

This test is closed book. Calculators are not allowed.

**Instructions**

**Part I - Basic Skills**

Problem	1	2	3	4	5	6	7	Total
Value	5	5	5	5	5	5	5	35
Earned								

**Part II**

Problem	8		9	10	11	12			13	14		Total
Value	6	6	12	12	12	10	10	8	12	6	6	100
Earned												

**Please Circle your Section**

A01 Berezovski, M (8:00)

A02 Berezovski, M (9:00)

A03 Doytchinova, T. (9:00)

A04 Malone, JJ (10:00)

A05 Doytchinova, T. (10:00)

A06 Abraham, J (1:00)

A07 Blais, M (1:00)

A08 Farr, W. (2:00)

A09 Broker, R. (3:00)

A10 Sarkis-Martins, M. (2:00)

## Part I - Basic Skills

Work the following problems and write your answers in the space provided. Use the scratch paper provided for your work. You need not simplify your answers. No partial credit will be given for these problems. Work carefully and check your work.

1. Find  $\frac{dy}{dx}$  if  $y = x^3 + \sqrt[3]{x^7} - \frac{2}{x^3} + \pi^2$       Ans. \_\_\_\_\_

2. Find  $\frac{dy}{dx}$  if  $y = x^7 \sec(x)$       Ans. \_\_\_\_\_

3. Find  $\frac{dy}{dx}$  if  $y = \frac{x^3 - e^x}{\tan(x)}$       Ans. \_\_\_\_\_

4. Find  $\frac{dy}{dx}$  if  $y = \cos^5(3x^2)$       Ans. \_\_\_\_\_

5. Find  $\frac{dy}{dx}$  if  $y = \ln(x^3 - 17x + 4)$       Ans. \_\_\_\_\_

6. Find  $\frac{dy}{dx}$  if  $y = (x^2 + 2x)^{2/3}$       Ans. \_\_\_\_\_

7. Find an equation for the tangent line to the curve  $y = x^3 - 3x - 9$  at the point (3,9)  
Ans. \_\_\_\_\_

## Part II

Work all of the following problems. Show your work in the space provided. You need not simplify your answers, but remember that on this part of the exam your work and your explanations are graded, not just the final answers

8. Evaluate each limit or show it does not exist. You must show your work to receive full credit. **Important note:** No credit will be given for using L'Hôpital's Rule to solve these problems.

a.  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - x - 2}$

b.  $\lim_{x \rightarrow 5} \frac{\sqrt{x+4} - 3}{x-5}$

9. For  $f(x) = \frac{1}{x-3}$ , find  $f'(x)$  by using the limit definition of the derivative.

*Note that since we know from the Power Rule that  $f'(x) = \frac{-1}{(x-3)^2}$ , no credit will be given for answers that do not use the limit definition of the derivative.*

10. Given the equation  $2x^2 - xy^2 - y^3 + e^{2y} + (1 - 2y)e^2 = 0$ , use implicit differentiation to find an equation of the tangent line at the point  $(1, 1)$ .

11. A car leaves the city at 1:00 pm, traveling north at 30 mph. A second car leaves the city at 2:00 pm, traveling east at 80 mph.
- How far apart are the two cars at 3:00 pm?
  - How fast is the distance between the cars changing at 3:00 pm?

12. This is the first part of a three-part question

[a] You are given the following information about a function  $f(x)$

$$f(x) = 3x^4 - 16x^3 - 66x^2 + 360x + 5$$

$$f'(x) = 12(x - 2)(x + 3)(x - 5)$$

$$f''(x) = 12(x + 1)(3x - 11)$$

Describe all intervals on which  $f(x)$  is increasing or decreasing. Identify any maxima or minima (x-coordinates only), and indicate whether they are local or global extrema. **Be sure to support your answers.**

12. This is the second part of a three-part question

[b] You are given the following information about a function  $g(x)$

$$g(x) = \frac{1}{3}x^2(2x^4 - 12x^3 - 15x^2 + 140x - 240)$$

$$g'(x) = 4x(x^4 - 5x^3 - 5x^2 + 35x - 40)$$

$$g''(x) = 20(x-1)^2(x+2)(x-4)$$

Describe all intervals on which  $g(x)$  is concave up or concave down.

Identify any inflection points (x-coordinates only). **Be sure to support your answers.**



12. This is the third part of a three-part question

[c] Draw a graph of a function  $h(x)$  that passes through the points  $(-2, 2)$  and  $(5,0)$  and meets the following requirements.

*Note that there are many possible graphs that would meet these conditions; you are only being asked to sketch one such graph.*

$$h'(x) > 0 \text{ over } (-\infty, -2) \cup (1, 3)$$

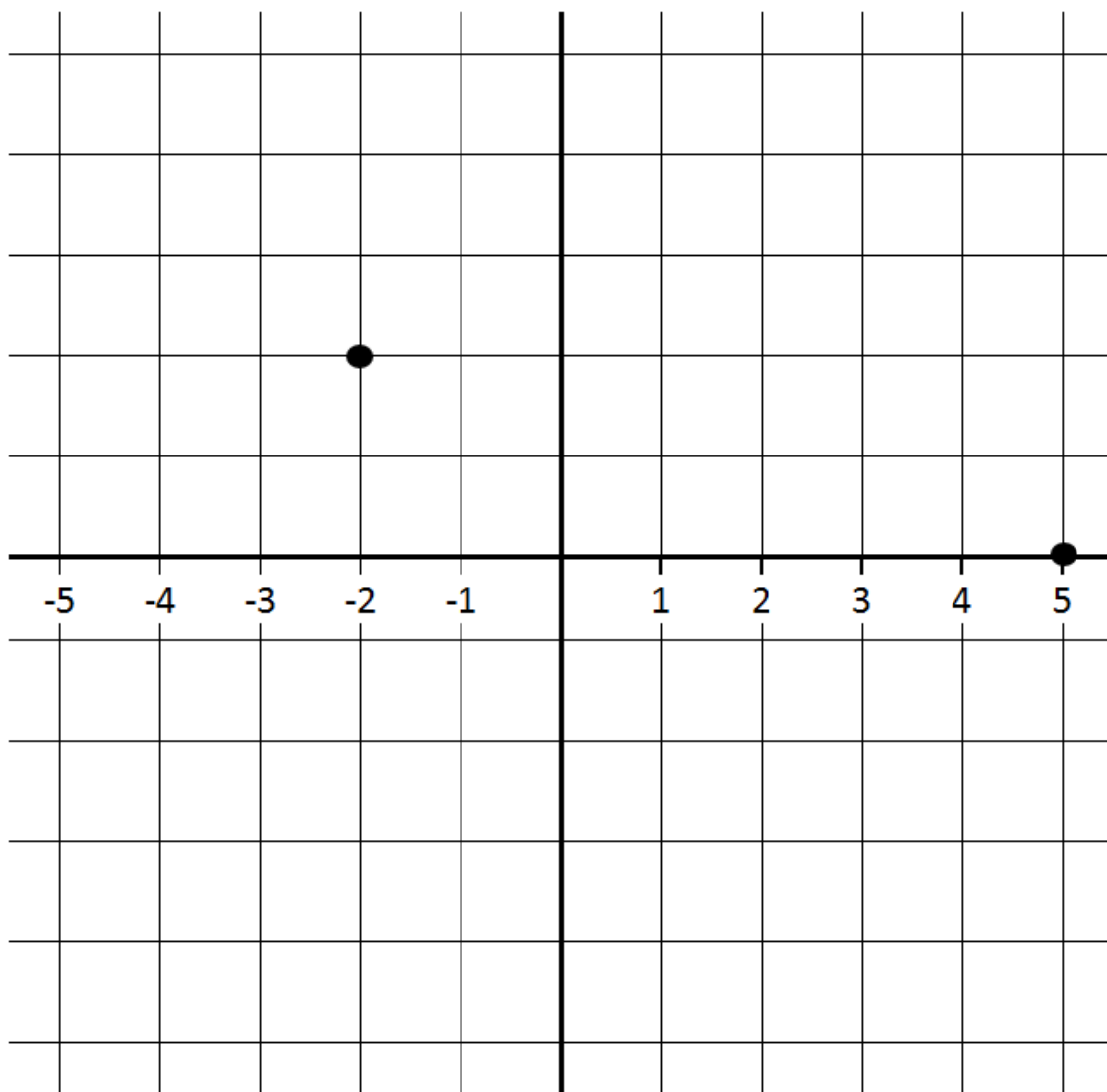
$$h'(x) < 0 \text{ over } (-2, 1) \cup (3, +\infty)$$

$$h'(-2) = h'(1) = h'(3) = 0$$

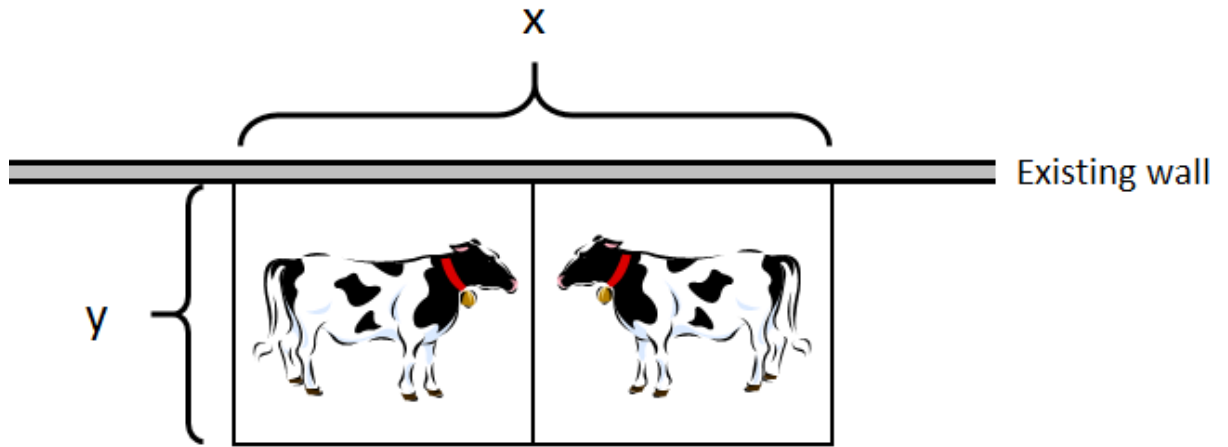
$$h''(x) < 0 \text{ over } (-\infty, 0) \cup (2, 5)$$

$$h''(x) > 0 \text{ over } (0, 2)$$

$$h''(0) = h''(2) = 0$$



13. You are building two equal-sized rectangular pens for your prize cows. You have 60 feet of fencing material, and you can use an existing long wall, as shown below. Letting “ $x$ ” be the amount of the existing wall that you use, find the largest combined area of the two pens.



14. For each of the following, find  $f'(x)$ . You **must** show your work to receive full credit.

a.  $f(x) = e^{-x} + x^{-e} + [\sin(x)]^x + x^2$

b.  $f(x) = \arctan(\sec(3x^2))$