***NOTE: EVALUATION IS AT THE BOTTOM OF THIS DOCUMENT**

Problem Statement:

A puck with a mass of 72g begins at rest on an inclined plane (θ = 38 degrees above the horizontal) with a length (L) of 2.9m. The coefficient of friction between the puck and the ramp is 0.17. The ramp is situated on top of a 1.6m tall counter (h) which will cause the puck to fall and land a certain distance away from the counter when it reaches the ground. Calculate the distance (X_{BC}) away from the counter that the puck will land based on the given information and then use a spreadsheet or algorithm to determine the angle of the inclined plane that would maximize the distance away from the counter that the puck would land.

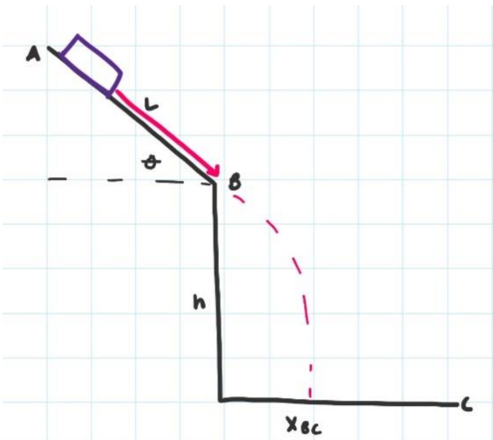


Figure 1: Diagram of Problem. The puck starts at the top of the ramp (A) and travels the length of the ramp (L) to the end (B). Then, the puck will fall from the end of the ramp (B) a certain distance from the ramp (X_{BC}).

Process:

For part A of the problem, we began by splitting the problem into two stages: motion on the inclined plane and projectile motion (Fig. 2).

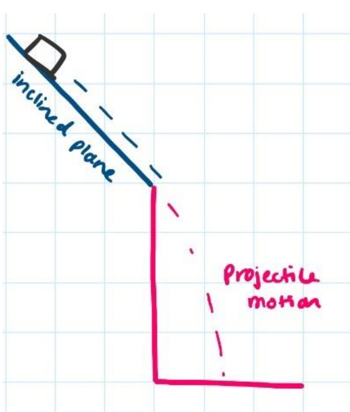


Figure 2: Diagram of Phases. The inclined plane phase is depicted in blue, and the projectile motion phase is depicted in red.

To begin solving the first stage, we drew a free body diagram for the puck, while it is on the ramp, which allowed us to visualize the different forces exerted on the puck (Figure 3). The forces exerted on the puck were the normal force (F_N), gravity (mg), and the force of friction (F_F). The force of gravity must be split into components (mgsinθ and mgcosθ) because this allows us to solve for the parallel and perpendicular forces relative to the incline separately.

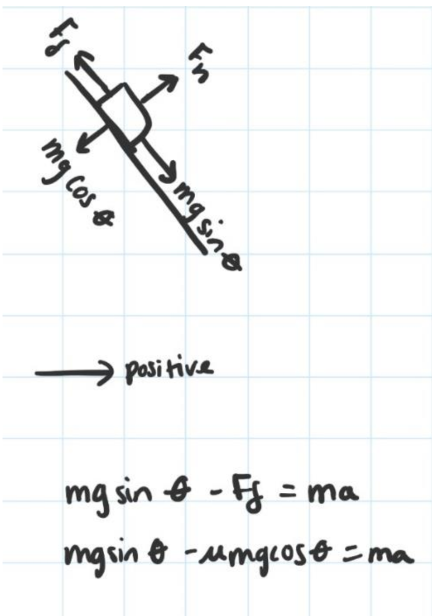


Figure 3: Free Body Diagram of Inclined Plane Phase. The positive direction is to the right and the equations are derived from the free body diagram are listed.

Our final goal for this stage was to find the velocity of the puck as it left the ramp, as this value would allow us to calculate the projectile motion in the next stage. In order to accomplish this, we started by finding the acceleration with the following equation:

$$mg\sin\theta - F_f = ma$$

$$\frac{(mg\sin\theta - F_f)}{m} = a$$

Because we do not directly know the force of friction, we have to use the relationship between friction and the normal force to calculate it using the given coefficient of friction.

$$F_f = \mu F_N$$

The normal force is equal to the force of mgcosθ because there is no acceleration perpendicular to the ramp, so we can substitute mgcosθ for normal force to solve for the force of friction.

$$F_N = \text{mg}\cos\theta$$

$$F_f = \mu\text{mg}\cos\theta$$

These relationships allow us to form the final equation to solve for acceleration by substituting $\mu\text{mg}\cos\theta$ for F_f in the equation for the forces parallel to the inclined plane.

$$\frac{(mg\sin\theta - \mu mg\cos\theta)}{m} = a$$

Once we have used this relationship to solve for acceleration, we can solve for the final velocity of the puck traveling down the ramp. We used the "no t" kinematic equation to do this.

$$v^2 = v_o^2 + 2a\Delta x$$

The final velocity of the ramp phase will be the initial velocity when the puck becomes a projectile. To solve for the distance away from the counter that the puck will land, we used two kinematic equations for projectile motion.

$$\Delta x = v_x t \ (for\ horizontal\ motion)$$

$$y = y_o + v_{oy}t + \frac{1}{2}at^2 (for\ vertical\ motion)$$

Using the equation for vertical motion, we solved for t with the given values as a quadratic equation. v_x *and* v_{oy} break down into $v_o\ cos\theta$ and $v_o\ sin\theta$, respectively. Then, we plugged t back into the horizontal motion equation to find Δx. This number was our final answer to the first section of the problem.

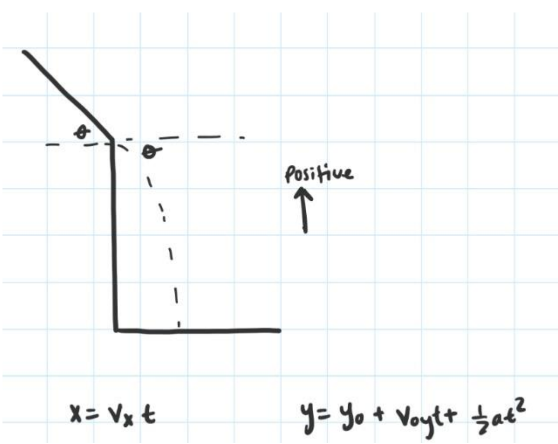


Figure 4: Diagram of Projectile Motion Phase. The puck will fall from the end of the ramp to the ground.

The second part of the question asks us to find the angle θ that would maximize the distance the puck would land away from the base of the counter. We used two methods to find this maximum angle. The first method consisted of us putting equations into Excel, which we then tested on every integer from 1 to 90. After we tested all the integers, we sorted the data and found the two integer angles that maximized the distance X_{BC}. Once we had these two integer angles, we tested every value that had a precision of .01 degrees between them before reaching our final answer.

The second method that we used was much more efficient, since we only had to type in one equation. We used Desmos Graphing Calculator to find the x that would maximize our function, since Desmos can calculate it automatically.

Solution:

As explained in the Process section above, we must first solve for the final velocity of the puck traveling down the ramp. To find the velocity, we must first find the acceleration, and then use the "no t" kinematic equation.

Using the equation derived in our process above and the given values, we solved for acceleration.

Given values:

$$m = 72g \ or \ 0.072kg$$

$$g = 9.8 \ m/s^2$$

$$\theta = 38°$$

$$\mu = 0.17$$

Solving for a:

$$\frac{(mgsin\theta - \mu mgcos\theta)}{m} = a$$

$$\frac{0.072(9.8)\sin(38°) - 0.17(0.072)(9.8)\cos(38°)}{0.072} = a$$

$$4.72\frac{m}{s^2} = a$$

Now, we can use the "no t" kinematic equation to solve for the final velocity.

Given or calculated values:

$$v_o = 0\frac{m}{s}$$

$$a = \ 4.72\frac{m}{s^2}$$

$$\Delta x = 2.9m$$

Solving for v:

$$v^2 = v_o^2 + 2a\Delta x$$

$$v^2 = 2(4.72)(2.9)$$

$$v = 5.23\ m/s$$

Knowing this final velocity, we can use it as the initial velocity when solving for projectile motion using the equations from our process.

Given or calculated values:

$$v \quad = 5.23\ m/s$$

$$\theta = 38°$$

$$a = -9.8\frac{m}{s^2}\ (acceleration\ due\ to\ gravity)$$

$$y = 0m\ (on\ the\ ground)$$

$$y_o = 1.6m\ (height\ of\ countertop)$$

Solving for t:

$$y = y_o + v_{oy}t + \frac{1}{2}at^2$$

$$y = y_o + v_o(sin\theta)t + \frac{1}{2}at^2$$

$$0 = 1.6 + (5.23)\sin(38°)\ t + \frac{1}{2}at^2$$

$$t = 0.33s$$

Solving for x:

$$\Delta x = v_x t$$

$$\Delta x = v_0 cos\theta t$$

$$\Delta x = 1.363\ m$$

After sliding down the ramp and falling off the end, the puck will land 1.363m away from the countertop.

Our first, less efficient solution for part B used an Excel spreadsheet to find the distance X_{BC} for any given angle theta. We were able to find (as shown in Fig. 5) that the values of 27 and 26 degrees were the two most optimal values to maximize X_{BC}.

Angle (Degrees)	Angle (Radians)	mass (kg)	mu	Ramp Distance (m)	a	v (End of Ram[	delta y	Time (sec)	delta x	Angle
27	0.471238898	0.072	0.17	2.9	2.964690028	4.1467098	-1.6	0.410754978	1.5176351	27
26	0.453785606	0.072	0.17	2.9	2.798646357	4.028914106	-1.6	0.418954052	1.517101335	26

Figure 5: Our excel spreadsheet, where we found the maximum "delta x" or X_{BC}.

With this in mind, we now looked at values between 26 and 27 (i.e. 26, 26.01, 26.02, ..., 26.98, 26.99, 27) and found that (as shown in Fig. 6), the optimal value of the angle x was 26.65 degrees, and that the maximum distance X_{BC} was roughly 1.518 m.

26.65	0.46513	0.072	0.17	2.9	2.906675	4.105936	-1.6	0.413611116	1.517843076

Figure 6: The optimal angle x calculated in Excel.

Our second, and most efficient solution for part B relied on Desmos Graphic Calculator. As shown in Fig. 7, we typed in the equation of X_{BC} in terms of the angle x (in radians) and Desmos graphed the function for us. We are able to click on the maximum, showing us that the x-coordinate (or angle in radians) that maximizes X_{BC} is roughly .46513 radians. Converting this to degrees gives us 26.64998... which is approximately 26.65 degrees. As shown again on the graph, the maximum distance X_{BC} would be the y-coordinate of the point shown, or roughly 1.518 m, the same as we got in our Excel solution.

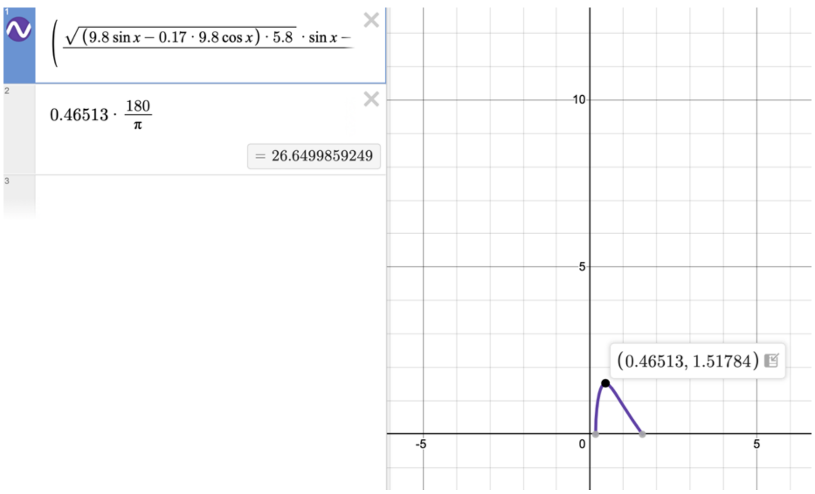
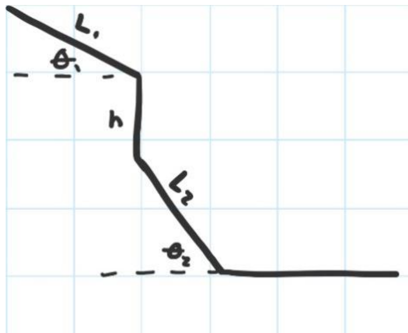


Figure 7: Using Desmos to find the maximum angle x. The x-coordinate of the maximum is shown.

Therefore, the angle theta that would maximize the distance X_{BC} would be 26.65 degrees, and the maximum distance X_{BC} would be roughly 1.518 m.

Extensions:

1. A puck with a mass of 72g begins at rest on an inclined plane (θ_1 = 38 degrees above the horizontal) with a length (L_1) of 2.9m. The coefficient of friction between the puck and the first ramp is 0.17. The first ramp is situated on top of a 1.6m tall counter (h) which will cause the puck to fall and land a certain distance away from the counter when it reaches the second ramp. This second inclined plane (θ_2 = 45 degrees above the horizontal) will have a length (L_2) of 3.4m. The coefficient of friction between this second ramp and the puck would be 0.20. Find the velocity of the puck when it leaves the second ramp and determine the angle of the first ramp that would maximize the final velocity of the second ramp.



2. Find the mass of the puck which would maximize the distance away from the counter that the puck would land (X_{BC}) in the original problem (using θ = 38 degrees above the horizontal).
    a. Taking a look at the equation we used to solve for a (shown below), we can see that the masses (represented by the variable m) actually cancel each other out, which means that no matter the mass of the puck, the distance the puck would land would stay the same.

$$\frac{mg \cdot sin\theta \; - \; \mu mg \; \cdot \; cos\theta}{m} = a$$

Evaluation

Overall, I really enjoyed this problem. Particularly, I enjoyed part B, since it isn't every day that you do modeling like this in physics. My favorite part of the problem was how I felt after seeing that the optimal value of theta we found in Excel and in Desmos were the same, since it was super cool to see multiple different ways of finding the same answer. I think the problem was a pretty good difficulty, since part A was pretty easy, whereas part B made you think a bit more. As for changing the problem, I thought it would be interesting if there was maybe a part in between parts A and B where you have to find an equation for X_{BC} in terms of theta, but at the same time, I figure this may be a bit complicated. I also think my group was able to work very well together, as we talked through our steps together to actually solve the problem, and when it came to divvying up the work for the write-up, I think we were also able to do a good job.