Assignment IV **Due: Wenesday 30th November, 2016**

1. Section 3.4: #11 The problem begins *A natural cubic spline S on [0, 2] is defined by*
2. Section 3.4: #12 The problem begins *A clamped cubic spline s for a function f ⋯*

See end of next

3. What constant c makes the quantity

$$\sum_{i=0}^{m}[f(x_i) - ce^{x_i}]^2$$

as small as possible?

4. Section 8.1: #7 The problem begins *In the lead example of this chapter, an experiment was described*

5. Section 8.1: #5 a, b, c The problem begins *Given the data:*
Feel free to use the start with the code `LeastSquares.m` and modify it to suit the problem. Turn in the code with your results.

6. In modelling an oil reservoir, it may be necessary to find a relationship between the equilibrium constant of a reaction and the pressure, at constant temperature. The data shown in the table relate the equilibrium constants (K-values) to pressure p and were obtained from an experimental PVT (pressure-volume-temperature) analysis.

Pressure	0.635	1.035	1.435	1.835	2.235	2.635	3.035	3.435	3.835	4.235	4.635	5.035	5.435
K-value	7.5	5.58	4.35	3.55	2.97	2.53	2.2	1.93	1.7	1.46	1.28	1.11	1.0

We look for an approximating function of the form $K = e^{ap+b}$. Write a Matlab code that does the following:

(a) Obtain the function that approximates the data by finding the best log-linear fit.

(b) Plot the data ($\star$) along with the approximating exponential function.

(c) Produce a **table** listing the log-linear errors at each data point. That is, find the errors $|\log(K_i) - (ap_i + b)|$ for each i. What is the total L_2 error? That is to say, what is

$$\sum_{i=1}^{m}[\log(K_i) - (ap_i + b)]^2\,?$$

(d) Produce a **table** listing the errors in the fit to K at each data point. That is, find the errors $|K_i - e^{(ap_i+b)}|$ for each i. What is the total L_2 error?

7. The Legendre polynomials $P_0(x), P_1(x), P_2(x), \cdots$ are defined as

$$P_0(x) = 1, \qquad P_n(x) = \frac{(-1)^n}{2^n n!}\frac{d^n}{dx^n}(1-x^2)^n \quad \text{for } n \geq 1.$$

The polynomials may also be generated recursively:

$$(n+1)P_{n+1}(x) - (2n+1)xP_n(x) + nP_{n-1}(x) = 0, \qquad \text{where } P_0(x) = 1, \quad P_1(x) = x.$$

(a) Write the first four Legendre polynomials out explicitly.

(b) Plot the first four Legendre polynomials on the same graph for $-1 \le x \le 1$.

(c) Verify that P_0, P_1 and P_2 are orthogonal functions on $[-1, 1]$. Are they orthonormal?

(d) Find the least squares polynomial approximation of (i) degree 2, and (ii) degree 3 on the interval $[-1, 1]$ to $f(x) = \ln(x + 2)$.

(e) Plot $f(x)$ and its quadratic and cubic approximations on the same graph.

---

10. Repeat Exercise 6 using the clamped cubic splines constructed in Exercise 8.

Section 3.4

11. A natural cubic spline S on $[0, 2]$ is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3, & \text{if } 0 \le x < 1, \\ S_1(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^3, & \text{if } 1 \le x \le 2. \end{cases}$$

Find b, c, and d.

12. A clamped cubic spline s for a function f is defined on $[1, 3]$ by

$$s(x) = \begin{cases} s_0(x) = 3(x-1) + 2(x-1)^2 - (x-1)^3, & \text{if } 1 \le x < 2, \\ s_1(x) = a + b(x-2) + c(x-2)^2 + d(x-2)^3, & \text{if } 2 \le x \le 3. \end{cases}$$

5. Given the data:

x_i	4.0	4.2	4.5	4.7	5.1	5.5	5.9	6.3	6.8	7.1
y_i	102.56	113.18	130.11	142.05	167.53	195.14	224.87	256.73	299.50	326.72

a. Construct the least squares polynomial of degree 1, and compute the error.
b. Construct the least squares polynomial of degree 2, and compute the error.
c. Construct the least squares polynomial of degree 3, and compute the error.
d. Construct the least squares approximation of the form be^{ax}, and compute the error.
e. Construct the least squares approximation of the form bx^a, and compute the error.

Section 8.1

6. Repeat Exercise 5 for the following data.

x_i	0.2	0.3	0.6	0.9	1.1	1.3	1.4	1.6
y_i	0.050446	0.098426	0.33277	0.72660	1.0972	1.5697	1.8487	2.5015

7. In the lead example of this chapter, an experiment was described to determine the spring constant k in Hooke's law:

$$F(l) = k(l - E).$$

The function F is the force required to stretch the spring l units, where the constant $E = 5.3$ in. is the length of the unstretched spring.

a. Suppose measurements are made of the length l, in inches, for applied weights $F(l)$, in pounds, as given in the following table.

$F(l)$	2	4	6
l	7.0	9.4	12.3

Find the least squares approximation for k.

b. Additional measurements are made, giving more data:

$F(l)$	3	5	8	10
l	8.3	11.3	14.4	15.9

Compute the new least squares approximation for k. Which of (a) or (b) best fits the total experimental data?

8. The following list contains homework grades and the final-examination grades for 30 numerical