B'02 MA1022

Basic Skills Exam

1. $\int_0^2 (x^2 + \pi^2)dx = \frac{8}{3} + 2\pi^2$

2. $\int_0^1 \frac{1}{(e^x)^2} dx = -\frac{1}{2} e^{-2x}\Big|_0^1 = \frac{1}{2} - \frac{1}{2} e^{-2}$

3. $\int_1^e \frac{\ln(3x)}{x} dx = \frac{1}{2}(\ln 3x)^2\Big|_1^e = \frac{1}{2}[(\ln 3e)^2 - (\ln 3)^2]$
   $u = \ln(3x)$

4. $\frac{d}{dx}\int_1^{x^2} \arcsin(2\pi\theta)d\theta = 2x\arcsin(2\pi x^2)$

5. $\frac{d}{dx}\arctan(e^{2x}) = \frac{2e^{2x}}{1+(e^{2x})^2}$

6. $\int x(x-1)^{1/3}dx = \int (u+1)u^{1/3}du = \int (u^{4/3} + u^{1/3})du = \frac{3}{7}(x-1)^{7/3} + \frac{3}{4}(x-1)^{4/3} + C$
   $u = x - 1$
   or: $\ldots = 3\int u^6 du + 3\int u^3 du = \ldots$
   $u = (x-1)^3;\ u^3 = x-1;\ 3u^2du = dx;\ x = u^3 + 1$

7. $\int xe^{-2x}dx = -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}dx = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$
   $u = x, \quad dv = e^{-2x}$
   $du = dx,\ v = -\frac{1}{2}e^{-2x}$

Regular Problems

8. Integration y-wise; checking the upper limit of integration: $y = -y^2 + 6,\ y_1 = -3,\ y_2 = 2$

$$\text{Area} = \int_0^2 [f(y) - g(y)]dy = \int_0^2 (6 - y - y^2)dy = \frac{22}{3}$$

9. Integration x-wise; limits of integration: $3 = \sqrt{25 - x^2};\ x_1 = -4,\ x_2 = 4$

$$\text{Volume} = \int_{-4}^{4} \pi[f^2(x) - g^2(x)]dx = 2\int_0^4 \pi(25 - x^2 - 3^2)dx = \frac{256\pi}{3}$$

10. Work = Work$|_{\text{gold}}$ + Work$|_{\text{cable}}$ = $50 \cdot 400 + \int_0^{400} \frac{1}{3}(400 - y)dy = \frac{140{,}000}{3}$ [lb·foot]

11. $m = \int_1^2 [f(x) - g(x)]dx = \int_1^2 (x^4 - x)dx$

$$M_x = \int_1^2 \frac{1}{2}[f(x) - g(x)]dx = \frac{1}{2}\int_1^2 (x^8 - x^2)dx$$

$$M_y = \int_1^2 x[f(x) - g(x)]dx = \int_1^2 (x^5 - x^2)dx$$

12. $y = x^{e^x};\ \ln y = e^x \ln x;\ \frac{1}{y}\frac{dy}{dx} = e^x \ln x + \frac{e^x}{x};\ \frac{dy}{dx} = x^{e^x}(e^x \ln x + \frac{e^x}{x})$

13a. $\int \frac{x}{1+x}dx = \int \frac{x+1-1}{x+1}dx = x - \ln(x+1) + C$

13b. $\int \frac{x}{1+x^2}dx = \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln(x^2+1) + C$

$u = 1 + x^2$

13c. $\int \frac{[\tan^{-1} x]^2}{1+x^2}dx = \int u^2 du = \frac{1}{3}[\tan^{-1} x]^3 + C$

$u = \tan^{-1} x;\ du = \frac{1}{1+x^2}dx$