MA 1021 A '15 **Final Exam** **Name** ANSWER KEY

This test is closed book. Notes are not allowed. Calculators are not allowed.

Instructions

Part I - Basic Skills

Problem	1	2	3	4	5	6	7	Total
Value	5	5	5	5	5	5	5	35
Earned								

Part II

Problem	8		9	10	11	12			13	14		Total
Value	6	6	12	12	12	10	10	8	12	6	6	100
Earned												

Please Circle your Section

A01 Muddamallappa, M (8:00)   A02 Farr, W (9:00)   A03 Broker, R (9:00)

A04 Malone, JJ (10:00)   A05 Lui, R (10:00)   A06 Abraham, J (1:00)

A07 Posterro, B (1:00)   A08 Lui, R (2:00)   A09 Muddamallappa, M (3:00)

Part I - Basic Skills

Work the following problems and write your answers in the space provided. Use the scratch paper provided for your work. You need not simplify your answers. No partial credit will be given for these problems. Work carefully and check your work.

1. Find $\frac{dy}{dx}$ if $y=\frac{7}{x^6}-\sqrt[3]{x^5}+e^3$ Ans. $\frac{-42}{x^7}-\frac{5}{3}x^{2/3}$

2. Find $\frac{dy}{dx}$ if $y=x^6\cos(x)$ Ans. $(6x^5)[\cos(x)]+(x^6)[-\sin(x)]$

3. Find $\frac{dy}{dx}$ if $y=\frac{x^2+5}{\sin(x)}$ Ans. $\frac{(2x)[\sin(x)]-(x^2+5)[\cos(x)]}{\sin^2(x)}$

4. Find $\frac{dy}{dx}$ if $y=\tan^4(x)$ Ans. $4\tan^3(x)\ \sec^2(x)$

5. Find $\frac{dy}{dx}$ if $y=\ln(x^4+8x^2+5)$ Ans. $\frac{4x^3+16x}{x^4+8x^2+5}$

6. Find $\frac{dy}{dx}$ if $y=e^{\sin(3x)}$ Ans. $\left[e^{\sin(3x)}\right]\left[3\cos(3x)\right]$

7. Find an equation for the tangent line to the curve $y=x^2-7x+16$ at the point (5, 6)

Ans. $y-6=3(x-5)$

OR

$y=3x-9$

Part II

Work all of the following problems. Show your work in the space provided. You need not simplify your answers, but remember that on this part of the exam your work and your explanations are graded, not just the final answers

8. Evaluate each limit or show it does not exist. You must show your work to receive full credit. **Important note**: No credit will be given for using L'Hôpital's Rule to solve these problems.

a. $\lim_{x \to -1} \frac{x^2 + 2x + 1}{x^2 - 1}$

$$= \lim_{x \to -1} \frac{(x+1)(x+1)}{(x+1)(x-1)} = \lim_{x \to -1} \frac{x+1}{x-1} = 0$$

b. $\lim_{x \to 0} \frac{\tan(2x)}{x}$

$$= \lim_{x \to 0} \frac{\sin(2x)}{x \cos 2x} = \lim_{x \to 0} \frac{\sin(2x)}{2x} \frac{2}{\cos(2x)} = 2$$

9. For $f(x) = x^2 + 4$, find $f'(x)$ by using the limit definition of the derivative.

Note that since we know from the Power Rule that $f'(x) = 2x$, no credit will be given for answers that do not use the limit definition of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{[(x+h)^2 + 4] - [x^2 + 4]}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 4 - x^2 - 4}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} (2x + h) = 2x$$

10. Given the equation $xy = e^{-xy}$, use implicit differentiation to find the derivative $\dfrac{dy}{dx}$.

$$(1)(y) + xy' = e^{-xy}\left[-y - xy'\right]$$

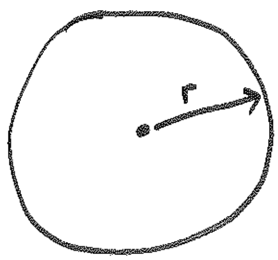
$$xy' + xe^{-xy}y' = -ye^{-xy} - y$$

$$y'\left[x + xe^{-xy}\right] = -\left[y + ye^{-xy}\right]$$

$$y' = \frac{-y\left[1+e^{-xy}\right]}{x\left[1+e^{-xy}\right]} = -\frac{y}{x}$$

11. A pebble is dropped into a still pond, causing a circular ripple to expand outward from the point where the pebble fell in. At the instant when the radius of this circular ripple is $\frac{10}{\pi}$ inches, the radius is growing at the rate of 3 inches per second.

How fast is the area of the circle enclosed by the ripple growing at this instant?



$\frac{dr}{dt} = \frac{10}{\pi}$

$r = 3$ now

$A = \pi r^2$ $\quad$ $A = 9\pi$ now

Turns out not to be important

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = (2\pi)(3)\left(\frac{10}{\pi}\right) = 60 \text{ inches}^2/\text{second}$$

12. [a] You are given the following information about a function $f(x)$

$$y = x^5 - 5x^4 - 5x^3 + 45x^2$$

$$y' = 5x(x+2)(x-3)^2$$

$$y'' = 10(x-3)(2x^2-3)$$

Describe all intervals on which $f(x)$ is increasing or decreasing. Find any critical points (x-coordinates only), and indicate whether they correspond to local (relative) or absolute (global) extrema.

Be sure to support your answers.

Critical points occur when $y' = 0$ or DNE

$x = -2, 0,$ or 3

interval	$5x(x+2)(x-3)(x-3)$				$f'(x)$		
$-\infty, -2$	$-$	$-$	$-$	$-$	$+$	inc	$\nearrow$
$-2, 0$	$-$	$+$	$-$	$-$	$-$	dec	$\searrow$
$0, 3$	$+$	$+$	$-$	$-$	$+$	inc	$\nearrow$
$3, +\infty$	$+$	$+$	$+$	$+$	$+$	inc	$\nearrow$

$f(x)$ increases on $(-\infty, -2) \cup (0, 3) \cup (3, \infty)$

$f(x)$ decreases on $(-2, 0)$

$x = -2$ is a local max

$x = 0$ is a local min

We know these are local because $\lim_{x \to \infty} f(x) = \infty$ & $\lim_{x \to -\infty} f(x) = -\infty$

[b] You are given the following information about a function $g(x)$

$$g(x) = x^4 - 3x^3 + 3x^2 - x$$

$$g'(x) = 4x^3 - 9x^2 + 6x - 1$$

$$g''(x) = 6(2x-1)(x-1)$$

Describe all intervals on which $g(x)$ is concave up or concave down. Identify any inflection points (x-coordinates only). **<u>Be sure to support your answers</u>**.

$g''(x) = 0$ when $x = \frac{1}{2}, 1$

interval	$6(2x-1)(x-1)$		$g''(x)$
$-\infty, \frac{1}{2}$	$-$	$-$	$+$
$\frac{1}{2}, 1$	$+$	$-$	$-$
$1, \infty$	$+$	$+$	$+$

$g(x)$ has inflection points at $x = \frac{1}{2}$ and $x = 1$

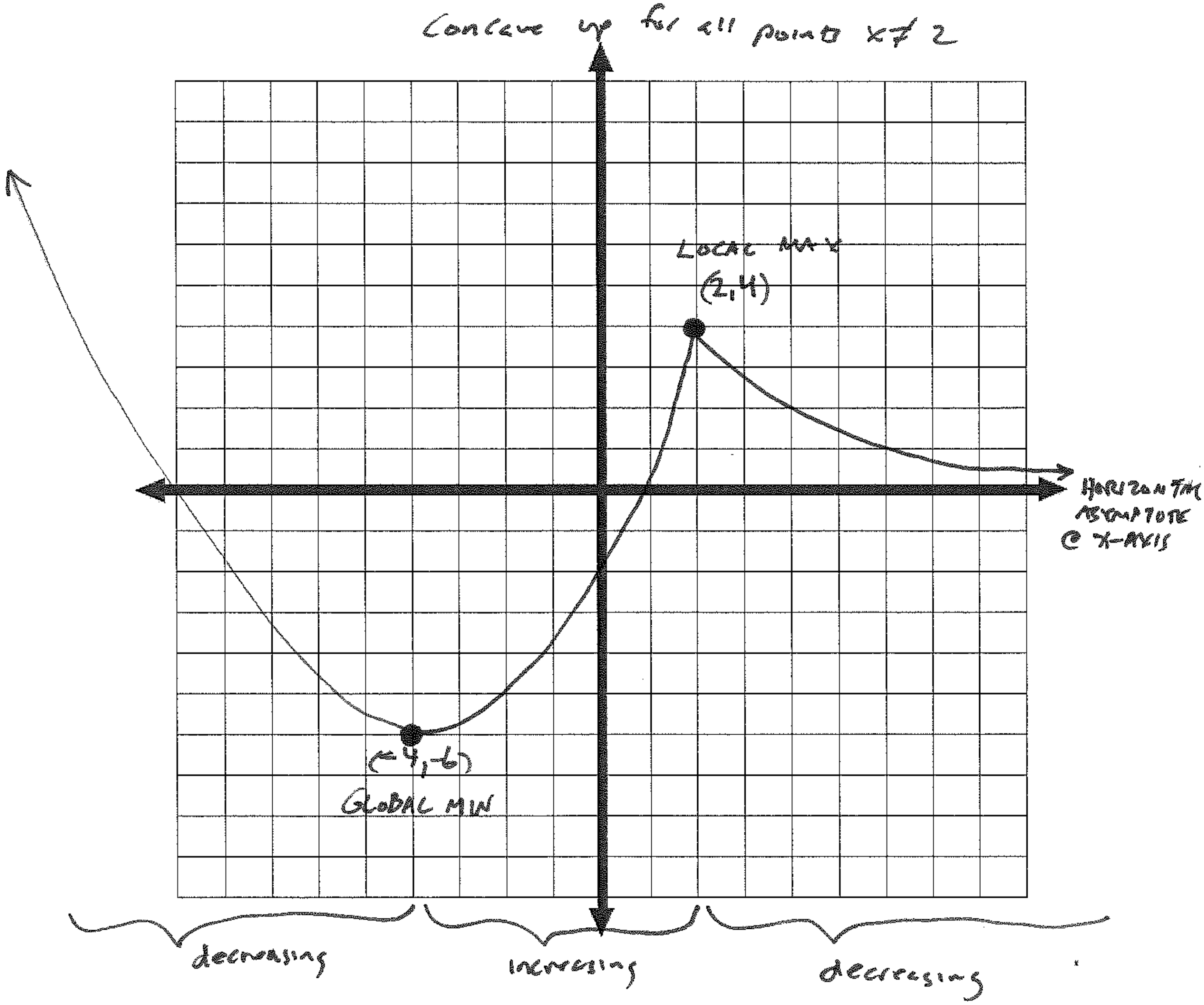
$g(x)$ is concave up on $(-\infty, \frac{1}{2}) \cup (1, \infty)$

$g(x)$ is concave down on $(\frac{1}{2}, 1)$

[c] Draw a graph of a continuous function $h(x)$ that meets the following conditions

$h(-4)=-6$ and $h(2)=4$	$h''(x)>0$ for $x\neq 2$
$h'(x)>0$ on $(-4,2)$	$\lim\limits_{x\to\infty} h(x)=0$
$h'(x)<0$ on $(-\infty,-4)\cup(2,+\infty)$	

Note that there are many possible graphs that would meet these conditions; you are only being asked to sketch one such graph.



13. Find two non-negative numbers x and y whose sum is 9 and such that xy^3 is maximized.

$x+y=9$

$S=xy^3$

$S(x)=x(9-x)^3$

$0 \le x \le 9$

$S'(x) = (9-x)^3 + 3x(9-x)^2(-1)$

$= (9-x)^2[9-x-3x]$

critical points $x=\frac{9}{4}$ or $x=9$

x	$S(x)$
0	0
9/4	$\frac{177,147}{256}$
9	0

Wasn't necessary to calculate this

$\boxed{x=\frac{9}{4}, \; y=\frac{27}{4}}$

$x+y=9$

$S=xy^3$

$S(y)=(y-9)y^3$

$0 \le y \le 9$

$S'(y) = 4y^3 - 27y^2$

$= y^2(4y-27)$

critical points $y=0$ or $y=\frac{27}{4}$

y	$S(y)$
0	0
$\frac{27}{4}$	$\frac{177,147}{256}$
9	0

$\boxed{x=\frac{9}{4}, \; y=\frac{27}{4}}$

14. For each of the following, find $f'(x)$. You **must** show your work to receive full credit.

a. $f(x) = x^{(4x^2)} + \arctan(3x) + x - \sec(2x) + 7$

$$y = x^{(4x^2)}$$
$$\ln y = 4x^2 \ln x$$
$$\frac{1}{y}\frac{dy}{dx} = 8x\ln x + \frac{4x^2}{x}$$
$$\frac{dy}{dx} = x^{(4x^2)}[8x\ln x + 4x]$$

$$f'(x) = (4x)(x^{(4x^2)})[2\ln x + 1] + \frac{3}{1+9x^2} + 1 - 2\sec(2x)\tan(2x)$$

b. $f(x) = \cos^5\left[3\tan(x^2)\right] - 3$

$$f'(x) = 5\cos^4[3\tan(x^2)]\,[3\sec^2(x^2)](2x)[-\sin(3\tan(x^2))]$$
$$= 30x\cos^4[3\tan(x^2)]\cdot[\sec^2(x^2)][-\sin(3\tan(x^2))]$$