Decision Analysis

Decisions made in environments with uncertainty.

eg. investment strategy in stock market;
eg bidding on a contract w/ other companies also bidding.

Working Example

The Texahoma family owns land that has a 1 in 4 chance of having oil underground.
¿Do they sell the land to an ~~interested~~ party who has offered them \$90,000?
¿Do they hold the land and drill themselves?
Cost of drilling is \$100,000.
If they drill and find oil, the expected profit will be \$800,000.

Version 1

Version 2

Get a detailed seismic survey done to obtain a better estimate of the chance of finding oil. – DECISION MAKING w/ EXPERIMENTATION.

Decision maker must choose an alternative from a set of decision alternatives.

Choice of alternative is made in the face of ________ because the outcome will be affected by random factors.
Each of these possible situations is called a possible ________.

We don't have rational player 2 promoting own welfare } Game theory 2 person zero-sum game

2 players → ________ vs ________

Strategies → ________ vs ________

Need to find OPTIMAL SOLN FOR DECISION MAKER.

Also, PRIOR DISTRIBUTION gives the relative likelihood of possible states of nature.
This may depend on experience or intuition of an individual.

PRIOR PROBABILITIES = probability for each possible state.

	State of Nature	
	Oil	Dry
Drill for oil	700	-100
Sell the land	90	90
Prior prob.	0.25	0.75

Units are in ฿ 1,000.

How to make the decision?

a) MAXIMIN payoff criterion

b) MAXIMUM LIKELIHOOD criterion

c) BAYES' DECISION RULE.

Maximin payoff – pick alternative that gives the maximum minimum payoff.

Maximum Likelihood - focus on the most likely state of nature and pick the one with the alternative that would give the largest payoff in that state.

BAYES' DECISION RULE

Using the prior probabilities, calculate the expected payoff for each alternative. Choose the decision w/ maximum expected payoff.

Expected value of a discrete random variable X =

$$P(X = \text{state } 1) \cdot (\text{Value of state } 1) +$$
$$P(X = \text{state } 2) \cdot (\text{Value of state } 2) +$$
$$\vdots$$
$$+ P(X = \text{state } m) \cdot (\text{Value of state } m)$$

Expected value of X

$$= \sum_{j=1}^{m} P(X = \text{state } j) \cdot (\text{Value of state } j)$$

Thus, expected payoff for alternative 1 (drill)

$= P$ (you drill and get oil) $\cdot$ (Payoff of drilling + getting oil)

$+ P$ (you drill and don't get oil) $\cdot$ (Payoll of drilling but not getting oil)

$$= \frac{1}{4} \cdot 700 + \frac{3}{4} \cdot (-100)$$

$$= \frac{700 - 300}{4} = \$100{,}000$$

Expected payoff for alternative 2 (selling)

$= P$ (you sell and there is oil) $\cdot$ (Payoff of selling & oil underground)
$+ P$ (you sell and there is no oil) $\cdot$ (Payoff of selling & there is no oil underground)

$$= \frac{1}{4} \cdot 90 + \frac{3}{4} 90 = \$90{,}000$$

So pick alternative 1 = keep the land & drill yourself

DECISION MAKING WITH EXPERIMENTATION

Additional testing can be done to get better estimates of the probabilities of the states of nature provided by the prior probabilities

The improved estimates are called
POSTERIOR PROBABILITIES.

The Texahoma family can conduct a detailed seismic survey of the land to get a better estimate of the probability of oil. The cost will be $30,000.

A seismic survey gives one of two results:

USS - unfavourable seismic surroundings.
$\implies$ oil fairly unlikely

FSS - favourable seismic surroundings
$\implies$ oil fairly likely.

These are NOT 100% accurate readings.
Sometimes when there is actually oil, one gets an USS.
Sometimes when there is no oil, it can give an FSS reading

Conditional Probabilities

"probability the reading is USS given that we know there is oil"

Suppose

$P(USS \mid oil) = 0.4$

$P(FSS \mid oil) = 0.6$

If you knew there was oil, what fraction of the time would you get USS or FSS

These sum to 1

"probability the reading is FSS given that there is oil"

and

$P(USS \mid dry) = 0.8$

$P(FSS \mid dry) = 0.2$

If it is given that there is no oil, what fraction of time does the reading say USS vs FSS.

these sum to 1.

We want to find out how to make our decisions if reading gives FSS vs if reading gives USS

We also want to findout whether it is even worth spending the \$30K on the test.

7 Do the posterior probabilities

We have already PRIOR PROBABILITIES:

$$P(\text{State} = \text{Oil})$$
$$P(\text{State} = \text{Dry})$$

We want to do a seismic survey and then get POSTERIOR PROBABILITIES:

$$P(\text{State} = \text{Oil} \mid \text{Finding} = USS)$$

$$P(\text{State} = \text{Dry} \mid \text{Finding} = USS)$$

$$P(\text{State} = \text{Oil} \mid \text{Finding} = FSS)$$

$$P(\text{State} = \text{Dry} \mid \text{Finding} = FSS)$$

so we will build a new payoff table

	State of Nature		Expected Payoff if USS	Expected Payoff if FSS
	Oil	Dry		
Drill	700	-100	*	*
Sell	90	90	*	*

These values will take into account the probabilities that USS reading can still result in oil & FSS reading can still result in no oil using the POSTERIORS

DEFN

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

P(A and B occuring at the same time)

$$\Longrightarrow P(A \cap B) = P(A|B) \cdot P(B)$$

Also

DEFN $$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Longrightarrow P(A \cap B) = P(B|A)P(A).$$

So, for posterior probability, we must find
P(State = Oil | Finding = USS), etc.

and all we know are the PRIORS: P(State = Oil) &
P(State = Dry)

along with P(Finding = USS | State = Oil)
P(Finding = USS | State = Dry)
P(Finding = FSS | State = Oil)
P(Finding = FSS | State = Dry)

from past experiments

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

BAYES' THM

For us

$P(\text{State} = \text{Oil} \mid \text{Finding} = \text{USS})$

we know this

we know this

$$= \frac{P(\text{Finding} = \text{USS} \mid \text{State} = \text{Oil}) \cdot P(\text{State} = \text{Oil})}{P(\text{Finding} = \text{USS})}$$

What is this?

$$\boxed{\begin{aligned} P(\text{Finding} = \text{USS}) &= P(\text{Finding} = \text{USS and State} = \text{Oil}) \\ &\quad + P(\text{Finding} = \text{USS and State} = \text{Dry}) \\ &= P(\text{Finding} = \text{USS} \mid \text{State} = \text{Oil}) \cdot P(\text{State} = \text{Oil}) \\ &\quad + P(\text{Finding} = \text{USS} \mid \text{State} = \text{Dry}) \cdot P(\text{State} = \text{Dry}) \end{aligned}}$$

So $P(\text{State} = \text{Oil} \mid \text{Finding} = \text{USS}) = \frac{(0.4)(0.25)}{(0.4)(0.25) + (0.8)(0.75)}$

$$= \frac{0.1}{(0.1 + 0.6)} = \frac{0.1}{0.7} = 1/7$$

We found $P(\text{Finding} = \text{USS}) = 0.7$

$$\Rightarrow P(\text{Finding} = \text{FSS}) = 0.3$$

Since there are only 2 choices for findings, so if the chance of the finding comes out to be USS is 0.7 then the chance of the finding not being USS (i.e. being FSS) is $1-0.7=0.3$.

$$P(\text{State} = \text{Oil} \mid \text{Finding} = \text{USS}) = \frac{1}{7}$$

$$\Rightarrow P(\text{State is not Oil} \mid \text{Finding} = \text{USS}) = 6/7$$

$$\text{i.e. } P(\text{State} = \text{Dry} \mid \text{Finding} = \text{USS}) = 6/7$$

What is $P(\text{State} = \text{Oil} \mid \text{Finding} = \text{FSS})$ and $P(\text{State} = \text{Dry} \mid \text{Finding} = \text{FSS})$?

Expected payoff of drilling if finding = USS

$$= P(\text{oil} \mid \text{USS}) \cdot (\text{Payoff if drill \& oil is found})$$

$$+ P(\text{dry} \mid \text{USS}) \cdot (\text{Payoff of drill \& dry})$$

$$= \frac{1}{7}(700) + \frac{6}{7}(-100)$$

$$= \frac{100}{7}$$

Expected payoff of selling if finding = USS

$$= P(\text{oil} \mid \text{USS}) \cdot (\text{Payoff if sold \& oil})$$

$$+ P(\text{dry} \mid \text{USS}) \cdot (\text{Payoff if sold \& dry})$$

$$= \frac{1}{7}(90) + \frac{6}{7}(90)$$

$$= 60$$

Expected payoff of drilling if finding = FSS

$= P(\text{oil} \mid \text{finding} = \text{FSS}) \cdot (\text{Payoff of drilling + getting oil})$

$+ P(\text{dry} \mid \text{finding} = \text{FSS}) \cdot \begin{pmatrix}\text{Payoff of drilling} \\ \text{\& dry}\end{pmatrix}$

$= \frac{1}{2} \cdot (700) + \frac{1}{2}(-100)$

$= \frac{1}{2}(700 - 100)$

$= 300$

Expected payoff of selling if finding = FSS

$= \frac{1}{2}(90) + \frac{1}{2}(90)$

$= 90$

	Posterior probs			
			$(1/7, 6/7)$	$(1/2, 1/2)$
	STATE OF NATURE		Expected payoff	Expected payoff
	Oil	Dry	if USS	if FSS
Drill	700	−100	$100/7 = 14\frac{2}{7}$	300
Sell	90	90	90	90
	0.25	0.75	0.7	0.3

If finding is USS, then sell and the venture would have an expected value of $90 - 30 = 60$ \$60,000

cost of test.

If finding is FSS then one would decide to keep land & drill and the venture would have an expected value of

$300 - 30 = 270$ \$270,000.

OPTIMAL EXPECTED VALUE w/ EXPERIMENTATION (excluding cost of survey) is

$=$ (Optimal Expected payoff if finding = FSS) $\cdot$ P(finding = FSS)
$+$ (Optimal Expected payoff if finding = USS) $\cdot$ P(finding = USS)
$= 300(0.3) + 90(0.7) = 153$ \$153,000

Is it worth spending \$30,000 on the seismic test?

Let's see:

no seismic survey

Optimal expected value w/o experimentation

(page 5)

is keep land and drill for yourself

\$100,000

get seismic survey

Optimal expected value w/ experimentation

(page 14)

is \$153,000

So Expected Value of Experimentation (EVE)

= \$153,000 − \$100,000

= \$53,000

Thus, it is worth paying \$30,000 for the seismic reading.