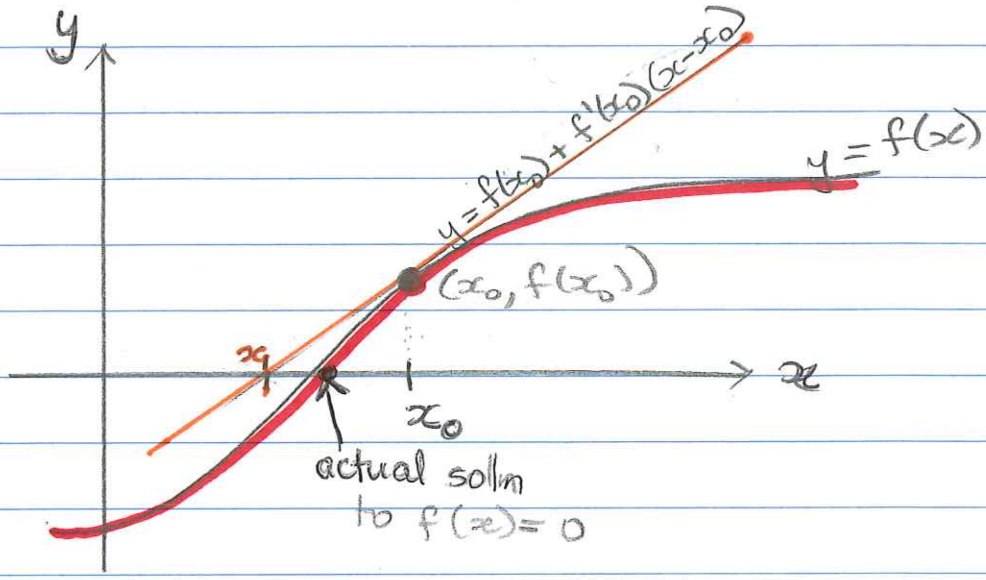
Newton's Method to solve $f(x)=0$



Consider the tangent line to curve $y=f(x)$ at the point $(x_0, f(x_0))$

The slope of the line is $f'(x_0)$.

The equation of the line is therefore

$$y - f(x_0) = f'(x_0)\cdot(x - x_0)$$

$$\Longrightarrow \quad y = f(x_0) + f'(x_0)(x - x_0).$$

Treat the tangent line to the curve at $(x_0, f(x_0))$ as an approximation to the actual curve $y=f(x)$.

Then just as we would ask where is $y=f(x)$ equal to 0, instead ask where is $y=f(x_0)+f'(x_0)(x-x_0)$ equal to 0.

$$f(x_0) + f'(x_0)(x - x_0) = 0$$

$$\Longrightarrow \quad x - x_0 = \frac{-f(x_0)}{f'(x_0)}$$

$$\Longrightarrow \quad x = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Call this x_1

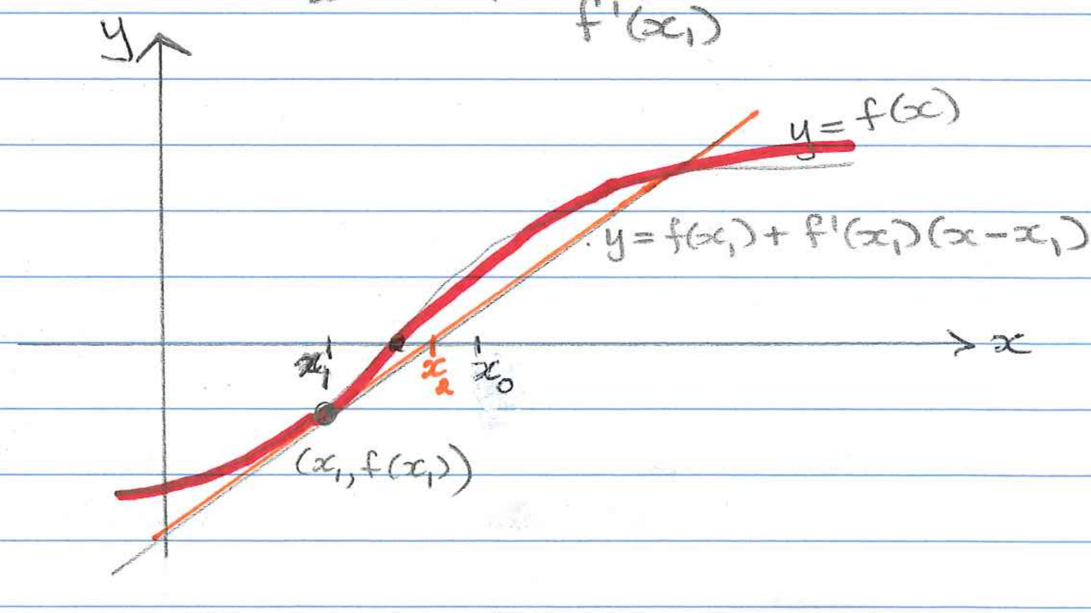
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Is $f(x_1)$ close enough to zero?

No? Then try this again and look at

$$y = f(x_1) + f'(x_1)(x - x_1)$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$



etc.

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}.$$

Better hope $f'(x_k) \neq 0$

So you have <u>iterates</u> $x_0, x_1, x_2, x_3, \ldots$ & employ stopping criteria

Newton's method doesn't always converge

<u>but</u> if you pick a good enough x_0,

it will converge and it will do so quickly.

r = actual solution

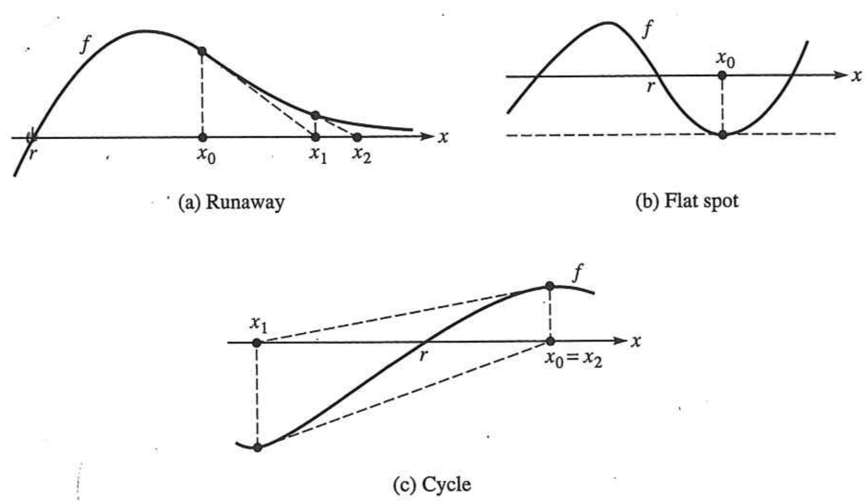


FIGURE 3.4
Failure of Newton's method due to bad starting points

③ Scenarios where Newtons's method does not converge to solution