Additional Class Notes

September 8, 2016

Continuity

Example 3. Where the function $f(z) = 1/(z^2 - 1)$ is continuous?

Solution: Our first observation here is the function is quotient of continuous functions (there are polynomials which are continuous in the numerator and denominator), then this function is continuous wherever its denominator is nonzero.

Therefore, $f(z)$ is continuous on its domain – which is the set of all real numbers other than ± 1.

Limits Involving Infinity

Example 4.

$$\lim_{x\to\infty}\frac{x^3-2x+1}{2x+4}=\lim_{x\to\infty}\frac{x^3/x-2x/x+1/x}{2x/x+4/x}=\lim_{x\to\infty}\frac{x^2-2+1/x}{2+4/x}=\infty$$

Example 5.

$$\lim_{x\to\infty}\frac{10x^3-3x^2+8}{\sqrt{25x^6+x^4+2}}=\lim_{x\to\infty}\frac{10x^3/x^3-3x^2/x^3+8/x^3}{\sqrt{25x^6/x^6+x^4/x^6+2/x^6}}=\lim_{x\to\infty}\frac{10-3/x+8/x^3}{\sqrt{25+1/x^2+2/x^6}}=\frac{10}{\sqrt{25}}=2$$

Example 6: $\lim_{x\to\infty}\ln x = ?$

We know that $y = \ln x$ is increasing function, and it grows infinitely large with x growing infinitely large. So, we can say:

$$\lim_{x\to\infty}\ln x=\infty$$

To fully convince ourselves that this answer is correct and eliminate the doubts whether the graph of $\ln x$ approaches a horizontal asymptote, or whether the functions grows without bound as $x\to\infty$, we can recall the inverse relation between e^x and $\ln x$. The domain of e^x $(-\infty, \infty)$ implies the range of $\ln x$ is also $(-\infty, \infty)$. So the limit is indeed ∞.

Example 7: $\lim_{x\to\infty}\cos x = ?$

The answer to this question is straightforward: the cosine function oscillates between -1 and 1 as x approaches ∞; therefore, $\lim_{x\text{->oo}} \cos x$ does not exist. (The same can be said about $\lim_{x\text{->-oo}} \cos x$, and about the limits of $\sin x$.)