MA 1021 A '14 **Final Exam** **Name** ______________________

This test is closed book. Calculators are not allowed.

ANSWER KEY

Instructions

Part I - Basic Skills

Problem	1	2	3	4	5	6	7	Total
Value	5	5	5	5	5	5	5	35
Earned								

Part II

Problem	8		9	10	11	12			13	14		Total
Value	6	6	12	12	12	10	10	8	12	6	6	100
Earned												

Please Circle your Section

A01 Berezovski, M (8:00)

A02 Berezovski, M (9:00)

A03 Doytchinova, T. (9:00)

A04 Malone, JJ (10:00)

A05 Doytchinova, T. (10:00)

A06 Abraham, J (1:00)

A07 Blais, M (1:00)

A08 Farr, W. (2:00)

A09 Broker, R. (3:00)

A10 Sarkis-Martins, M. (2:00)

Part I - Basic Skills

Work the following problems and write your answers in the space provided. Use the scratch paper provided for your work. You need not simplify your answers. No partial credit will be given for these problems. Work carefully and check your work.

1. Find $\dfrac{dy}{dx}$ if $y = x^3 + \sqrt[3]{x^7} - \dfrac{2}{x^3} + \pi^2$ Ans. $3x^2 + \frac{7}{3}x^{4/3} + 6x^{-4}$

2. Find $\dfrac{dy}{dx}$ if $y = x^7 \sec(x)$ Ans. $(7x^6)[\sec(x)] + (x^7)(\sec(x)\tan(x))$

3. Find $\dfrac{dy}{dx}$ if $y = \dfrac{x^3 - e^x}{\tan(x)}$ Ans. $\dfrac{(3x^2 - e^x)(\tan(x)) - (x^3 - e^x)\sec^2(x)}{\tan^2(x)}$

4. Find $\dfrac{dy}{dx}$ if $y = \cos^5(3x^2)$ Ans. $-5[\cos^4(3x^2)][\sin(3x^2)](6x)$

5. Find $\dfrac{dy}{dx}$ if $y = \ln(x^3 - 17x + 4)$ Ans. $\dfrac{3x^2 - 17}{x^3 - 17x + 4}$

6. Find $\dfrac{dy}{dx}$ if $y = (x^2 + 2x)^{2/3}$ Ans. $\frac{2}{3}(x^2 + 2x)^{-1/3}(2x + 2)$

7. Find an equation for the tangent line to the curve $y = x^3 - 3x - 9$ at the point (3,9)

$\dfrac{dy}{dx} = 3x^2 - 3$

@ $x = 3$, $\dfrac{dy}{dx} = 24$

Ans. $y - 9 = 24(x - 3)$

OR

$y = 24x - 63$

Part II

Work all of the following problems. Show your work in the space provided. You need not simplify your answers, but remember that on this part of the exam your work and your explanations are graded, not just the final answers

8. Evaluate each limit or show it does not exist. You must show your work to receive full credit. **Important note**: No credit will be given for using L'Hôpital's Rule to solve these problems.

a. $\lim_{x \to 2} \frac{x^2 - 4}{x^2 - x - 2}$

$$= \lim_{x \to 2} \frac{(x+2)(x-2)}{(x+1)(x-2)} = \lim_{x \to 2} \frac{x+2}{x+1} = \frac{4}{3}$$

b. $\lim_{x \to 5} \left(\frac{\sqrt{x+4} - 3}{x - 5} \right) \left[\frac{\sqrt{x+4} + 3}{\sqrt{x+4} + 3} \right]$

$$= \lim_{x \to 5} \left(\frac{x+4-9}{x-5} \right) \left(\frac{1}{\sqrt{x+4} + 3} \right)$$

$$= \lim_{x \to 5} \frac{1}{\sqrt{x+4} + 3} = \frac{1}{6}$$

9. For $f(x)=\dfrac{1}{x-3}$, find $f'(x)$ by using the limit definition of the derivative.

Note that since we know from the Power Rule that $f'(x)=\dfrac{-1}{(x-3)^2}$, *no credit will be given for answers that do not use the limit definition of the derivative.*

① $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

General definition of a derivative

② $$f'(x)=\lim_{h\to 0}\frac{\frac{1}{x+h-3}-\frac{1}{x-3}}{h}$$

Apply definition to the particular function to be evaluated

③ $$f'(x)=\lim_{h\to 0}\frac{\frac{(x-3)-(x+h-3)}{(x+h-3)(x-3)}}{h}=\lim_{h\to 0}\frac{\frac{-h}{(x+h-3)(x-3)}}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{-1}{(x+h-3)(x-3)}$$

Do algebraic manipulation to get the expression to a place where you can evaluate the limit

④ $$f'(x)=\frac{-1}{(x-3)^2}$$

Evaluate the limit

10. Given the equation $2x^2 - xy^2 - y^3 + e^{2y} + (1-2y)e^2 = 0$, use implicit differentiation to find an equation of the tangent line at the point $(1,1)$.

$$4x - y^2 - 2xyy' - 3y^2y' + 2y'e^{2y} - 2y'e^2 = 0$$

$$y'(-2xy - 3y^2 + 2e^{2y} - 2e^2) = -4x + y^2$$

$$y' = \frac{-4x+y^2}{-2xy - 3y^2 + 2e^{2y} - 2e^2}$$

@ $x=1,\ y=1$

$$y' = \frac{-4+1}{-2-3+2e^2-2e^2} = \frac{3}{5}$$

$$y - 1 = \frac{3}{5}(x-1)$$

OR

$$y = \frac{3}{5}x + \frac{2}{5}$$

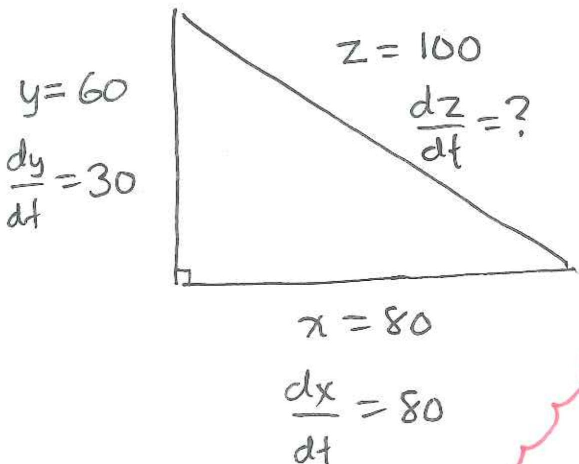
11. A car leaves the city at 1:00 pm, traveling north at 30 mph. A second car leaves the city at 2:00 pm, traveling east at 80 mph.

a. How far apart are the two cars at 3:00 pm?

At 3:00 pm, the first car is 60 miles north of the city (two hrs at 30 mph), and the second car is 80 miles east of the city – so they are 100 miles apart

b. How fast is the distance between the cars changing at 3:00 pm?

Draw a picture to illustrate



Find an equation to link the variables which are changing

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{x}{z}\frac{dx}{dt} + \frac{y}{z}\frac{dy}{dt}$$

$$= \left(\frac{80}{100}\right)(80) + \left(\frac{60}{100}\right)(30)$$

$$\frac{dz}{dt} = 82 \text{ mph}$$

12. This is the <u>first</u> part of a three-part question

[a] You are given the following information about a function $f(x)$

$$f(x) = 3x^4 - 16x^3 - 66x^2 + 360x + 5$$
$$f'(x) = 12(x-2)(x+3)(x-5)$$
$$f''(x) = 12(x+1)(3x-11)$$

Describe all intervals on which $f(x)$ is increasing or decreasing. Identify any maxima or minima (x-coordinates only), and indicate whether they are local or global extrema. **Be sure to support your answers**.

Use the First Derivative test, and $f'(x)$

Critical points are $x = 2$, $x = -3$, $x = 5$

interval	$f'(x) = 12(x-2)(x+3)(x-5)$	$f'(x)$	function
$(-\infty, -3)$	$-\ \ -\ \ -$	$-$	decreasing
$(-3, 2)$	$-\ \ +\ \ -$	$+$	increasing
$(2, 5)$	$+\ \ +\ \ -$	$-$	decreasing
$(5, +\infty)$	$+\ \ +\ \ +$	$+$	increasing

$f(x)$ increases on $(-3, 2) \cup (5, +\infty)$

$f(x)$ decreases on $(-\infty, -3) \cup (2, 5)$

$x = -3$ corresponds to a local minimum

$x = 2$ corresponds to a local maximum

$x = 5$ corresponds to a local minimum

12. This is the <u>second</u> part of a three-part question

[b] You are given the following information about a function $g(x)$

$$g(x) = \frac{1}{3}x^2\left(2x^4 - 12x^3 - 15x^2 + 140x - 240\right)$$

$$g'(x) = 4x(x^4 - 5x^3 - 5x^2 + 35x - 40)$$

$$g''(x) = 20(x-1)^2(x+2)(x-4)$$

Describe all intervals on which $g(x)$ is concave up or concave down. Identify any inflection points (x-coordinates only). **Be sure to support your answers**.

Use the Second Derivative Test and $g''(x)$

$g''(x) = 0$ when $x = -2$, $x = 1$, or $x = 4$

interval	$20(x-1)^2$	$(x+2)$	$(x-4)$	$g''(x)$	$g(x)$
$(-\infty, -2)$	+	−	−	+	concave up
$(-2, 1)$	+	+	−	−	concave down
$(1, 4)$	+	+	−	−	concave down
$(4, +\infty)$	+	+	+	+	concave up

$g(x)$ is concave up on $(-\infty, -2) \cup (4, +\infty)$

$g(x)$ is concave down on $(-2, 1) \cup (1, 4)$

$x = -2$ and $x = 4$ are inflection points because the concavity of $g(x)$ changes at these points

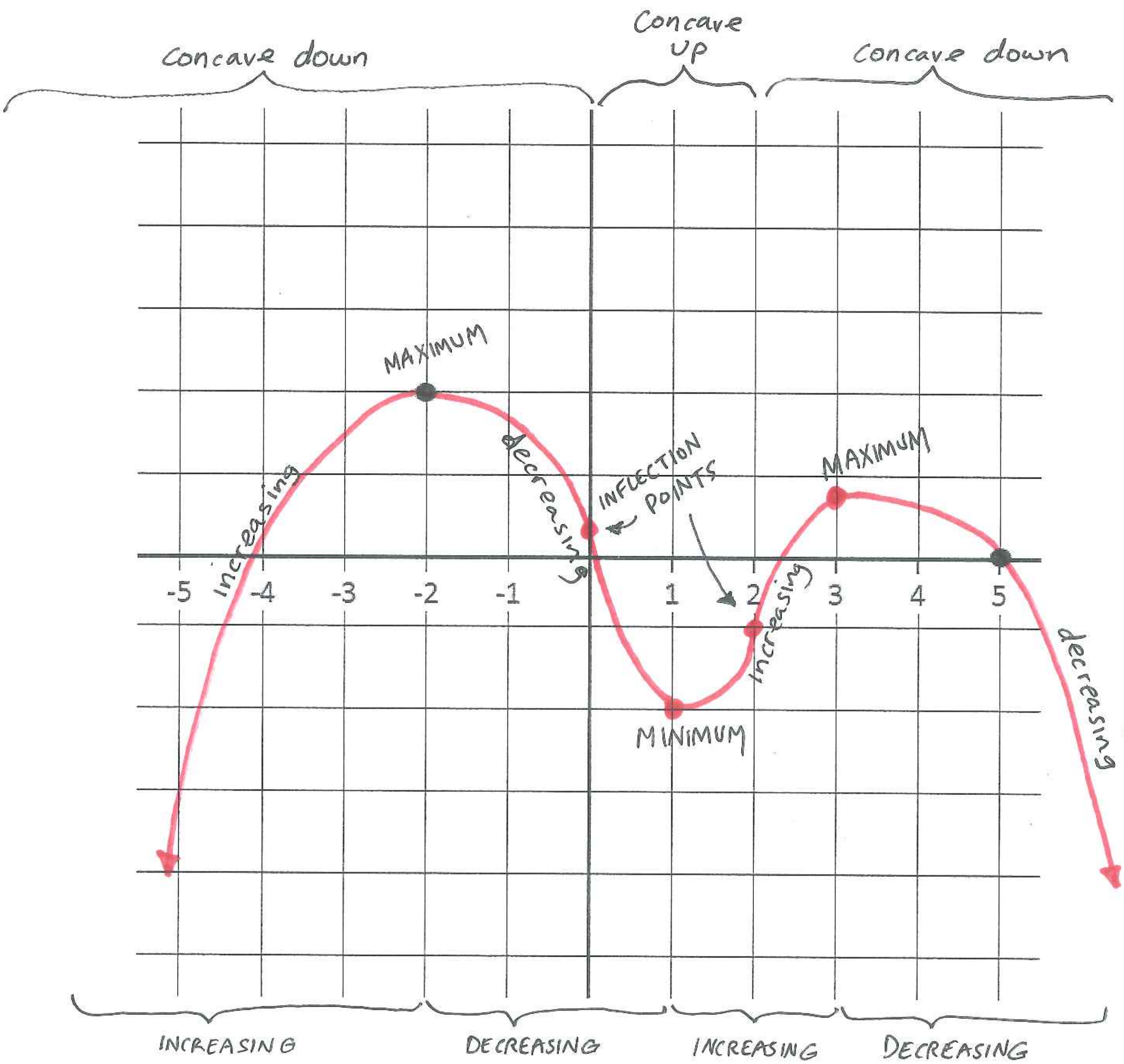
$x = 1$ is <u>not</u> an inflection point because the concavity does not change there

12. This is the third part of a three-part question

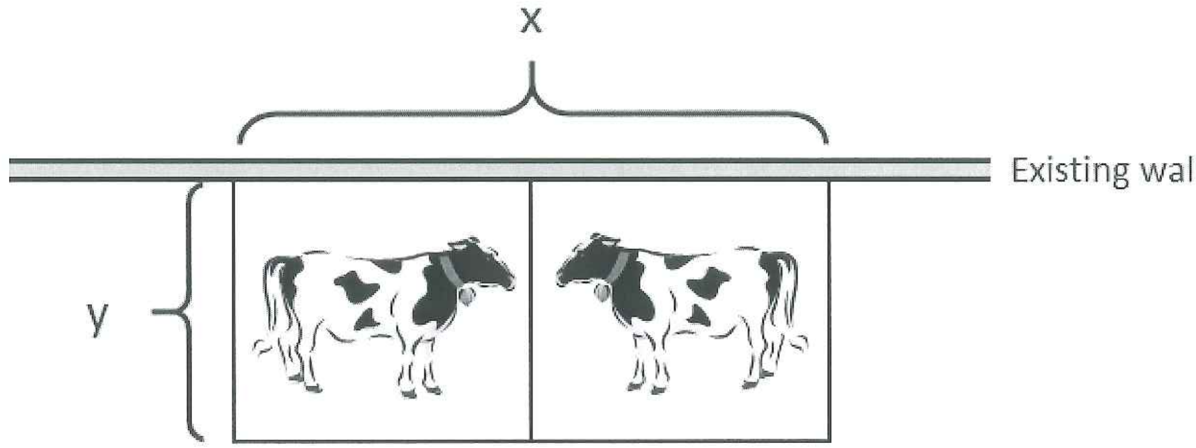
[c] Draw a graph of a function $h(x)$ that passes through the points (-2, 2) and (5,0) and meets the following requirements.

Note that there are many possible graphs that would meet these conditions; you are only being asked to sketch one such graph.

$h'(x) > 0$ over $(-\infty, -2) \cup (1,3)$	$h''(x) < 0$ over $(-\infty, 0) \cup (2,5)$
$h'(x) < 0$ over $(-2,1) \cup (3,+\infty)$	$h''(x) > 0$ over $(0,2)$
$h'(-2) = h'(1) = h'(3) = 0$	$h''(0) = h''(2) = 0$



13. You are building two equal-sized rectangular pens for your prize cows. You have 60 feet of fencing material, and you can use an existing long wall, as shown below. Letting "x" be the amount of the existing wall that you use, find the largest combined area of the two pens.



$x+3y=60$

$y=20-\frac{1}{3}x$

Area $= xy$

$A(x) = x\left(20-\frac{1}{3}x\right) \qquad 0 \le x \le 60$

$A'(x) = 20 - \frac{2}{3}x$

$A'(x) = 0$ when $x = 30$

Minimum and Maximum of $A(x)$ occurs at endpoints or critical pts

x	$A(x)$
0	0
30	300
60	0

Maximum area (combined) of pens occurs when $x=30$ and $y=10$

Area $= 300 \text{ ft}^2$

14. For each of the following, find $f'(x)$. You **must** show your work to receive full credit.

a. $f(x)=e^{-x}+x^{-e}+[\sin(x)]^{x}+x^{2}$

$$y=[\sin(x)]^{x}$$
$$\ln y = x\ln[\sin(x)]$$
$$\frac{1}{y}\frac{dy}{dx}=\ln[\sin(x)]+x\frac{\cos(x)}{\sin(x)}$$
$$\frac{dy}{dx}=[\ln[\sin(x)]+x\cot(x)][\sin(x)]^{x}$$

do this part first for the third term in f(x)

$$f'(x)=-e^{-x}-ex^{-e-1}+[\ln[\sin(x)]+x\cot(x)][\sin(x)]^{x}+2x$$

b. $f(x)=\arctan(\sec(3x^{2}))$

$$\frac{dy}{dx}=\left[\frac{1}{1+\sec^{2}(3x^{2})}\right]\left(\sec(3x^{2})\tan(3x^{2})\right)(6x)$$

****** *End of exam* ******