MA1021	INTERMEDIATE EXAM # 2	A'16

Section A06 NAME ______________________

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Point Total 100 (105 with the *bonus*)

1. (12 pts) Using the definition, find the derivative of the function $f(x) = \sqrt{4x+1}$.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{4(x+h)+1}-\sqrt{4x+1}}{h} = \lim_{h\to 0}\frac{\sqrt{4x+4h+1}-\sqrt{4x+1}}{h}\cdot\frac{\sqrt{4x+4h+1}+\sqrt{4x+1}}{\sqrt{4x+4h+1}+\sqrt{4x+1}}$$

$$= \lim_{h\to 0}\frac{4x+4h+1-(4x+1)}{h(\sqrt{4x+4h+1}+\sqrt{4x+1})} = \lim_{h\to 0}\frac{4h}{h(\sqrt{4x+4h+1}+\sqrt{4x+1})} = \lim_{h\to 0}\frac{4}{\sqrt{4x+4h+1}+\sqrt{4x+1}}$$

$$= \frac{4}{\sqrt{4x+1}+\sqrt{4x+1}} = \frac{4}{2\cdot\sqrt{4x+1}} = \frac{2}{\sqrt{4x+1}}.$$

2. Find the derivatives of the functions:

(a) (10 pts) $f(v) = [v^2(2\sqrt{v}+1)]$

$$f'(v) = \left(\frac{d}{dv}(v^2)\right)\cdot(2\sqrt{v}+1) + v^2\cdot\frac{d}{dv}(2\sqrt{v}+1) = 2v\cdot(2\sqrt{v}+1) + v^2\cdot\left(2\cdot\frac{1}{2\sqrt{v}}+0\right)$$

$$= 4v\cdot v^{\frac{1}{2}} + 2v + v^2\cdot v^{-\frac{1}{2}} = 4v^{\frac{3}{2}} + 2v + v^{\frac{3}{2}} = 5v^{\frac{3}{2}} + 2v.$$

(b) (10 pts) $f(x) = \dfrac{x^2+3x+4}{x^2-1}$

$$f'(x) = \frac{(x^2-1)\cdot(x^2+3x+4)' - (x^2+3x+4)\cdot(x^2-1)'}{(x^2-1)^2} = \frac{(x^2-1)(2x+3)-(x^2+3x+4)\cdot 2x}{(x^2-1)^2}$$

$$= \frac{2x^3+3x^2-2x-3-(2x^3+6x^2+8x)}{(x^2-1)^2} = \frac{-3x^2-10x-3}{(x^2-1)^2}.$$

(c) (10 pts) $f(\theta) = \sin\theta - \cos\theta - \theta\sin\theta - \theta\cos\theta$

$$f'(\theta) = \cos\theta - (-\sin\theta) - [\theta\cdot(\sin\theta)' + 1\cdot\sin\theta] - [\theta\cdot(\cos\theta)' + 1\cdot\cos\theta]$$

$$= \cos\theta + \sin\theta - (\theta\cdot\cos\theta + \sin\theta) - [\theta\cdot(-\sin\theta) + \cos\theta]$$

$$= \cos\theta + \sin\theta - \theta\cos\theta - \sin\theta + \theta\sin\theta - \cos\theta = \theta(\sin\theta - \cos\theta)$$

(d) (10 pts) $f(t) = \left(\dfrac{5t^2}{3t^2+2}\right)^3$

$$f'(t) = 3\cdot\left(\frac{5t^2}{3t^2+2}\right)^2\cdot\frac{d}{dt}\left(\frac{5t^2}{3t^2+2}\right) = 3\cdot\left(\frac{5t^2}{3t^2+2}\right)^2\cdot\frac{(3t^2+2)\cdot(5t^2)' - 5t^2\cdot(3t^2+2)'}{(3t^2+2)^2}$$

$$= 3\cdot\frac{25t^4}{(3t^2+2)^2}\cdot\frac{(3t^2+2)\cdot 10t - 5t^2\cdot 6t}{(3t^2+2)^2} = 3\cdot\frac{25t^4}{(3t^2+2)^2}\cdot\frac{20t}{(3t^2+2)^2} = \frac{1500t^5}{(3t^2+2)^4}.$$

(e) (10 pts) $f(z) = 5^{\sin z}$

$$f'(z) = 5^{\sin z} \cdot \ln 5 \cdot \frac{d}{dz}(\sin z) = 5^{\sin z} \cdot \ln 5 \cdot \cos z$$

(f) (10 pts) $f(x) = \frac{\ln x^2}{x^2}$

$$f'(x) = \frac{x^2 \cdot (\ln x^2)' - (\ln x^2)\cdot(x^2)'}{(x^2)^2} = \frac{x^2 \cdot (\frac{1}{x^2} \cdot 2x) - (\ln x^2) \cdot 2x}{x^4} = \frac{2x - 2x \ln x^2}{x^4} = \frac{2(1-\ln x^2)}{x^3}.$$

3. (14 pts) Find y' when $\sin xy = x^2 + y$.

$$\frac{d}{dx}(\sin xy) = \frac{d}{dx}(x^2+y)$$

$$\Rightarrow (\cos xy) \cdot \frac{d}{dx}(xy) = 2x + y'$$

$$\Rightarrow (\cos xy) \cdot (x \cdot y' + y) = 2x + y'$$

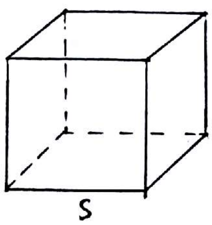
$$\Rightarrow (x \cos xy) \cdot y' + y \cos xy = 2x + y'$$

$$\Rightarrow (x \cos xy) y' - y' = 2x - y \cos xy$$

$$\Rightarrow (x \cos xy - 1) y' = 2x - y \cos xy$$

$$\Rightarrow y' = \frac{2x - y\cos xy}{x \cos xy - 1}.$$

4. (14 pts) A sample of a new polymer is of the shape of a cube. It is subjected to heat so that its expansion properties may be studied. The surface area of the cube is increasing at the rate of 6 in^2/min. How fast is the volume increasing at the moment when the surface area is 150 in^2?



Surface Area: $A = 6s^2$

Volume: $V = s^3$.

We know that $\frac{dA}{dt} = 6$,

and $A = 150$ at that moment.

So, $\frac{dA}{dt} = 12s \cdot \frac{ds}{dt} = 6$, (1)

$A = 150 = 6s^2$, (2)

From (1), $\frac{ds}{dt} = \frac{6}{12s} = \frac{1}{2s}$.

From (2), $s = \sqrt{\frac{150}{6}} = \sqrt{25} = 5$.

Hence, $\frac{ds}{dt} = \frac{1}{2s} = \frac{1}{2\times5} = \frac{1}{10}$ at that moment.

Then, $\frac{dV}{dt} = 3s^2 \cdot \frac{ds}{dt} = 3 \cdot 5^2 \cdot \frac{1}{10} = 3 \cdot 25 \cdot \frac{1}{10} = 7.5$

Therefore, the volume is increasing at the rate of 7.5 in^2/min at that moment.

5. (*5 bonus pts*) **(Do not address this until you have solved all the problems above!)** Find the linear approximation to $f(x) = \sin x$ at $x = 0$ and use it to approximate $\sin 2.5°$. (Note that $2.5° \sim 0.04363$ rad.)

$f'(x) = \cos x \Rightarrow f'(0) = \cos 0 = 1$.

$f(0) = \sin 0 = 0$

The linear approximation is: $L(x) = f(0) + f'(0)(x-0) = 0 + 1\cdot(x-0) = x$.

Then, $\sin 2.5° \approx L(0.04363) = 0.04363$.