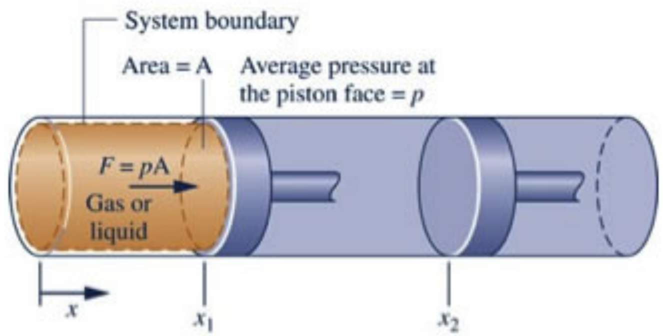
Modeling Expansion and Compression Work

► A case having many practical applications is a gas (or liquid) undergoing an expansion (or compression) process while confined in a piston-cylinder assembly.



► During the process, the gas exerts a normal force on the piston, $F = p\mathrm{A}$, where p is the pressure at the interface between the gas and piston and A is the area of the piston face.

Modeling Expansion and Compression Work

► From mechanics, the work done by the gas as the piston face moves from x_1 to x_2 is given by

$$W = \int F dx = \int p A dx$$

► Since the product $A dx = dV$, where V is the volume of the gas, this becomes

$$W = \int_{V_1}^{V_2} p dV \qquad \textbf{(Eq. 2.17)}$$

► For a compression, dV is negative and so is the value of the integral, in keeping with the sign convention for work.

Modeling Expansion and Compression Work

►To perform the integral of **Eq. 2.17** requires a relationship between gas pressure at the interface between the gas and piston and the total gas volume.

►During an actual expansion of a gas such a relationship may be difficult, or even impossible, to obtain owing to non-equilibrium effects during the process – for example, effects related to combustion in the cylinder of an automobile engine.

►In most such applications, the work value can be obtained only by experiment.

Modeling Expansion and Compression Work

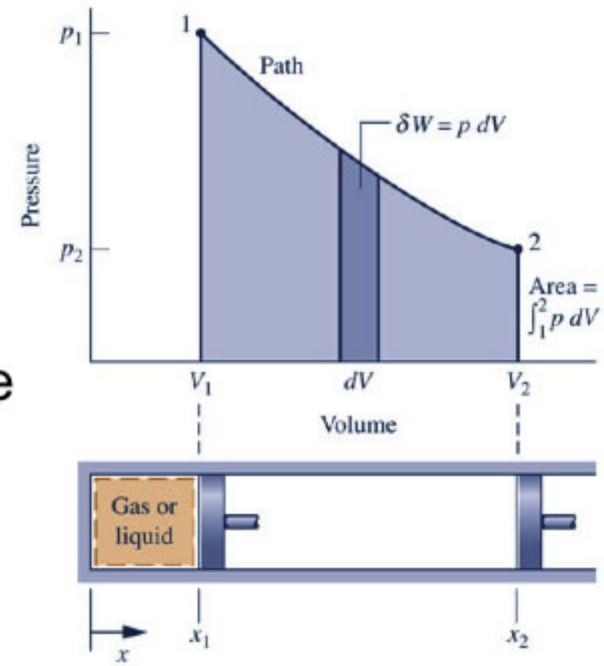
►**Eq. 2.17** can be applied to evaluate the work of idealized processes during which the pressure p in the integrand is the pressure of the entire quantity of the gas undergoing the process and not only the pressure at the piston face.

►For this we imagine the gas undergoes a sequence of equilibrium states during the process. Such an idealized expansion (or compression) is called a ***quasiequilibrium*** process.

Modeling Expansion and Compression Work

► In a quasiequilibrium expansion, the gas moves along a pressure-volume curve, or path, as shown.

► Applying **Eq. 2.17**, the work done by the gas on the piston is given by the **area** under the curve of pressure versus volume.



Modeling Expansion and Compression Work

► When the pressure-volume relation required by **Eq. 2.17** to evaluate work in a quasiequilibrium expansion (or compression) is expressed as an equation, the evaluation of expansion or compression work can be simplified.

► An example is a quasiequilibrium process described by $pV^n = constant$, where n is a constant. This is called a *polytropic process*.

► For the case $n = 1$, $pV = constant$ and **Eq. 2.17** gives

$$W = (constant)\ln\left(\frac{V_2}{V_1}\right)$$ **where** $constant = p_1V_1 = p_2V_2$.

Modeling Expansion and Compression Work

► Since non-equilibrium effects are invariably present during actual expansions (and compressions), the work determined with quasiequilibrium modeling can at best approximate the actual work of an expansion (or compression) between given end states.

You will need to analyze the integral of PdV in order to estimate the work associated with volume expansions/compressions.

$$W = \int_{V_1}^{V_2} p\,dV$$

So, for polytropic situations: pV^n = constant, then

$$W = \int_{V_1}^{V_2} \frac{constant}{V^n} dV = constant \int_{V_1}^{V_2} V^{-n} dV$$

But, most gas situations will use Tables that have combined the Internal Energy (U) with the pV and a thermodynamic term (Enthalpy, H or h) is documented.

That is: H = U + pV

h = u + pv

(more on this when we reach Chapter 3)

Table A-4 (Continued)

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	p = 5.0 bar = 0.50 MPa (T_{sat} = 151.86°C)				p = 7.0 bar = 0.70 MPa (T_{sat} = 164.97°C)			
Sat.	0.3749	2561.2	2748.7	6.8213	0.2729	2572.5	2763.5	6.7080
180	0.4045	2609.7	2812.0	6.9656	0.2847	2599.8	2799.1	6.7880
200	0.4249	2642.9	2855.4	7.0592	0.2999	2634.8	2844.8	6.8865
240	0.4646	2707.6	2939.9	7.2307	0.3292	2701.8	2932.2	7.0641
280	0.5034	2771.2	3022.9	7.3865	0.3574	2766.9	3017.1	7.2233
320	0.5416	2834.7	3105.6	7.5308	0.3852	2831.3	3100.9	7.3697
360	0.5796	2898.7	3188.4	7.6660	0.4126	2895.8	3184.7	7.5063
400	0.6173	2963.2	3271.9	7.7938	0.4397	2960.9	3268.7	7.6350
440	0.6548	3028.6	3356.0	7.9152	0.4667	3026.6	3353.3	7.7571
500	0.7109	3128.4	3483.9	8.0873	0.5070	3126.8	3481.7	7.9299
600	0.8041	3299.6	3701.7	8.3522	0.5738	3298.5	3700.2	8.1956
700	0.8969	3477.5	3925.9	8.5952	0.6403	3476.6	3924.8	8.4391

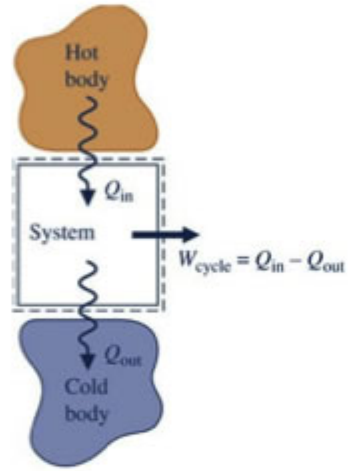
Modes of Heat Transfer

- For any particular application, energy transfer by heat can occur by one or more of three modes:
  - conduction
  - radiation
  - convection

Thermodynamic Cycles

- A *thermodynamic cycle* is a sequence of processes that begins and ends at the same state.
- Examples of thermodynamic cycles include
  - **Power cycles** that develop a net energy transfer by work in the form of electricity using an energy input by heat transfer from hot combustion gases.
  - **Refrigeration cycles** that provide cooling for a refrigerated space using an energy input by work in the form of electricity.
  - **Heat pump cycles** that provide heating to a dwelling using an energy input by work in the form of electricity.

Power Cycle



► A system undergoing a power cycle is shown at right.

► The energy transfers by heat and work shown on the figure are each positive in the direction of the accompanying arrow. This convention is commonly used for analysis of thermodynamic cycles.

► W_{cycle} is the **net energy transfer by work** from the system per cycle of operation – in the form of electricity, typically.

► Q_{in} is the **heat transfer of energy to the system** per cycle from the hot body – drawn from hot gases of combustion or solar radiation, for instance.

► Q_{out} is the **heat transfer of energy from the system** per cycle to the cold body – discharged to the surrounding atmosphere or nearby lake or river, for example.

Power Cycle

►Applying the closed system energy balance to each cycle of operation,

$$\Delta E_{\text{cycle}} = Q_{\text{cycle}} - W_{\text{cycle}} \qquad \textbf{(Eq. 2.39)}$$

►Since the system returns to its initial state after each cycle, there is no net change in its energy: $\boldsymbol{\Delta E_{\text{cycle}} = 0}$, and the energy balance reduces to give

$$W_{\text{cycle}} = Q_{\text{in}} - Q_{\text{out}} \qquad \textbf{(Eq. 2.41)}$$

►In words, the ***net*** energy transfer by work from the system equals the ***net*** energy transfer by heat to the system, each per cycle of operation.

Power Cycle

► The performance of a system undergoing a power cycle is evaluated on an energy basis in terms of the extent to which the energy added by heat, Q_{in}, is converted to a net work output, W_{cycle}. This is represented by the ratio

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} \quad \text{(power cycle)} \qquad \textbf{(Eq. 2.42)}$$

called the thermal efficiency.

► Introducing **Eq. 2.41**, an alternative form is obtained

$$\eta = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} \quad \text{(power cycle)} \qquad \textbf{(Eq. 2.43)}$$

Power Cycle

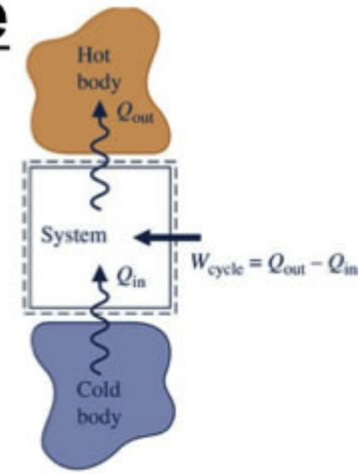
►Using the second law of thermodynamics (Chapter 5), we will show that the value of thermal efficiency must be less than unity: $\eta < 1$ (< 100%). That is, only a portion of the energy added by heat, Q_{in}, can be obtained as work. The remainder, Q_{out}, is discharged.

Example: A system undergoes a power cycle while receiving 1000 kJ by heat transfer from hot combustion gases at a temperature of 500 K and discharging 600 kJ by heat transfer to the atmosphere at 300 K. Taking the combustion gases and atmosphere as the hot and cold bodies, respectively, determine for the cycle, the net work developed, in kJ, and the thermal efficiency.

► Substituting into **Eq. 2.41**, W_{cycle} **= 1000 kJ – 600 kJ = 400 kJ**.

► Then, with **Eq. 2.42**, η **= 400 kJ/1000 kJ = 0.4 (40%)**. Note the thermal efficiency is commonly reported on a percent basis.

Refrigeration Cycle



► A system undergoing a refrigeration cycle is shown at right.

► As before, the energy transfers are each positive in the direction of the accompanying arrow.

- ► W_{cycle} is the **net energy transfer by work** to the system per cycle of operation, usually in the form of electricity.
- ► Q_{in} is the **heat transfer of energy to the system** per cycle from the cold body – drawn from a freezer compartment, for example.
- ► Q_{out} is the **heat transfer of energy from the system** per cycle to the hot body – discharged to the space surrounding the refrigerator, for instance.

Refrigeration Cycle

► Since the system returns to its initial state after each cycle, there is no net change in its energy: $\Delta E_{\text{cycle}} = 0$, and the energy balance reduces to give

$$W_{\text{cycle}} = Q_{\text{out}} - Q_{\text{in}} \qquad \textbf{(Eq. 2.44)}$$

► In words, the ***net*** energy transfer by work to the system equals the ***net*** energy transfer by heat from the system, each per cycle of operation.

Refrigeration Cycle

► The performance of a system undergoing a refrigeration cycle is evaluated on an energy basis as the ratio of energy drawn from the cold body, Q_{in}, to the net work required to accomplish this effect, W_{cycle}:

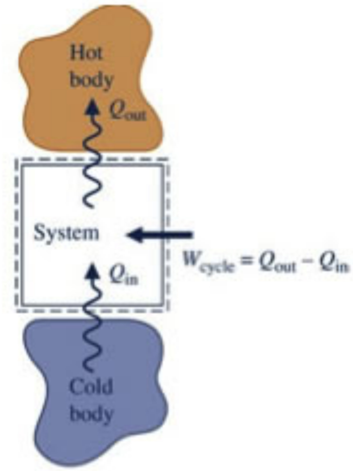
$$\beta = \frac{Q_{in}}{W_{cycle}} \quad \text{(refrigeration cycle)} \qquad \textbf{(Eq. 2.45)}$$

called the coefficient of performance for the refrigeration cycle.

► Introducing **Eq. 2.44**, an alternative form is obtained

$$\beta = \frac{Q_{in}}{Q_{out} - Q_{in}} \quad \text{(refrigeration cycle)} \qquad \textbf{(Eq. 2.46)}$$

Heat Pump Cycle



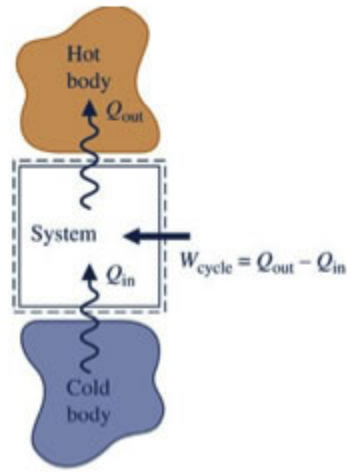
► The heat pump cycle analysis closely parallels that given for the refrigeration cycle. The same figure applies:

► But now the focus is on Q_{out}, which is the **heat transfer of energy from the system** per cycle to the hot body – such as to the living space of a dwelling.

► Q_{in} is the **heat transfer of energy to the system** per cycle from the cold body – drawn from the surrounding atmosphere or the ground, for example.

Heat Pump Cycle

► As before, W_{cycle} is the **net energy transfer by work** to the system per cycle, usually provided in the form of electricity.



► As for the refrigeration cycle, the energy balance reads

$$W_{cycle} = Q_{out} - Q_{in} \quad \textbf{(Eq. 2.44)}$$

Heat Pump Cycle

► The performance of a system undergoing a heat pump cycle is evaluated on an energy basis as the ratio of energy provided to the hot body, Q_{out}, to the net work required to accomplish this effect, W_{cycle}:

$$\gamma = \frac{Q_{out}}{W_{cycle}} \quad \text{(heat pump cycle)} \qquad \textbf{(Eq. 2.47)}$$

called the coefficient of performance for the heat pump cycle.

► Introducing **Eq. 2.44**, an alternative form is obtained

$$\gamma = \frac{Q_{out}}{Q_{out} - Q_{in}} \quad \text{(heat pump cycle)} \qquad \textbf{(Eq. 2.48)}$$

Heat Pump Cycle

Example: A system undergoes a heat pump cycle while discharging 900 kJ by heat transfer to a dwelling at 20^{o}C and receiving 600 kJ by heat transfer from the outside air at 5^{o}C. Taking the dwelling and outside air as the hot and cold bodies, respectively, determine for the cycle, the net work input, in kJ, and the coefficient of performance.

▶ Substituting into **Eq. 2.44**, W_{cycle} = **900 kJ – 600 kJ = <u>300 kJ</u>**.

▶ Then, with **Eq. 2.47**, γ = **900 kJ/300 kJ = <u>3.0</u>**. Note the coefficient of performance is reported as its numerical value, as calculated here.

Go to the: [next lecture | chapter top | previous lecture]