

MA1022:
Calculus II

Supplementary Class Notes

PART I

B'19
2019 – 2020

Calc II as a Part of *Calculus*

MA1022
(Calc II)
=
Integral
Calculus

MA1021
MA1022
MA1023
MA1024

Calculus

The course on fundamentals of mathematics, i.e., on some *elements of basic human knowledge and essential components of the universal world culture*



Key Introductory Points

Major Topics of the Course:

- **The Integral**
- **Applications of the Integral**
- **Techniques of Integration**

Important Advice – Important to Follow:

- ❑ *Be here - come to class - don't miss lecture meetings!*
- ❑ *Read the textbook!*
- ❑ *Do the homework!*

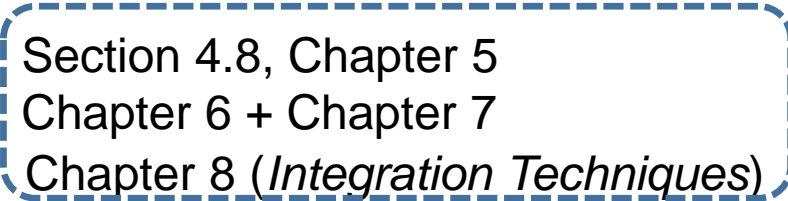
Structure of the Tests:

Midterm Test 1: 50 min

Midterm Test 2: 50 min

COMMON FINAL: 1 h **BASIC SKILLS**

1 h **COMPREHENSIVE EXAM** All Course Material



Section 4.8, Chapter 5
Chapter 6 + Chapter 7
Chapter 8 (*Integration Techniques*)



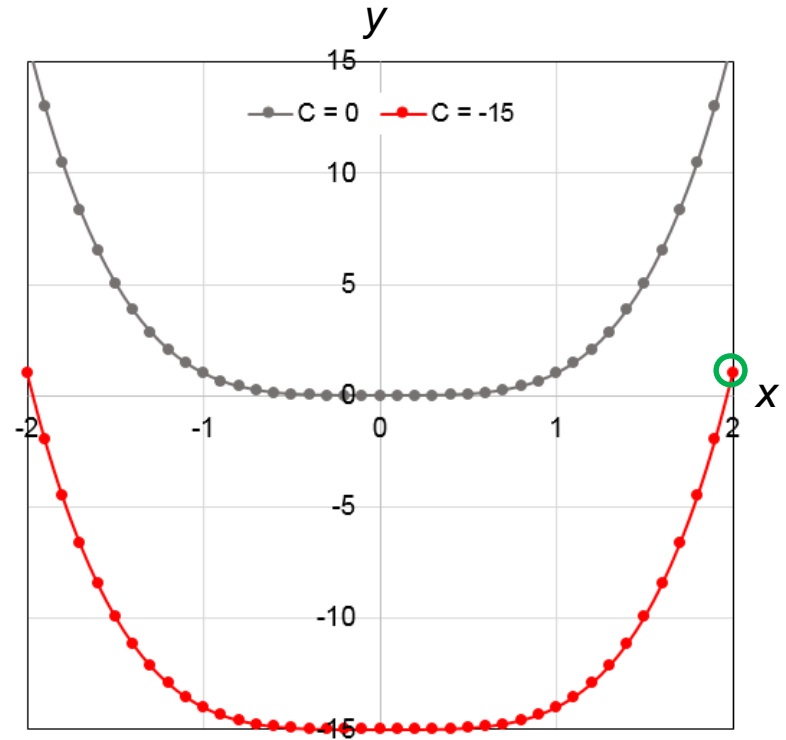
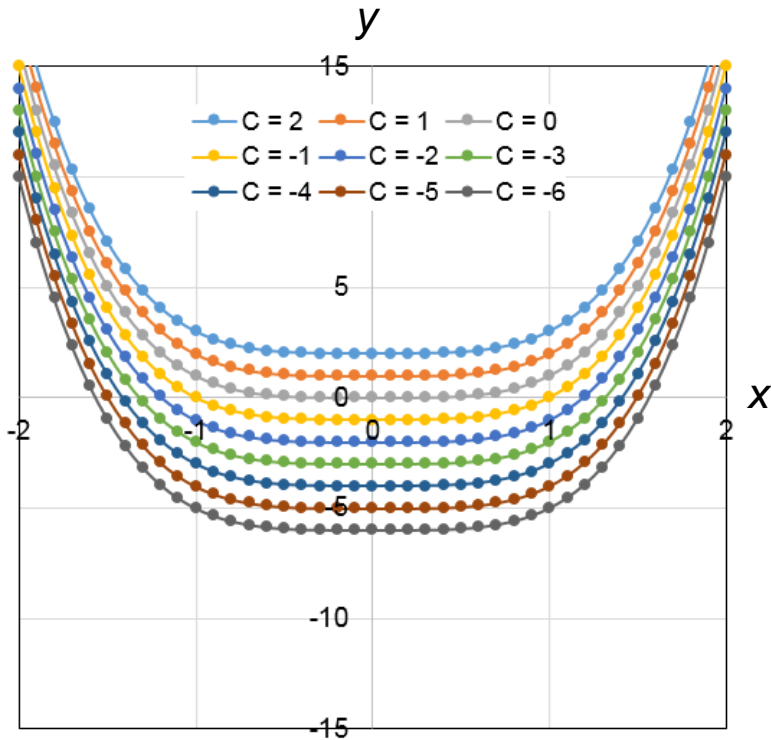
Differential Equations: General and Particular Solutions

$$\text{DE: } dy/dx = 4x^3$$

$$\text{General solution: } y = x^4 + C$$

$$\text{Initial condition: } y(2) = 1$$

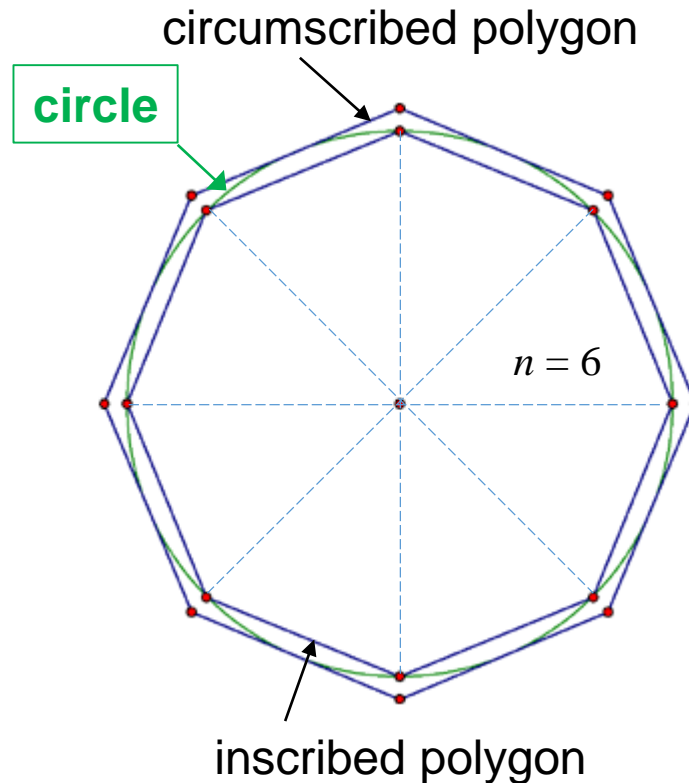
$$\text{Particular solution: } y = x^4 - 15$$



Polygon Area vs Circle Area

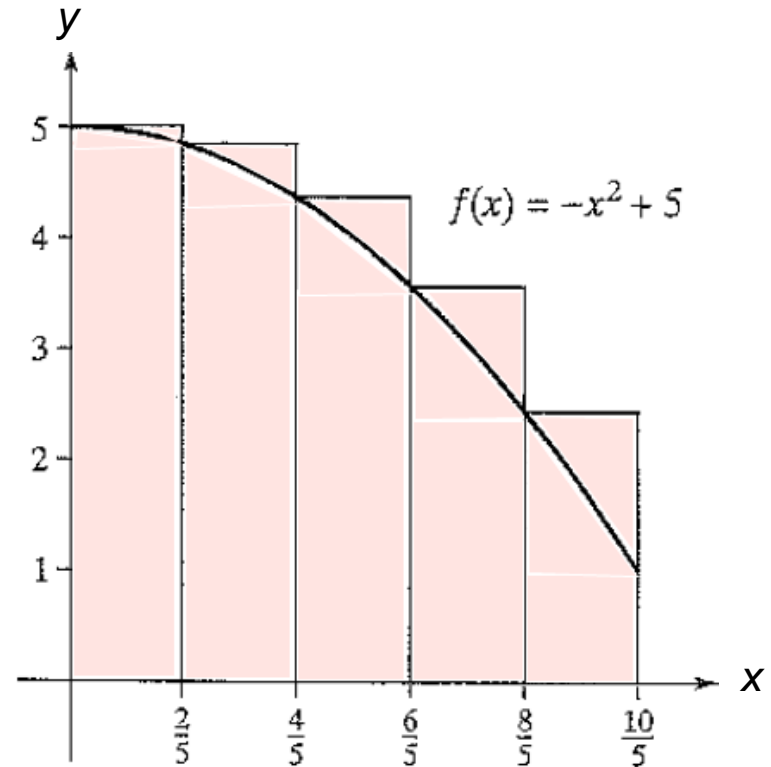
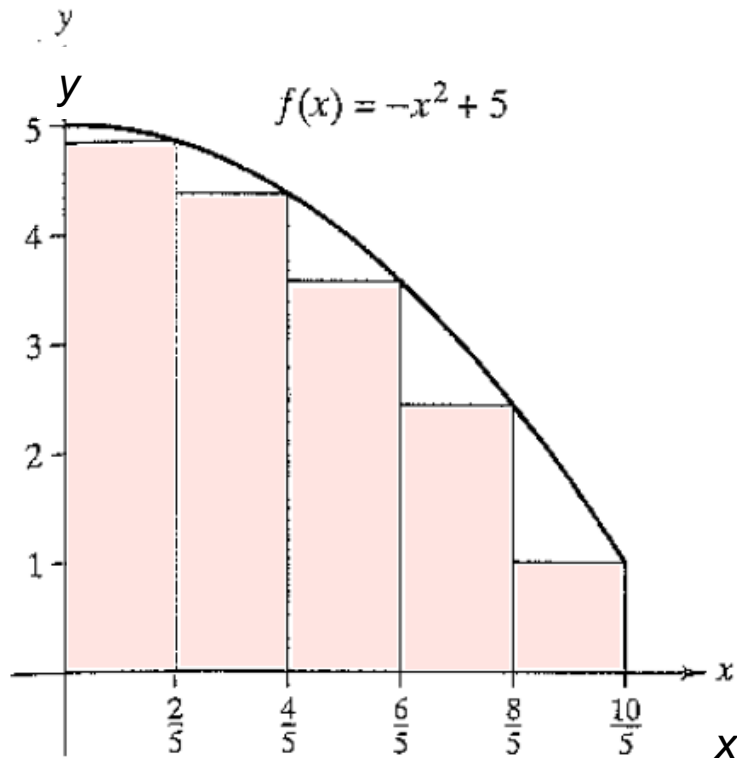


Archimedes
of Syracuse
(287 BC - 212 BC)



- Area of the *inscribed* polygon is less than the area of the circle;
- Area of the *circumscribed* polygon is greater than the area of the circle;
- As n goes up, the areas of both polygons becomes better and better approximations to the area of the circle 522.9.

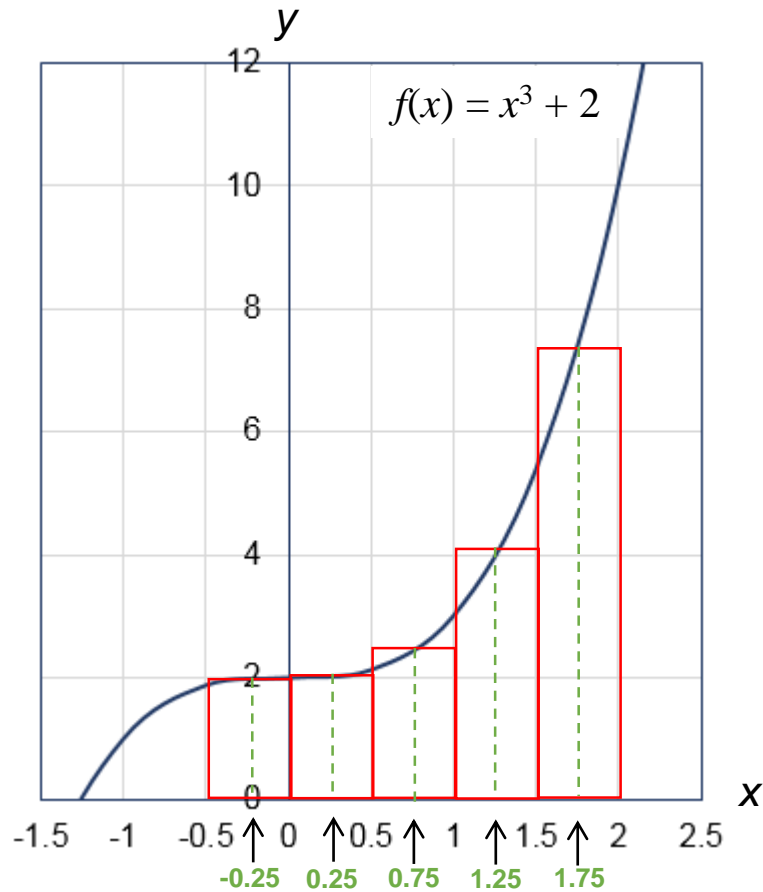
Area via Inscribed & Circumscribed Polygons



$$n = 5$$

Evaluation of the Riemann Sum

Function $f(x) = x^3 + 2$ on the interval $[-0.5, 2]$;
sample points in the **mid-points** of the sub-intervals



Elementary Integrals – Part 1

$$\int \cos u \, du = \sin u + C \quad (1)$$

$$\int \sin u \, du = -\cos u + C \quad (2)$$

$$\int \sec^2 u \, du = \tan u + C \quad (3)$$

$$\int \csc^2 u \, du = -\cot u + C \quad (4)$$

$$\int \sec u \tan u \, du = \sec u + C \quad (5)$$

$$\int \csc u \cot u \, du = -\csc u + C \quad (6)$$

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1 \quad (7)$$

Elementary Integrals – Part 2

$$\int \frac{1}{u} du = \ln|u| + C \quad (8)$$

$$\int e^u du = e^u + C \quad (9)$$

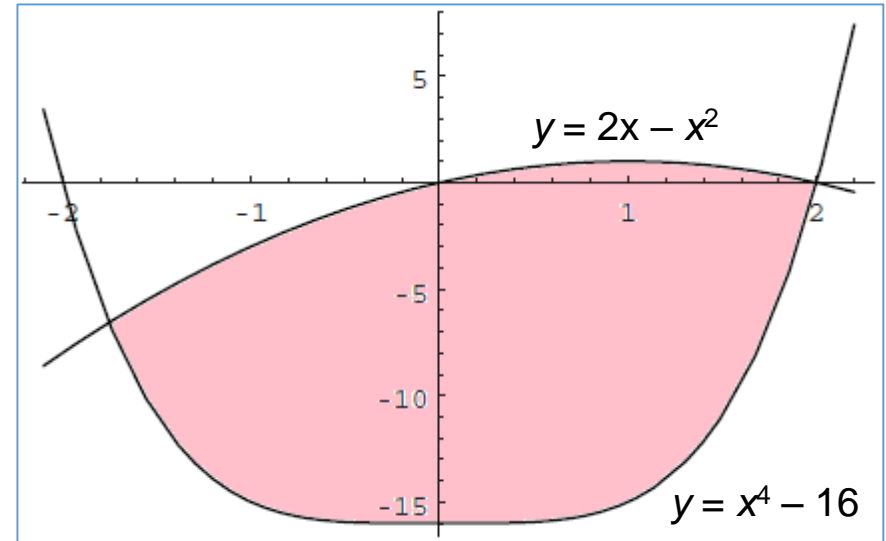
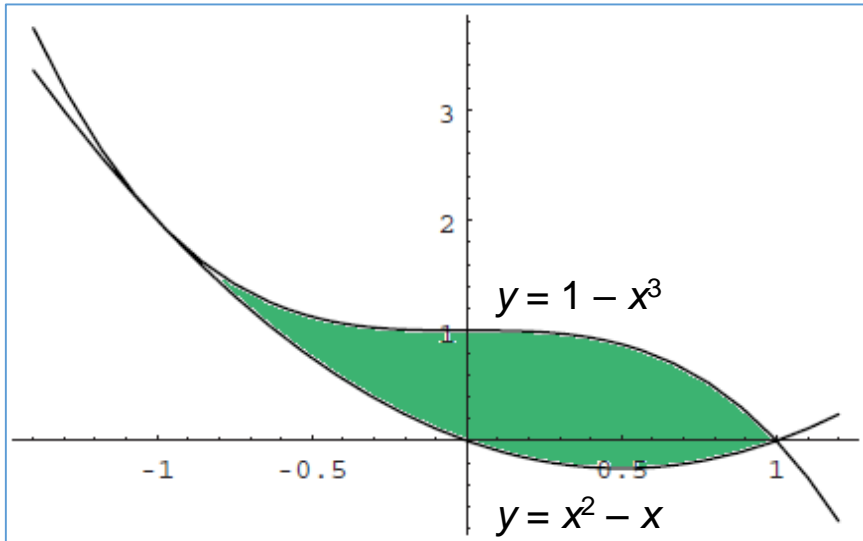
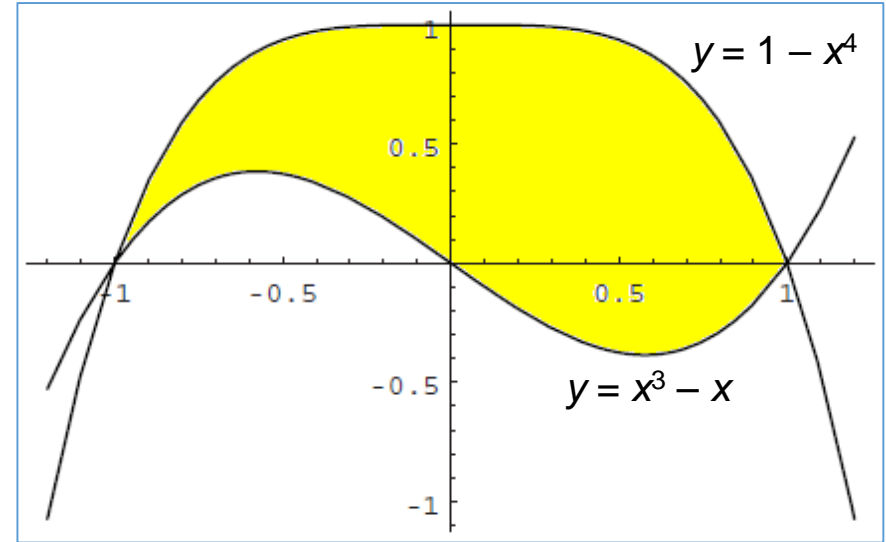
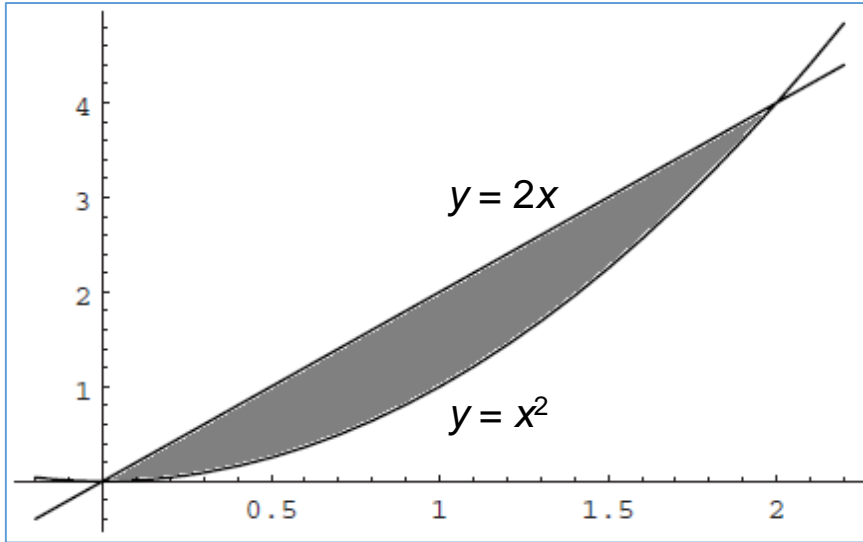
$$\int a^u du = \frac{a^u}{\ln a} + C \quad (10)$$

$$\int \frac{1}{\sqrt{a^2 - u^2}} du = \sin^{-1} \frac{u}{a} + C \quad (11)$$

$$\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \tan^{-1} \frac{u}{a} + C \quad (12)$$

$$\int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a} \sec^{-1} \left| \frac{u}{a} \right| + C \quad (13)$$

Areas of Plane Regions (1)



Areas of Plane Regions (2)

