TIE for leaving variable, DEGENERACY, & CYCLING

Consider the following tableau. The BFS is (0, 0, 0,3,2,1) and z=0.
Highlighted is the pivot column indicating that x_3 will be entering the basis on the next pivot step.

TABLEAU 1

	z	x1	x2	x3	x4	x5	x6	RHS	Ratio
z	1	-2	1	-8	0	0	0	0	
x4	0	2	-4	6	1	0	0	3	0.5
x5	0	-1	3	4	0	1	0	2	0.5
x6	0	0	0	2	0	0	1	1	0.5

The values of the ratio of RHS to x_3 coefficient (only computed when the coefficient is positive) is shown in the last column. We may choose any of x_4, x_5, and x_6 for the enteri ng variable as the ratios are all the same.
There is a **tie for the leaving variable.** **The solution is to simply choose one at random.**

The tableau below shows the results when x_4 was chosen to be the leaving variable when x_3 entered. The BFS associated with this tableau is (0, 0, 0.5, 0, 0, 0) and z = 4.
Why are there now zeroes in the RHS of the equations corresponding to the other basic variable?

TABLEAU 2a

	z	x1	x2	x3	x4	x5	x6	RHS
z	1	0.666667	-4.33333	0	1.333333	0	0	4
x3	0	0.333333	-0.66667	1	0.166667	0	0	0.5
x5	0	-2.33333	5.666667	0	-0.66667	1	0	0
x6	0	-0.66667	1.333333	0	-0.33333	0	1	0

The tableau below shows the results when x_5 is chosen to be the leaving variable instead. The BFS associated with this tableau is (0, 0, 0.5, 0, 0, 0) and z = 4.
Why are there now zeroes in the RHS of the equations corresponding to the other basic variable?

TABLEAU 2b

	z	x1	x2	x3	x4	x5	x6	RHS
z	1	-4	7	0	0	2	0	4
x4	0	3.5	-8.5	0	1	-1.5	0	0
x3	0	-0.25	0.75	1	0	0.25	0	0.5
x6	0	0.5	-1.5	0	0	-0.5	1	0

Alternatively, tableau below shows the results when x_6 is chosen to be the leaving variable. The BFS associated with this tableau is (0, 0, 0.5, 0, 0, 0) and z = 4.
Why are there now zeroes in the RHS of the equations corresponding to the other basic variable?

TABLEAU 2c

	z	x1	x2	x3	x4	x5	x6	RHS
z	1	-2	1	0	0	0	4	4
x4	0	2	-4	0	1	0	-3	0
x5	0	-1	3	0	0	1	-2	0
x3	0	0	0	1	0	0	0.5	0.5

When there is a tie for leaving variable, the one from the tie not chosen will remain basic but will have a value of 0. **If the BFS contains a basic variable that is 0, we call this a degenerate BFS.** All our choices above for the 2nd tableau are **degenerate tableaux** because they all give degenerate BFS.

Let's continue. Let's pick that last tableau (Tableau 2c) where x_6 leaves as our 2nd tableau in our simplex iteration. This is a degenerate tableau because there are 0 in the RHS which makes x_4 and x_5 zero in the BFS.

TABLEAU 2

	z	x1	x2	x3	x4	x5	x6	RHS	Ratio
z	1	-2	1	0	0	0	4	4	
x4	0	2	-4	0	1	0	-3	0	0
x5	0	-1 **	3	0	0	1	-2	0	**
x3	0	0 *	0	1	0	0	0.5	0.5	*

The coefficients in the z equation show that the entering variable must be x_1.

For the ratios, we note 2 things:

- (*) We don't bother taking the ratio when the coefficient in the pivot column is zero because the coefficient of the entering variable is zero and changing the value of that variable will therefore not have to be constrained. It will not cause any other variable to go negative.

- (**) We don't bother taking the ratio when the denominator is negative. For example, the tableau above says that -x_1 + 3x_2 + x_5 – 2x_6 = 0. The value of x_1 can be increased without bound (when x_2 and x_6 remain at 0) and it would not make things infeasible.

Thus, the leaving variable must be x_4.

A degenerate pivot is a pivot on a degenerate tableau that does not change the value of z or does not change the BFS. This happens when one of the ratios when calculating the leaving variable is 0.
This pivot from tableau 2 above to tableau 3 below is a degenerate pivot because the value of z does not change.

TABLEAU 3

	z	x1	x2	x3	x4	x5	x6	RHS	Ratio
z	1	0	-3	0	1	0	1	4	
x1	0	1	-2	0	0.5	0	-1.5	0	**
x5	0	0	1	0	0.5	1	-3.5	0	0
x3	0	0	0	1	0	0	0.5	0.5	*

The BFS associated with this tableau is (0, 0, 0.5, 0, 0, 0) and z = 4.
This tableau is degenerate.

Moving on, the entering variable must be x_2.
The leaving variable must be x_5. Note that there was only one ratio to consider.
This tableau is also degenerate and the pivot is degenerate because the value of z did not change.

This pivot from tableau 3 to tableau 4 below is a degenerate pivot because the value of z did not change.

TABLEAU 4

	z	x1	x2	x3	x4	x5	x6	RHS	Ratio
z	1	0	0	0	2.5	3	-9.5	4	
x1	0	1	0	0	1.5	2	-8.5	0	**
x2	0	0	1	0	0.5	1	-3.5	0	**
x3	0	0	0	1	0	0	0.5	0.5	1

The BFS associated with this tableau is (0, 0, 0.5, 0, 0, 0) and z is unchanged at 4.
This tableau is degenerate.

Moving on, the entering variable must be x_6.
The leaving variable must be x_3. Note that the ratio is non-zero!

TABLEAU 5

	z	x1	x2	x3	x4	x5	x6	RHS
z	1	0	0	19	2.5	3	0	13.5
x1	0	1	0	17	1.5	2	0	8.5
x2	0	0	1	7	0.5	1	0	3.5
X6	0	0	0	2	0	0	1	1

The BFS associated with this tableau is (8.5, 3.5, 0, 0, 0, 1) and z has increased to 13.5.

This tableau is **not degenerate.**

Also, we have reached the optimal solution as the coefficients in the z equation are now all non-negative.

We got stuck at (0, 0, 0.5) for a while with z=4 but eventually we got out.

However, there are rare times when a series of degenerate pivots go on ad infinitum – failure!
This is when we have cycling. **Cycling** occurs when the simplex method makes a sequence of degenerate pivots that return to a previously visited tableau in which case the sequence of pivots goes on again and again.
There are rules for avoiding cycling however. In particular, there is a rule called **Bland's Rule.**

TIE FOR ENTERING VARIABLE

Consider the tableau below.

There are actually **more than one choice for entering variable**, x_1 and x_6. Which one should be picked? The choice is yours. That is, **pick arbitrarily.**

TABLEAU 6

	z	x1	x2	x3	x4	x5	x6	RHS
	1	**-2**	**1**	**0**	**0**	**0**	**-2**	**4**
x4	0	3	-4	0	1	0	-3	8
x5	0	-1	3	0	0	1	-2	14
x3	0	0	0	1	0	0	0.5	0.5

UNBOUNDEDNESS

Consider the tableau below.

TABLEAU 7

	z	x1	x2	x3	x4	x5	x6	RHS	Ratio
	1	-2	1	0	0	0	-1	4	
x4	0	-3	-4	0	1	0	3	8	**
x5	0	-1	3	0	0	1	-2	14	**
x3	0	0	0	1	0	0	-8	0.5	*

The entering variable we choose is x_1.

However, every entry in the x_1 column is negative which means that we can increase x_1 without bound towards infinity (all the while keeping x_2 = x_6 =0) and increase z towards infinity and never violate any constraint.

For example, the equation for x_4 in the tableau reads: -3x_1 -4 x_2 + x_4 + 3 x_6 = 8.
That is, x_4 = 8 + 3x_1 + 4 x_2 - 3x_6. Keeping x_2 and x_6 non-basic hence equal to zero reduces this to x_4 = 8 + 3x_1. One can increase x_1 indefinitely without violating the non-negativity constraint on x_4.

If there was a 3 instead of -3, we would have x_4 = 8 - 3x_1 and we would not push x_1 beyond 8/3 and we would have put a ratio to consider in the Ratio column of the tableau.

The LP associated with this tableau is unbounded and there is no solution.

However, we could have picked x_6 as our entering variable and we would have had to consider some ratios, in this case, and could pivot to another tableau. We would have chosen x_4 as our leaving basic variable. See Tableau 8.

TABLEAU 8

	z	x1	x2	x3	x4	x5	x6	RHS	Ratio
	1	-2	1	0	0	0	-1	4	
x4	0	-3	-4	0	1	0	3	8	8/3
x5	0	-1	3	0	0	1	-2	14	**
x3	0	0	0	1	0	0	-8	0.5	**

However, the LP is still unbounded though looking at the x_6 column alone will not tell you that.

So, **one must look at the columns of each tableau and check for unboundedness.** **A column in a tableau containing no positive entries tells that the LP is unbounded.** The simplex method will be terminated then.