# Interpolation Error

The discrepancy bet'n a function and its Lagrange Interpolating Polynomial.

**THM** $f : [a,b] \longrightarrow \mathbb{R}$ $n$ times differentiable

$x_1, \dots, x_n \in [a, b]$

$\deg(p(x)) \leqslant n-1$ such that $p(x_i) = f(x_i)$

(the Lagrange Interp. Poly'n)

Then for $x \in [a,b]$, $\exists\, c \in [a,b]$

(c depends on $x$)

such that

$$f(x) - p(x) = \frac{f^{(n)}(c(x))}{n!} \prod_{j=1}^{n} (x - x_j)$$

We won't do the proof.

**Notes**

- $n = 1$ then $p(x) =$ constant $f(x_1)$

  $f(x) - p(x) = f'(c)(x - x_1)$ for some $c$

  i.e. $f(x) - f(x_1) = f'(c)(x - x_1)$

  $\Rightarrow \dfrac{f(x) - f(x_1)}{x - x_1} = f'(c)$ This is MV Thm!

- For $x \in [a,b]$ and $x_i \in [a,b]$, $f(x) - P(x)$ is small for large $n$ (i.e. $n!$ huge), and $f^{(n)}$ moderate size, and $x$ close to all or many $x_j$

When we use the polynomial $P(x)$ purely for interpolation of a table of data, we don't know $f(x)$ so this error formula is not useful.

However, these interpolating polynomials and their error formulae can be used to design techniques for numerical integration and differentiation of functions $f(x)$.

eg Finding technique for $\int_a^b f(x)\,dx$ when $f(x)$ does not have an antiderivative:

Then we know $f(x)$ but we need to know error in numerical calculus which depends on error in interpolating polynomial.

The degree of polynomial needed for desired accuracy is generally not known.

Use results from various polynomials until appropriate agreement is reached.

eg $P_k(x^*) \approx P_{k+1}(x^*)$ say if you want to have a good approx to $x^*$.

The way we did the L. I. Polynomial so far is such that the work we did getting $P_k(x^*)$ does not significantly lessen work for $P_{k+1}(x^*)$ though.

We can get formulations of approx polynomials that use previous calculations to a greater advantage, though