MA 1022 B 2018 Basic Skills Exam December 13, 2018

ANSWER KEY

Name ______________________________

Instructions

This test is closed book. Calculators are not allowed. On the basic skills part of the exam only the final answers are graded – it's "all or nothing".

Part I (Basic Skills Test)

Problem	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Total**
Value	5	5	5	5	5	5	5	35
Earned								

Please circle your section

B01Y (Tang, 9:00 lecture)

B02Y (Tang, 10:00 lecture)

B08Y (Yakovlev, 2:00 lecture)

B09Y (Yakovlev, 3:00 lecture)

ABRAHAM

B03Y (8:00 am conference with David)

B04Y (4:00 pm conference with Yuriy)

B05Y (2:00 pm conference with Rob)

B06Y (3:00 pm conference with Daniela)

B07Y (2:00 pm conference with Ryan)

Part I (Basic Skills Test) **Name** ______________________

There is no partial credit on these problems -- only your final answer will be graded. Work carefully and check your work. You need not simplify answers, except that an answer for a definite integral should be simplified as much as possible.

1. $\int \left(\frac{1}{x^{2/3}} - x^{-3} + 4x^5 \right) dx$ Ans: $3x^{1/3} + \frac{1}{2}x^{-2} + \frac{2}{3}x^6 + C$

$\int x^{-2/3} - x^{-3} + 4x^5 dx = \frac{x^{1/3}}{1/3} - \frac{x^{-2}}{-2} + \frac{4x^6}{6}$

2. $\int 5x\cos(x)\, dx$ $u = 5x$ $dv = \cos(x)dx$ $du = 5dx$ $v = \sin(x)$ Ans: $5x\sin(x) + 5\cos(x) + C$

$5x\sin x - \int 5\sin(x)dx$

3. $\int \frac{x-10}{(x+2)(x-4)} dx$ Ans: $2\ln|x+2| - \ln|x-4| + C$

$\frac{A}{x+2} + \frac{B}{x-4} = \frac{x-10}{(x+2)(x-4)} \Rightarrow A(x-4) + B(x+2) = x-10$

$x=4 \to B=-1;\ x=-2 \to A=2$

4. $\int \frac{e^{2x}}{e^{4x}+1} dx$ $u = e^{2x}$ $du = 2e^{2x}dx$ Ans: $\frac{1}{2}\arctan(e^{2x}) + C$

$\frac{1}{2}\int \frac{du}{u^2+1}$

5. $\int \frac{e^{4x}}{e^{4x}+9} dx$ $u = e^{4x}+9$ $du = 4e^{4x}dx$ Ans: $\frac{1}{4}\ln(e^{4x}+9) + C$

$\frac{1}{4}\int \frac{du}{u}$

6. $\int_0^{\pi/8} 3\tan(2x)\sec^2(2x)dx$ Ans: $3/4$

$u = \tan(2x)$

$du = 2\sec^2(2x)dx$

$\frac{3}{2}\int u\,du = \frac{3}{4}u^2 = \frac{3}{4}\tan^2(2x)\Big|_0^{\pi/8} = \frac{3}{4}$

7. $\frac{d}{dx}\int_5^{x^2} 3t^2\sin(2t)dt$ Ans: $3x^4 \cdot \sin(2x^2) \cdot 2x$

MA 1022 B 2018 Final Exam December 13, 2018

Name ___________________________________

Instructions
This test is closed book. Calculators are not allowed. Show your work in the space provided. On the general part of the exam your work and your explanations are graded, not just the final answers.

Part II (General Exam)

Problem	**8**	**9**	**10**		**11**	**12**						**13**	**Total**
Value	14	14	5	5	12	6	6	6	6	6	6	14	**100**
Earned													

Please circle your section

B01Y (Tang, 9:00 lecture)

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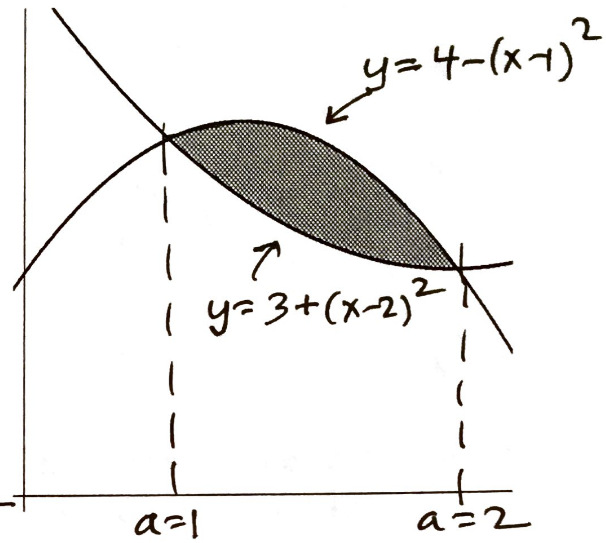
B07Y (2:00 pm conference with Ryan)

8. Find the area of the region which is bounded between the two curves $y=3+(x-2)^2$ and $y=4-(x-1)^2$



$3+(x-2)^2 = 4-(x-1)^2$

$3+x^2-4x+4 = 4-(x^2-2x+1)$

$2x^2-6x+4 = 0$

$2(x-2)(x-1)=0 \quad x=1 \text{ or } x=2$

$$\text{Area} = \int_a^b \text{upper}-\text{lower}\,dx = \int_1^2 [4-(x-1)^2]-[3+(x-2)^2]\,dx$$

$$A = \int_1^2 -2x^2+6x-4\,dx = \left.\frac{-2x^3}{3} + 3x^2-4x\right]_1^2$$

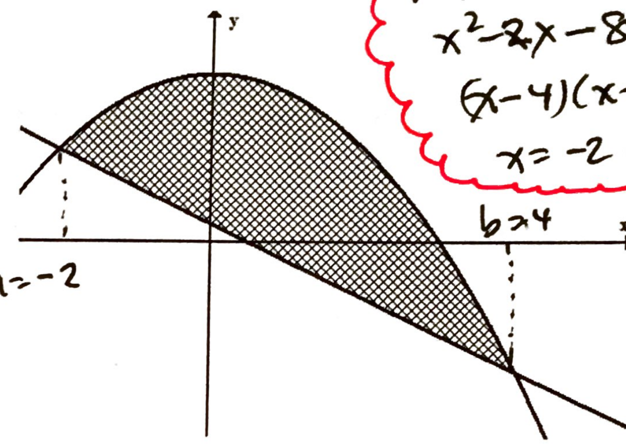
$$= \left[\left(-\tfrac{2}{3}\right)(8)+(3)(4)-(4)(2)\right] - \left[-\tfrac{2}{3}+3-4\right]$$

$$= \left(-\tfrac{16}{3}+12-8\right) - \left(-\tfrac{2}{3}+3-4\right)$$

$$= -\tfrac{4}{3} - \left(-\tfrac{5}{3}\right) = \tfrac{1}{3}$$

9. The region $\boldsymbol{R}$ is captured between the curve $y=9-x^2$ and the line $y=1-2x$.

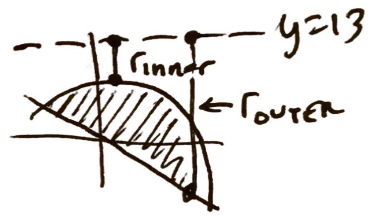
Set up but do not evaluate an integral expression for the volume of the solid which results when $\boldsymbol{R}$ is revolved around the line $y=13$.



$$9-x^2=1-2x$$
$$x^2-2x-8=0$$
$$(x-4)(x+2)$$
$$x=-2 \text{ or } x=4$$

a. **Set up but do not evaluate** an integral expression for the volume of the solid which results when $\boldsymbol{R}$ is revolved around the line $y=13$.

$$V=\int_a^b A(x)\,dx=\int_a^b \pi r_{OUTER}^2-\pi r_{INNER}^2\,dx$$

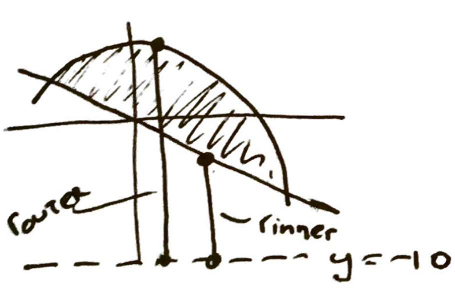


$$r_{OUTER}=13-(1-2x)=12+2x$$
$$r_{INNER}=13-(9-x^2)=4+x^2$$

$$\boxed{V=\int_{-2}^{4}\pi(12+2x)^2-\pi(4+x^2)^2\,dx}$$

b. **Set up but do not evaluate** an integral expression for the volume of the solid which results when $\boldsymbol{R}$ is revolved around the line $y=-10$

$$r_{OUTER}=(9-x^2)-(-10)=19-x^2$$
$$r_{INNER}=(1-2x)-(-10)=11-2x$$



$$\boxed{V=\int_{-2}^{4}\pi(19-x^2)^2-\pi(11-2x)^2\,dx}$$

10. Do each of the following:

a. Calculate the arc length of the curve $y = x^{3/2}$ between the points $(0,0)$ and $(4,8)$

$$\text{arc length} = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^4 \sqrt{1+\left(\frac{3}{2}x^{\frac{1}{2}}\right)^2}\,dx = \int_0^4 \sqrt{1+\frac{9x}{4}}\,dx$$

$u = 1 + \frac{9x}{4}$

$du = \frac{9}{4}\,dx$

$$\frac{4}{9}\int_{u=1}^{u=10} u^{1/2}\,du = \frac{8}{27}\,u^{3/2}\Big|_1^{10}$$

$x=0 \Rightarrow u = 1$

$x=4 \Rightarrow u = 10$

$$\boxed{\text{arc length} = \frac{8}{27}\left(10^{3/2} - 1\right)}$$

b. **<u>Set up but do not evaluate</u>** an expression for the surface area swept out when the curve $y = \cos(3x)$ between the points $(0,1)$ and $\left(\frac{\pi}{6}, 0\right)$ is rotated around the $x-axis$

$$\text{surface area} = \int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$= \int_0^{\pi/6} 2\pi\cos(3x)\sqrt{1+\left[-3\sin(3x)\right]^2}\,dx$$

11. A thin metallic plate covering the first quadrant region beneath the curve $y = x^3$ and between the lines $x = 1$ and $x = 3$ has a varying density given by $\delta(x) = \frac{3}{x}$

Find the coordinates of the center of mass of the plate.

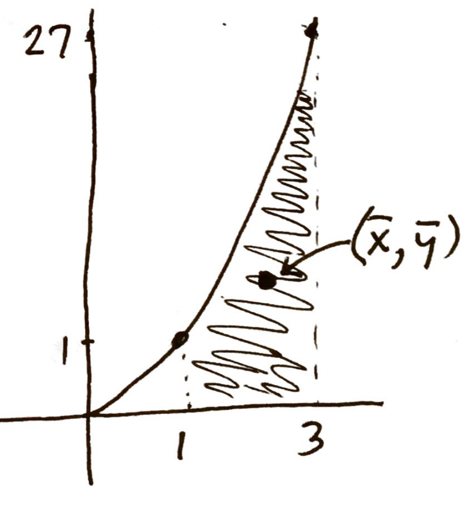
$$m = \int_a^b \delta(x)\, f(x)\, dx = \int_1^3 \left(\frac{3}{x}\right)(x^3)\, dx = x^3 \Big|_1^3 = 26$$

$$M_y = \int_a^b x\, \delta(x)\, f(x)\, dx = \int_1^3 (x)\left(\frac{3}{x}\right)(x^3)\, dx = \frac{3}{4} x^4 \Big|_1^3 = 60$$

$$M_x = \int_a^b \frac{f(x)}{2}\, \delta(x) f(x)\, dx = \int_1^3 \left(\frac{x^3}{2}\right)\left(\frac{3}{x}\right) x^3\, dx = \frac{1}{4} x^6 \Big|_1^3 = 182$$

$$\bar{x} = \frac{M_y}{m} = \frac{60}{26} = 2\tfrac{4}{13}$$

$$\bar{y} = \frac{M_x}{m} = \frac{182}{26} = 7$$



center of mass occurs at $(\bar{x}, \bar{y}) = (2\tfrac{4}{13}, 7)$

12. Evaluate each of the following integrals. Be sure to show your work; **unsupported answers will receive no credit**.

a. $\int \sec^4(x)\tan^3(x)\,dx$

TWO APROACHES

$u = \sec(x)$

$du = \sec(x)\tan(x)\,dx$

$\int \underbrace{\sec^3(x)}_{u^3}\ \underbrace{\tan^2(x)}_{(u^2-1)}\ \underbrace{\sec(x)\tan(x)\,dx}_{du}$

$\int u^3 \quad (u^2-1) \quad du$

$\frac{1}{6}u^6 - \frac{1}{4}u^4 + C$

$\frac{1}{6}\sec^6(x) - \frac{1}{4}\sec^4(x) + C$

OR

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$\int \underbrace{\sec^2(x)}_{(1+u^2)}\ \underbrace{\tan^3(x)}_{(u^3)}\ \underbrace{\sec^2(x)\,dx}_{du}$

$\int (1+u^2)(u^3) \quad du$

$\frac{1}{4}u^4 + \frac{1}{6}u^6 + C$

$\frac{1}{4}\tan^4(x) + \frac{1}{6}\tan^6(x) + C$

b. $\int x\arctan\left(x^2\right)dx$

$\theta = x^2$

$d\theta = 2x\,dx$

$\frac{1}{2}\int \arctan\theta \, d\theta$

$u = \arctan\theta \qquad dv = \frac{1}{2}d\theta$

$du = \frac{d\theta}{1+\theta^2} \qquad v = \frac{\theta}{2}$

$\int u\,dv = uv - \int v\,du$

$\frac{\theta}{2}\arctan\theta - \frac{1}{2}\int \frac{\theta\,d\theta}{1+\theta^2}$

$u = 1+\theta^2$

$du = 2\theta\,d\theta$

$\frac{\theta}{2}\arctan\theta - \frac{1}{4}\int \frac{du}{u} = \frac{\theta}{2}\arctan\theta - \frac{1}{4}\ln(1+\theta^2)$

$= \frac{x^2}{2}\arctan(x^2) - \frac{1}{4}\ln(1+x^4) + C$

c. $\int e^x \cos(x)dx$

$u = \cos(x)$	$dv = e^x dx$
$du = -\sin(x)dx$	$v = e^x$

$$\int e^x\cos(x)dx = e^x\cos(x) + \int e^x \sin(x)\,dx$$

$u = \sin(x)$	$dv = e^x dx$
$du = \cos(x)dx$	$v = e^x$

$$\int e^x\cos(x)dx = e^x\cos(x) + e^x\sin(x) - \int e^x\cos(x)\,dx$$

$$I = e^x(\cos(x) + \sin(x)) - I$$

$$I = \int e^x\cos(x)\,dx = \frac{1}{2}e^x(\cos(x) + \sin(x)) + C$$

d. $\int \frac{4x^2+7x-12}{x^3-x^2-6x}dx = \int \frac{4x^2+7x-12}{x(x-3)(x+2)}dx = \int \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+2}\,dx$

$$4x^2+7x-12 = A(x-3)(x+2) + Bx(x+2) + Cx(x-3)$$

$x=0 \Rightarrow -12 = -6A \Rightarrow A = 2$

$x=3 \Rightarrow 45 = 15B \Rightarrow B = 3$

$x=-2 \Rightarrow -10 = 10C \Rightarrow C = -1$

$$\int\left(\frac{2}{x} + \frac{3}{x-3} - \frac{1}{x+2}\right)dx$$

$$= 2\ln x + 3\ln(x-3) - \ln(x+2) + C$$

e. $\int \frac{e^x}{\sqrt{1-e^{2x}}}dx$

$u = e^x$
$du = e^x dx$

$$\int \frac{du}{\sqrt{1-u^2}} = \arcsin(u) + C$$

$$= \arcsin(e^x) + C$$

f. $\int \frac{1}{x^2+4x+5}dx$

$$\int \frac{1}{x^2+4x+4+1}dx = \int \frac{1}{(x+2)^2+1}dx$$

$u = x+2$
$du = dx$

$$\int \frac{du}{u^2+1} = \arctan(u) + C$$

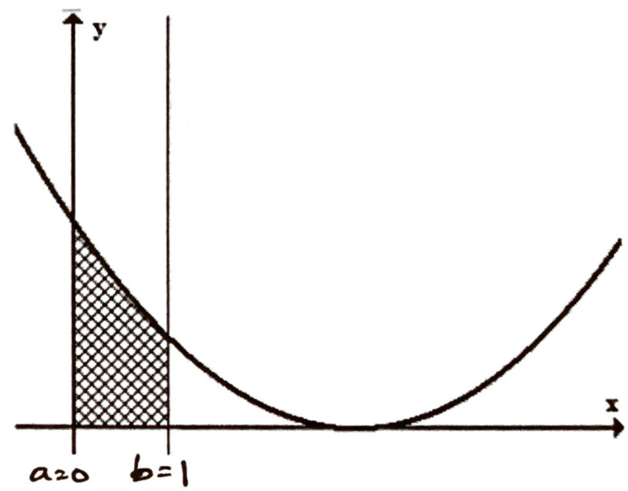
$$= \arctan(x+2) + C$$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$	$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

13. The region **R** is the first quadrant region beneath $y=(x-3)^2$ given $0 \le x \le 1$

(a) Write a Riemann sum approximating the area of the indicated region by dividing the interval of integration into **n** equal parts, and evaluating the function at the right endpoints of the subintervals.

(b) Using the expression obtained in part (a), let $\mathbf{n} \to \infty$, and determine a numerical value for the integral.



$R_n = \sum_{i=1}^{n}$ height * width

width $= \Delta x = \frac{b-a}{n} = \frac{1}{n}$

height $= f(x_i) = (x_i - 3)^2$

$x_i = a + \frac{b-a}{n} i = \frac{i}{n}$

$R_n = \sum_{i=1}^{n} \left(\frac{i}{n} - 3\right)^2 \left(\frac{1}{n}\right)$

$$R_n = \sum_{i=1}^{n} \left(\frac{i^2}{n^2} - \frac{6i}{n} + 9\right)\left(\frac{1}{n}\right) = \frac{1}{n^3}\sum_{i=1}^{n} i^2 - \frac{6}{n^2}\sum_{i=1}^{n} i + \frac{9}{n}\sum_{i=1}^{n} 1$$

$$R_n = \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] - \frac{6}{n^2}\left(\frac{n(n+1)}{2}\right) + \left(\frac{9}{n}\right)(n)$$

$$\text{area} = \lim_{n\to\infty} R_n = \frac{2}{6} - \frac{6}{2} + 9 = \frac{19}{3}$$

$$\text{check: } \int_0^1 (x-3)^2 dx = \frac{1}{3}(x-3)^3 \Big|_0^1 = \frac{1}{3}\left[-8-(-27)\right] = \frac{19}{3} \quad \checkmark$$