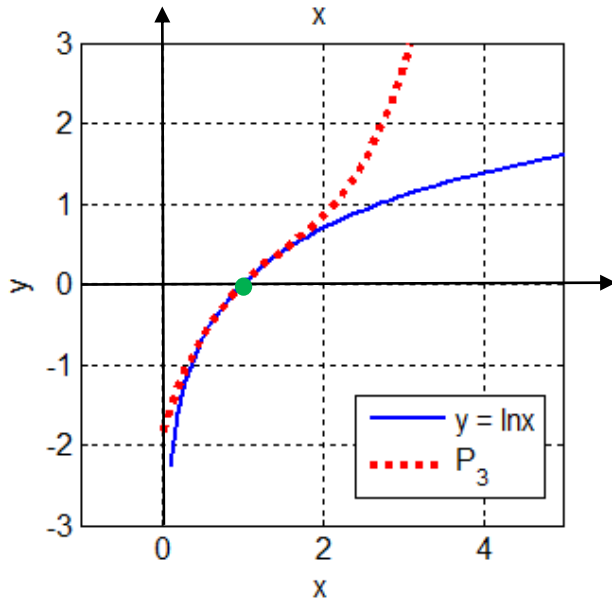
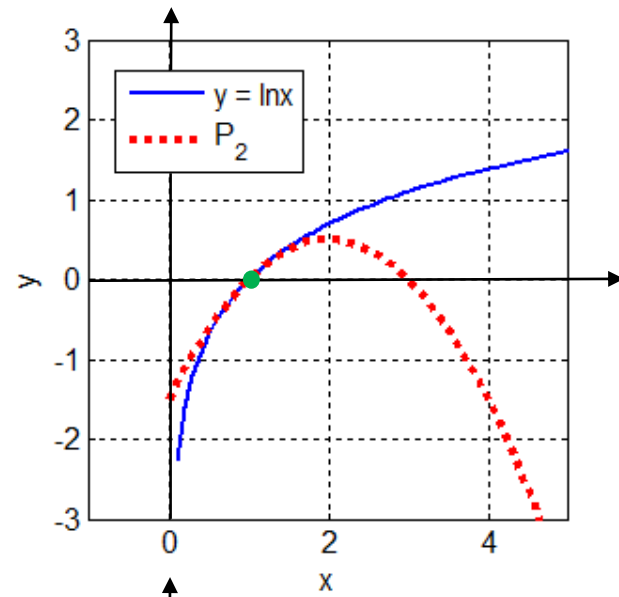
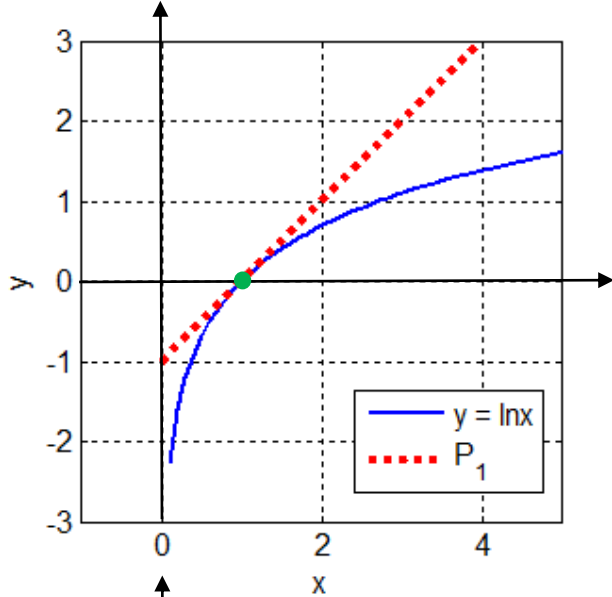
Test 1: 50 min, in class
Test 2: 50 min, in class
Test 3: 50 min, in class

Sections 4.5, 8.8, 10.1-10.4
Sections 10.5-10.9, 11.1-11.4
Sections 11.5, 12.1-12.5, 13.1-13.3

Taylor Polynomials

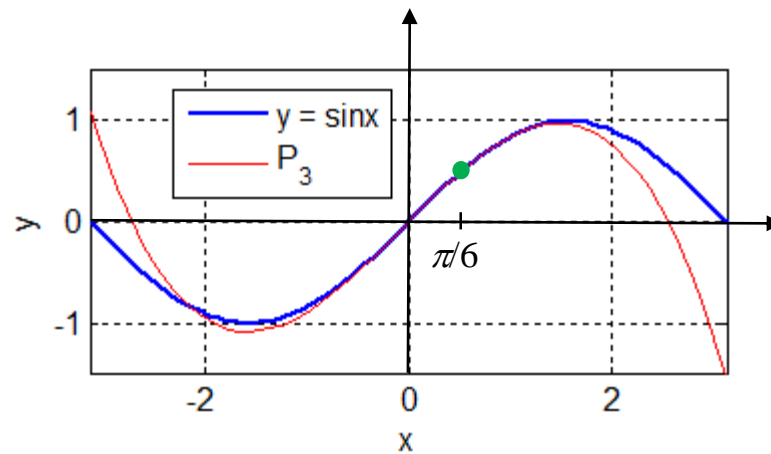
Function $y = \ln x$ and Its Approximations by the Taylor Polynomials of Orders 1, 2, 3, 4

• Approximation at $x = 1$



Taylor Polynomials

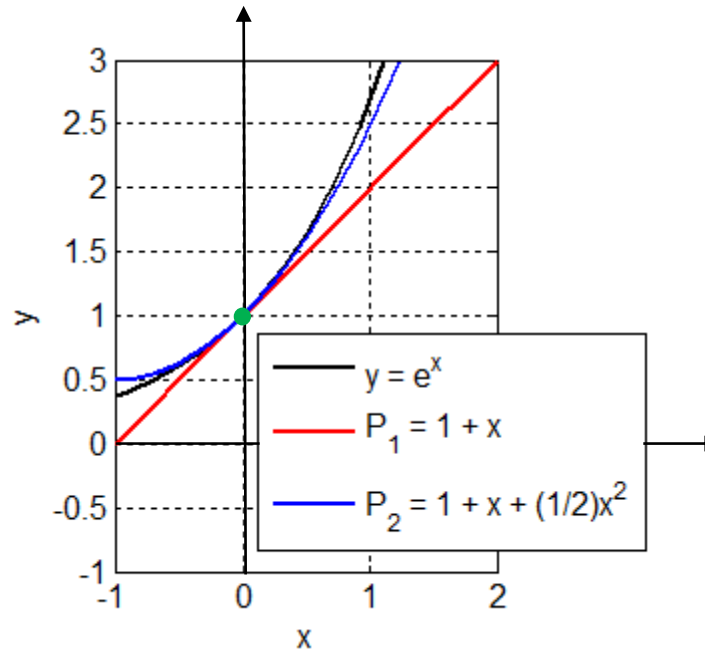
Function $y = \sin x$ and Its Approximations by the Taylor Polynomial of Order 3



- Approximation at $x = \pi/6$

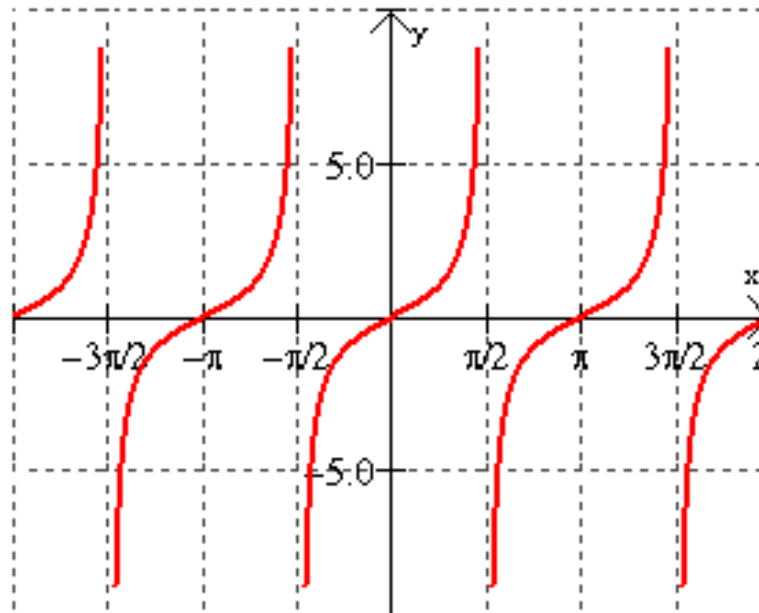
Taylor Polynomials

Function $y = e^x$ and Its Approximations by the Taylor Polynomials of Order 1 and 2



- Polynomials of order 1 (P_1) and order 2 (P_2)
- Approximation at $x = 0$

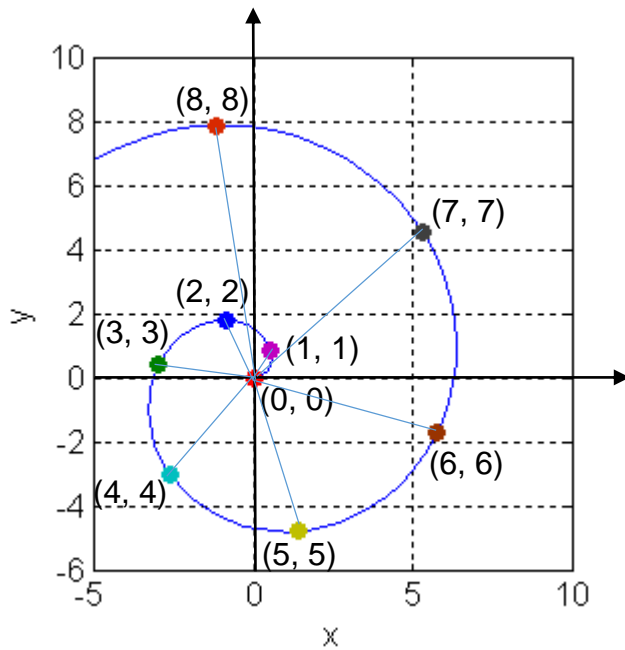
$\tan x$ - Reminder



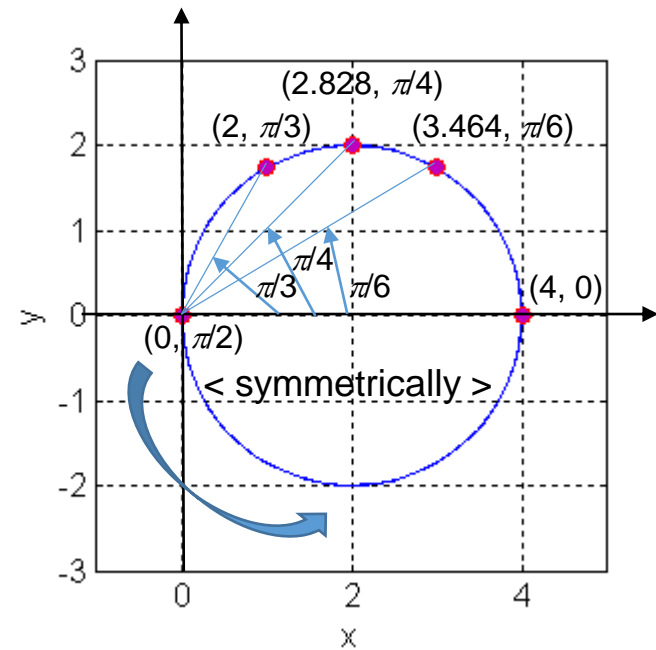
Polar Curves

Spiral curve

$$r = \theta$$

Circle centered at the x -axis

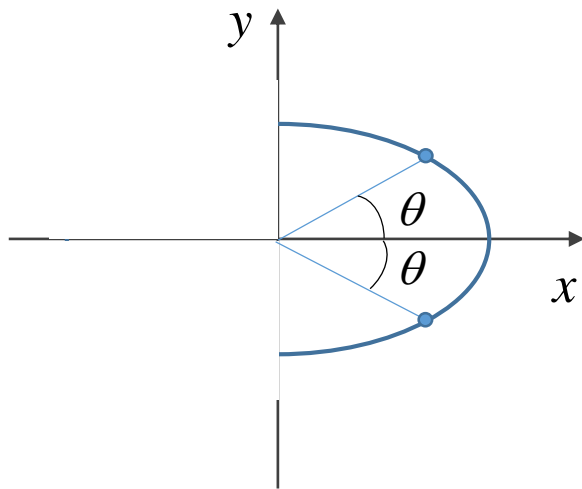
$$r = 4\cos\theta$$



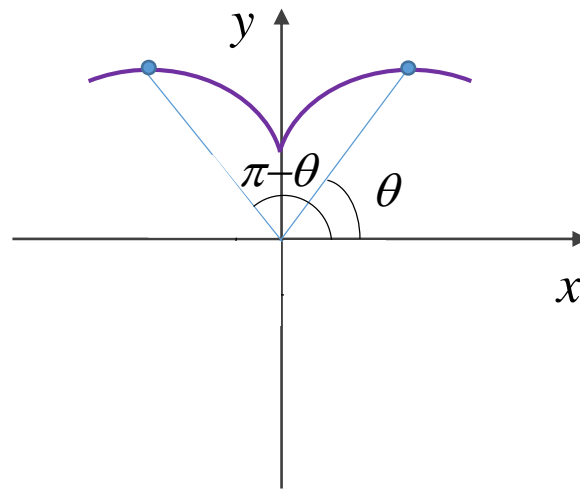
Polar Curves

Symmetry with respect to:

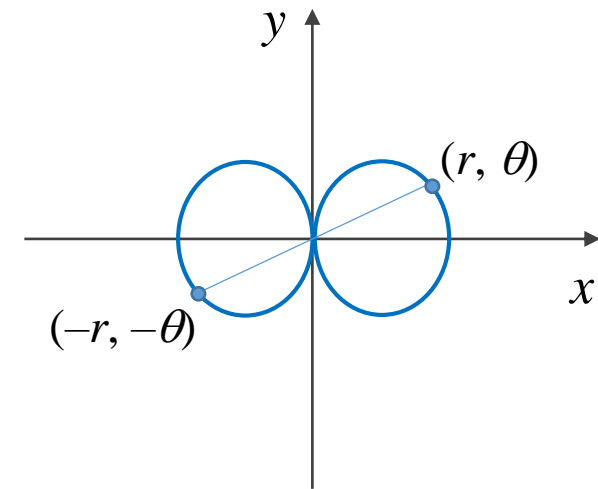
... the x -axis



... the y -axis



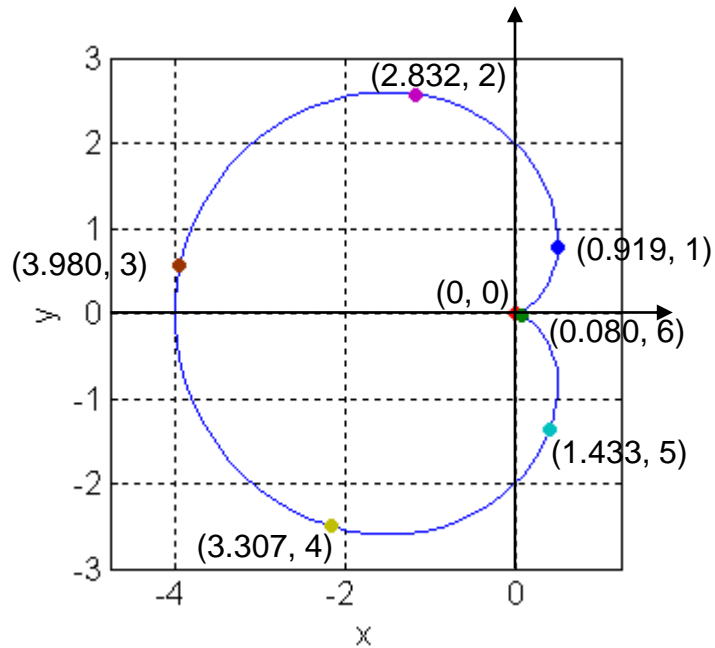
... the origin



Polar Curves

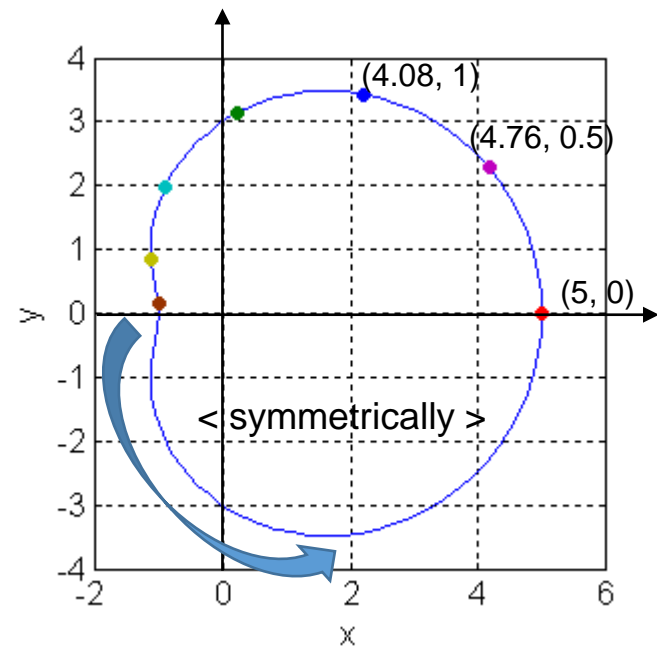
Cardioid

$$r = 2(1 - \cos\theta)$$



Symmetry w/r to the x -axis

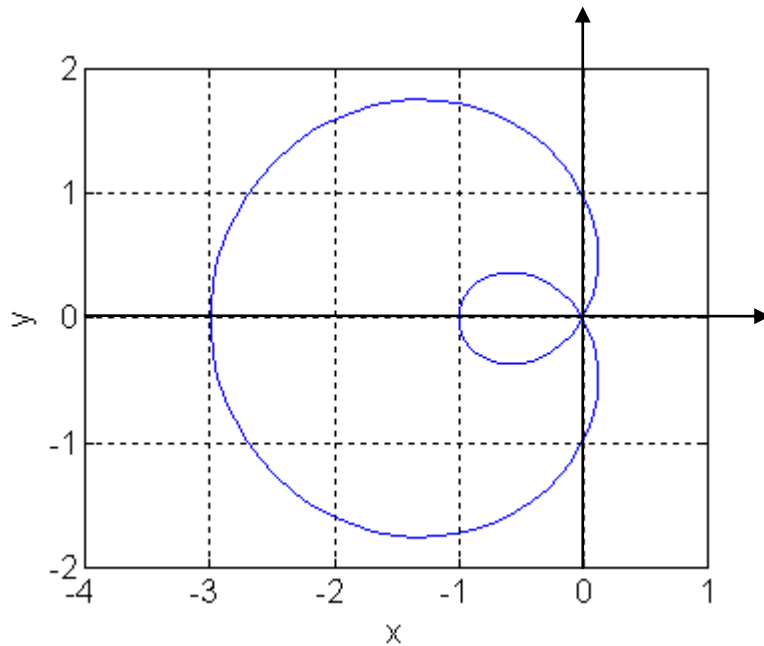
$$r = 3 + 2\cos\theta$$



Polar Curves

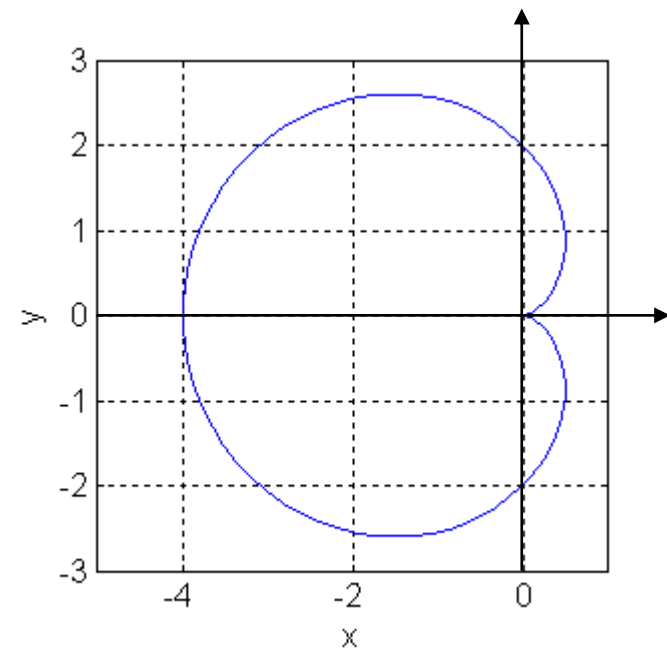
$$\text{Limaçon } r = b - a\cos\theta$$

$$r = 1 - 2\cos\theta$$



$b/a < 1$:
inner loop

$$r = 2 - 2\cos\theta$$

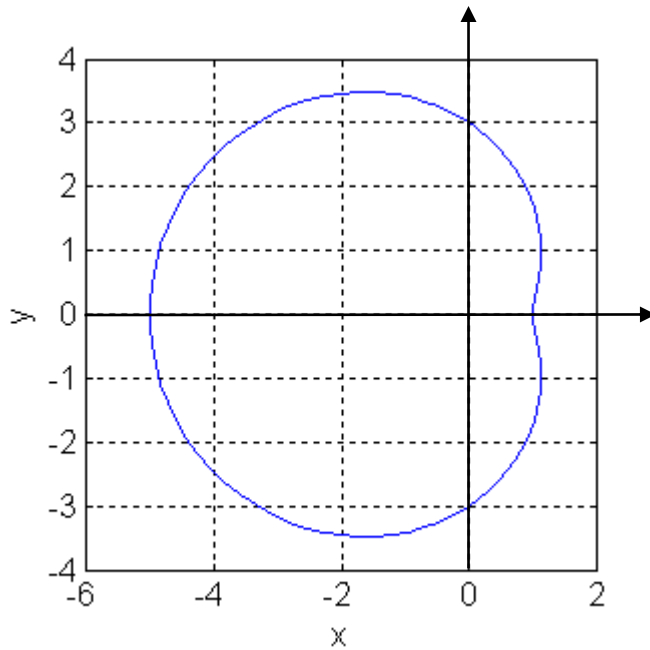


$b/a = 1$:
cardioid

Polar Curves

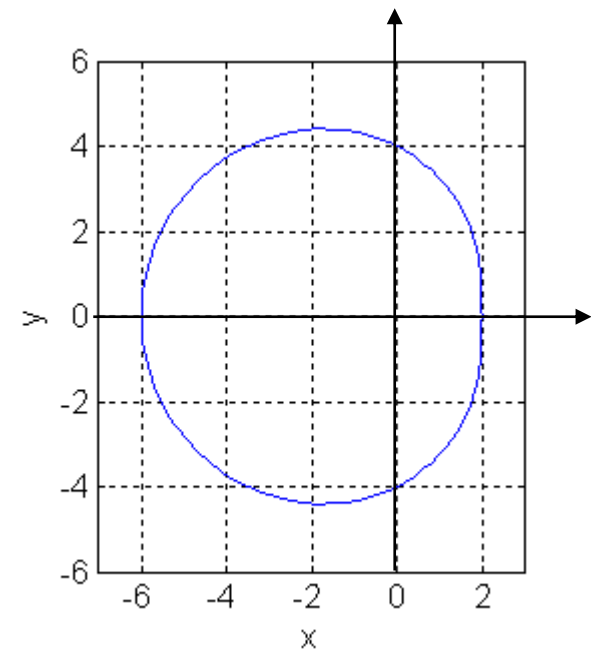
$$\text{Limaçon } r = b - a\cos\theta$$

$$r = 3 - 2\cos\theta$$



$1 < b/a < 2$:
dimple

$$r = 4 - 2\cos\theta$$

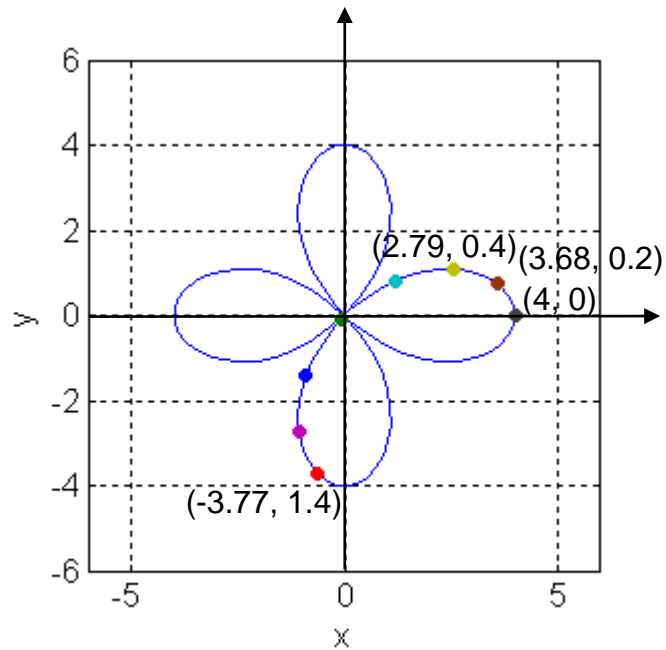


$b/a \geq 2$:
convex

Polar Curves

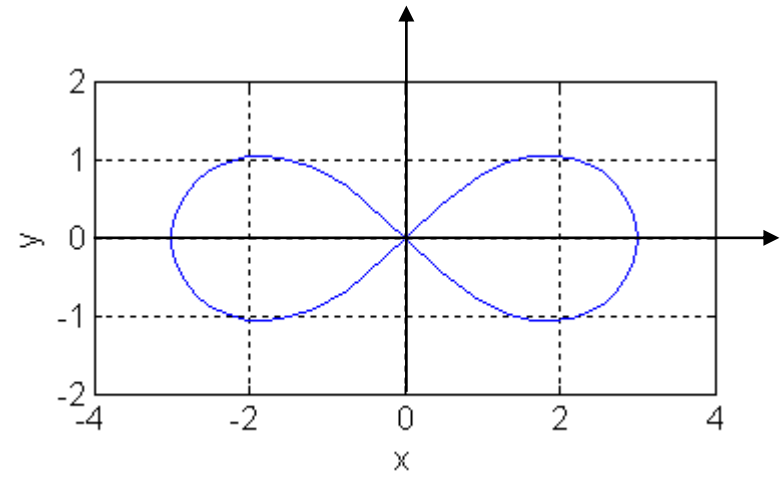
Rose curve

$$r = 4\cos 2\theta$$



Lemniscate

$$r^2 = 9\cos 2\theta$$



Polar Curves

Experiment with polar graphs at:

<https://www.wolframalpha.com/examples/mathematics/plotting-and-graphics/>