MA 1022 B '10 **Final Exam** Name ____________________

ANSWER KEY

Instructions

This test is closed book. Calculators are not allowed.

Part I - Basic Skills

Problem	1	2	3	4	5	6	7	Total
Value	5	5	5	5	5	5	5	35
Earned								

Part II

Problem	8	9	10	11	12	13	14	Total
Value	14	14	14	13	13	20	12	100
Earned								

Please Circle your Section

B01 Henry, G (8:00)

B02 Blais, M (9:00)

B03 Xie, W (9:00)

B04 Malone, JJ (10:00)

B05 Sulman, M (10:00)

B06 Tashjian, G (1:00)

B07 Xie, W (1:00)

B08 Abraham, J (2:00)

B09 Henry, G (3:00)

Part I - Basic Skills

Work the following problems and write your answers in the space provided. Use the scratch paper provided for your work. You need not simplify your answers.

1. $\int (3x^4 - \frac{6}{x^{1/3}} + \pi)\,dx$ Ans: $\frac{3}{5}x^5 - 9x^{2/3} + \pi x + C$

2. $\int x\sin(4x)\,dx$ Ans: $\frac{1}{16}\sin(4x) - \frac{1}{4}x\cos(4x) + C$

3. $\int \sec^5(x)\tan(x)\,dx$ Ans: $\frac{1}{5}\sec^5 x + C$

4. $\int \frac{7}{\sqrt{1-4x^2}}\,dx$ Ans: $\frac{7}{2}\arcsin(2x) + C$

5. $\int \frac{6}{(x-1)(x+1)}\,dx$ Ans: $3\ln\left|\frac{x-1}{x+1}\right| + C$

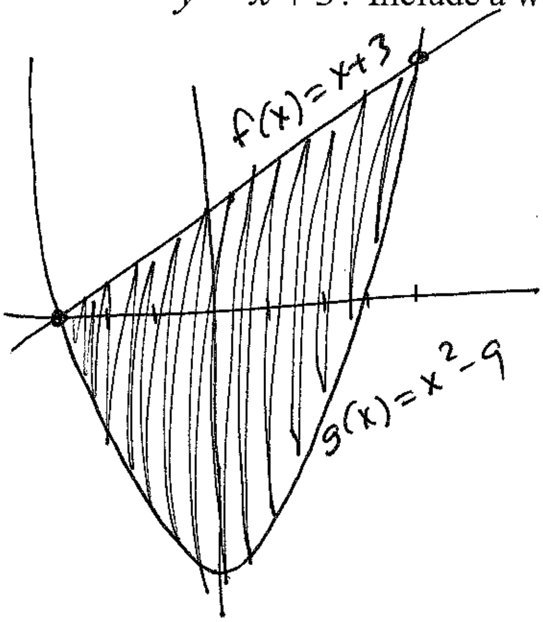
6. $\int_{-1}^{1} (x^2+3)^2\,dx$ Ans: $22\frac{2}{5}$

7. $\frac{d}{dx}\left(9^{(7x^2)}\right)$ Ans: $\left[9^{(7x^2)}\right](\ln 9)(14x)$

Part II

Work all of the following problems. Show your work in the space provided. You need not simplify your answers, but remember that on this part of the exam your work and your explanations are graded, not just the final answers

8. Find the area of the region bounded by the curves $y = x^2 - 9$ and $y = x + 3$. Include a well-labeled sketch of the region.



$x^2 - 9 = x + 3$

$x^2 - x - 12 = 0$

$(x-4)(x+3)$

$x = -3$ or $x = 4$

$$\text{Area} = \int_a^b f(x) - g(x)\ dx$$

$$\text{Area} = \int_{-3}^{4} (x+3) - (x^2-9)\ dx = \frac{x^2}{2} + 12x - \frac{x^3}{3} \Bigg|_{-3}^{4}$$

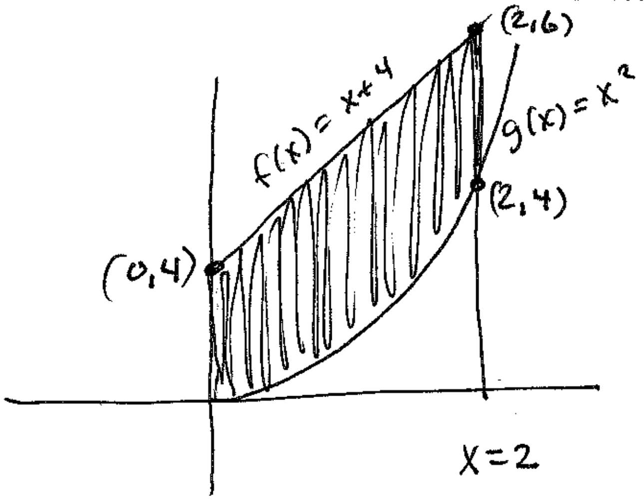
$$= \left(8 + 48 - \frac{64}{3}\right) - \left(\frac{9}{2} - 36 + 9\right)$$

$$= 34\tfrac{2}{3} - (-22.5)$$

$$= 57\tfrac{1}{6} \text{ or } \frac{343}{6}$$

9. $\boldsymbol{R}$ is the first quadrant region below the line $y = x + 4$, above $y = x^2$, to the right of the $y-axis$ and to the left of $x = 2$. Find the volume of the solid obtained by revolving $\boldsymbol{R}$ around the $x-axis$.

Include a well-labeled sketch of the region.



$$\text{Volume} = \int_a^b \left(\pi r_{\text{OUTER}}^2 - \pi r_{\text{INNER}}^2\right) dx$$

$$r_{\text{OUTER}} = f(x) = x+4$$

$$r_{\text{inner}} = g(x) = x^2$$

$$V = \int_0^2 \left[\pi (x+4)^2 - \pi (x^2)^2\right] dx$$

$$V = \int_0^2 \pi \left[x^2 + 8x + 16 - x^4\right] dx$$

$$V = \pi \left[\frac{x^3}{3} + 4x^2 + 16x - \frac{x^5}{5}\right]\Big|_0^2$$

$$V = \pi \left[\frac{8}{3} + 16 + 32 - \frac{32}{5}\right]$$

$$V = \frac{664\pi}{15}$$

10. Answer each of the following questions

a. **Set up but do not evaluate** an expression for the length of the arc $y = x^3$ on the interval $1 \le x \le 4$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

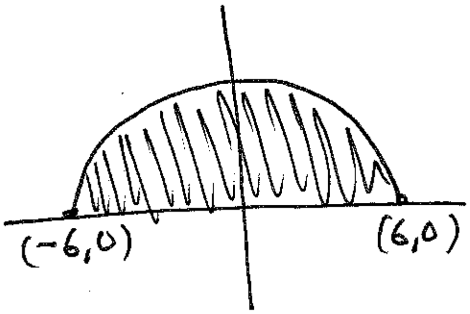
$$S = \int_1^4 \sqrt{1 + (3x^2)^2}\, dx$$

b. **Set up but do not evaluate** an expression for the surface area of revolution generated by rotating the arc $y = x^3$ on the interval $1 \le x \le 4$ around the $x - axis$

$$SA = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$SA = \int_1^4 2\pi x^3 \sqrt{1 + (3x^2)^2}\, dx$$

11. Find the centroid of the semi-circular region bounded above by $y = \sqrt{36 - x^2}$ and below by the $x - axis$.



$M =$ area of region $= \frac{1}{2}\pi r^2 = 18\pi$

$M_y = 0$ by symmetry

$$M_x = \int_{-6}^{6} \frac{1}{2}\left(\sqrt{36-x^2}\right)^2 dx = \int_{-6}^{6} 18 - \frac{1}{2}x^2 \, dx$$

$$M_x = 18x - \frac{x^3}{6}\Big|_{-6}^{6} = (108-36) \times 2 = 144$$

$$\bar{x} = \frac{M_y}{m} \qquad \bar{y} = \frac{M_x}{m} = \frac{144}{18\pi} = \frac{8}{\pi}$$

centroid $= \left(0, \frac{8}{\pi}\right)$

12. The half-life of Carbon 14 is 5700 years. Carbon taken from a relic found in the basement of Stratton Hall contains 70% as many ^{14}C atoms per gram as a present-day specimen of the same substance. Compute the approximate age of the relic.

$$N(t) = N(0)\, e^{kt}$$

$$N(5700) = \underbrace{\frac{1}{2} N(0) = N(0)\, e^{5700k}}_{k = \frac{\ln(1/2)}{5700}}$$

k will be negative

We want:

$$N(t) = 0.70\, N(0) = N(0)\, e^{-\frac{t \ln 2}{5700}}$$

or

$$\ln(0.70) = \frac{-t \ln 2}{5700}$$

$$t = -\frac{5700 \ln(0.70)}{\ln 2} \quad \text{or} \quad t = \frac{5700 \ln(0.70)}{\ln(0.50)}$$

13. Compute the following integrals

a. $\displaystyle\int \frac{x}{(x-3)^2}\,dx = \int \frac{1}{x-3} + \frac{3}{(x-3)^2}\,dx$

$\displaystyle\frac{x}{(x-3)^2} = \frac{A}{x-3} + \frac{B}{(x-3)^2}$ $\displaystyle = \ln|x-3| - \frac{3}{x-3} + C$

$A(x-3) + B = x$

$A = 1$

$B = 3$

OR

$u = x-3$

$du = dx$

$x = u+3$

Can use partial fractions or u-substitution

$\displaystyle\int \frac{u+3}{u^2}\,du = \ln|u| - \frac{3}{u} + C = \ln|x-3| - \frac{3}{x-3} + C$

b. $\displaystyle\int \frac{3}{x\left[1+(\ln(x))^2\right]}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\displaystyle\int \frac{3\,du}{1+u^2} = 3\arctan u + C$

$= 3\arctan(\ln|x|) + C$

c. $\int_0^{\pi/2} \cos^3 x \sin^7 x dx$

$u = \sin x$
$du = \cos x \, dx$

$$\int_{x=0}^{x=\pi/2} (1-u^2)(u^7)\, du = \left. \frac{u^8}{8} - \frac{u^{10}}{10} \right|_{x=0}^{x=\pi/2}$$

$$= \left(\frac{1}{8} - \frac{1}{10}\right) - (0-0) = \frac{1}{40}$$

d. $\int x^2 e^{-2x} dx$

$u = x^2 \quad dv = e^{-2x} dx$
$du = 2x dx \quad v = -\frac{1}{2} e^{-2x}$

$$-\frac{1}{2} x^2 e^{-2x} + \int x e^{-2x} dx$$

$u = x \quad dv = e^{-2x}$
$du = dx \quad v = -\frac{1}{2} e^{-2x}$

$$-\frac{1}{2} x^2 e^{-2x} - \frac{1}{2} x e^{-2x} + \int \frac{1}{2} e^{-2x} dx$$

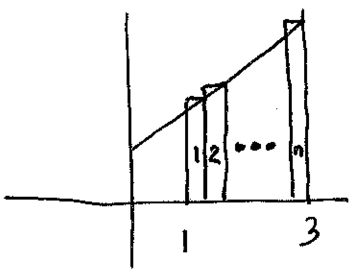
$$\int x^2 e^{-2x} dx = -\frac{1}{2} x^2 e^{-2x} - \frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C$$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$	$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

14. Consider the integral $\int_1^3 (2x+1)\, dx$ and given the above,

(a) Write a Riemann sum approximating the above integral by dividing the interval of integration into ***n*** equal parts, and evaluating the function at the right endpoints of the subintervals.

$$R_n = \sum_{i=1}^{n} f(x_i)\Delta x_i = \sum_{i=1}^{n} f\left(a + \frac{b-a}{n} i\right)\left(\frac{b-a}{n}\right)$$



$$R_n = \sum_{i=1}^{n}\left[2\left(1+\frac{2i}{n}\right)+1\right]\left(\frac{2}{n}\right)$$

$$R_n = \sum_{i=1}^{n} \frac{6}{n} + \frac{8i}{n^2} = 6 + \frac{8}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$\Delta x = \frac{b-a}{n} = \frac{2}{n}$

$x_i = a + \frac{b-a}{n} i = 1 + \frac{2i}{n}$

$$R_n = 6 + 4 + \frac{4}{n} = 10 + \frac{4}{n}$$

(b) Using the expression obtained in part (a), let $n \to \infty$, and demonstrate that the value of the integral as a limit of the Riemann sums is 10.

$$\lim_{n\to\infty} 10 + \frac{4}{n} = 10$$

****** *End of Exam* ******