Maximum Flow Problem

Problem Description

1. Given a directed, connected network.
2. All flow originates at one source node and terminates at one sink node.
3. All remaining nodes are transshipment nodes.
4. Flow through an edge is alled through the direction of the edge. The maximum flow is the capacity of the edge.
    - At the source all edges point away from the node.
    - At the sink all edges point into the node.
5. Objective: Maximize the total amount of flow from the source to the sink as measured by either the amount leaving the source or the amount entering the sink.

Definition: An **augmenting path** is a directed path from the source to the sink in the residual network such that every arc on this path has strictly positive residual capacity.

The minimum of these residual capacities is called the **residual capacity of the augmenting path** because it represents the amount of flow that can feasibly be added to the entire path.

- Each augmenting path provides an opportunity to further augment the flow through the network.

The Augmenting Path Algorithm

1. Identify an augmenting path by finding some directed path from the source to the sink in the residual network. If no such path exists then the current pattern is optimal.
2. Identify the residual capacity of this augmenting path c^* by finding the minimum of the residual capacities of the arcs on this path.

   Increase the flow in this path by c^*.
3. Update your residual network.
   - Decrease by c^* the residual capacity of each arc on this augmenting path.
   - Increase by c^* the residual capacity of each arc in the opposite direction on this augmenting path.
   - Return to step 1.