A polytropic process is a quasiequilibrium process described by

\[ pV^n = \text{constant} \]  

(Eq. 3.52)

The exponent, \( n \), may take on any value from \(-\infty\) to \(+\infty\) depending on the particular process.

- For any gas (or liquid), when \( n = 0 \), the process is a constant-pressure (isobaric) process.
- For any gas (or liquid), when \( n = \pm\infty \), the process is a constant-volume (isometric) process.
- For a gas modeled as an ideal gas, when \( n = 1 \), the process is a constant-temperature (isothermal) process.
Chapter 4

Control Volume Analysis Using Energy
Mass Rate Balance

\[
\frac{dm_{CV}}{dt} = \dot{m}_i - \dot{m}_e
\]  
(Eq. 4.1)
Energy Rate Balance

\[ \frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \dot{m}_i \left( u_i + \frac{V_i^2}{2} + gz_i \right) - \dot{m}_e \left( u_e + \frac{V_e^2}{2} + gz_e \right) \]  
(Eq. 4.9)
Evaluating Work for a Control Volume

The expression for work is

\[ \dot{W} = \dot{W}_{cv} + \dot{m}_e(p_ev_e) - \dot{m}_i(p_iv_i) \]  

(Eq. 4.12)

where

- \( \dot{W}_{cv} \) accounts for work associated with rotating shafts, displacement of the boundary, and electrical effects.
- \( \dot{m}_e(p_ev_e) \) is the flow work at exit \( e \).
- \( \dot{m}_i(p_iv_i) \) is the flow work at inlet \( i \).
Control Volume Energy Rate Balance
(One-Dimensional Flow Form)

Using Eq. 4.12 in Eq. 4.9

\[
\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i (u_i + p_i v_i + \frac{V_i^2}{2} + gz_i) - \dot{m}_e (u_e + p_e v_e + \frac{V_e^2}{2} + gz_e)
\]

(Eq. 4.13)

For convenience substitute enthalpy, \( h = u + pv \)

\[
\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i (h_i + \frac{V_i^2}{2} + gz_i) - \dot{m}_e (h_e + \frac{V_e^2}{2} + gz_e)
\]

(Eq. 4.14)
Control Volume Energy Rate Balance
(One-Dimensional Flow Form)

In practice there may be several locations on the boundary through which mass enters or exits. **Multiple inlets and exits** are accounted for by **introducing summations**:

\[
\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i (h_i + \frac{V_i^2}{2} + gz_i) - \sum_e \dot{m}_e (h_e + \frac{V_e^2}{2} + gz_e)
\]

(Eq. 4.15)

**Eq. 4.15** is the **accounting balance** for the energy of the control volume.
Control Volume Energy Rate Balance
(Steady-State Form, One-Inlet, One-Exit)

Many important applications involve one-inlet, one-exit control volumes at steady state.

The mass rate balance reduces to $m_1 = m_2 = \dot{m}$.

\[
0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right] \quad \text{(Eq. 4.20a)}
\]

or dividing by mass flow rate

\[
0 = \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \quad \text{(Eq. 4.20b)}
\]
Nozzles and Diffusers

► **Nozzle**: a flow passage of varying cross-sectional area in which the velocity of a gas or liquid **increases** in the direction of flow.

► **Diffuser**: a flow passage of varying cross-sectional area in which the velocity of a gas or liquid **decreases** in the direction of flow.
Nozzle and Diffuser Modeling

\[ 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right] \]  
\text{(Eq. 4.20a)}

\( \dot{W}_{cv} = 0. \)

\( \dot{Q}_{cv} = 0. \)

If the change in potential energy from inlet to exit is negligible, \( g(z_1 - z_2) \) drops out.

If the heat transfer with surroundings is negligible, \( \dot{Q}_{cv} \) drops out.

\[ 0 = (h_1 - h_2) + \left( \frac{V_1^2 - V_2^2}{2} \right) \]  
\text{(Eq. 4.21)}
Turbines

>Turbine: a device in which power is developed as a result of a gas or liquid passing through a set of blades attached to a shaft free to rotate.
Turbine Modeling

\[ 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right] \]

\[ \text{Eq. 4.20a} \]

► If the change in kinetic energy of flowing matter is negligible, \( \frac{1}{2}(V_1^2 - V_2^2) \) drops out.

► If the change in potential energy of flowing matter is negligible, \( g(z_1 - z_2) \) drops out.

► If the heat transfer with surroundings is negligible, \( \dot{Q}_{cv} \) drops out.

\[ \dot{W}_{cv} = \dot{m}(h_1 - h_2) \]
Compressors and Pumps

- Compressors and Pumps: devices in which work is done on the substance flowing through them to change the state of the substance, typically to increase the pressure and/or elevation.

- **Compressor**: substance is gas
- **Pump**: substance is liquid
Compressor and Pump Modeling

\[ 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right] \]

\[ \text{Eq. 4.20a} \]

► If the change in kinetic energy of flowing matter is negligible, \( \frac{1}{2}(V_1^2 - V_2^2) \) drops out.

► If the change in potential energy of flowing matter is negligible, \( g(z_1 - z_2) \) drops out.

► If the heat transfer with surroundings is negligible, \( \dot{Q}_{cv} \) drops out.

\[ \dot{W}_{cv} = \dot{m}(h_1 - h_2) \]
Heat Exchangers

► **Direct contact**: A mixing chamber in which hot and cold streams are mixed directly.

► **Tube-within-a-tube counterflow**: A gas or liquid stream is separated from another gas or liquid by a wall through which energy is conducted. Heat transfer occurs from the hot stream to the cold stream as the streams flow in opposite directions.
Heat Exchanger Modeling

\[ 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i m_i (h_i + \frac{V_i^2}{2} + g z_i) - \sum_e m_e (h_e + \frac{V_e^2}{2} + g z_e) \]  

- \( \dot{W}_{cv} = 0. \)
- If the kinetic energies of the flowing streams are negligible, \( m_i (V_i^2/2) \) and \( m_e (V_e^2/2) \) drop out.
- If the potential energies of the flowing streams are negligible, \( m_i g z_i \) and \( m_e g z_e \) drop out.
- If the heat transfer with surroundings is negligible, \( \dot{Q}_{cv} \) drops out.

\[ 0 = \sum_i m_i h_i - \sum_e m_e h_e \]  

(Eq. 4.18)
Throttling Devices

Throttling Device: a device that achieves a significant reduction in pressure by introducing a restriction into a line through which a gas or liquid flows. Means to introduce the restriction include a partially opened valve or a porous plug.
Throttling Device Modeling

\[
0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{\left(V_1^2 - V_2^2\right)}{2} + g(z_1 - z_2) \right]
\]  

\text{(Eq. 4.20a)}

► \(\dot{W}_{cv} = 0\).

► If the change in kinetic energy of flowing matter upstream and downstream of the restriction is negligible, \(\frac{1}{2}(V_1^2 - V_2^2)\) drops out.

► If the change in potential energy of flowing matter is negligible, \(g(z_1 - z_2)\) drops out.

► If the heat transfer with surroundings is negligible, \(\dot{Q}_{cv}\) drops out.

\[h_2 = h_1\]  

\text{(Eq. 4.22)}
System Integration

Engineers creatively combine components to achieve some overall objective, subject to constraints such as minimum total cost. This engineering activity is called system integration.

The simple vapor power plant of Fig 4.16 provides an illustration.
The Mass Balance
(Transient Analysis)

► **Transient**: state changes with time.

► **Integrate** mass rate balance (Eq. 4.2) from time 0 to a final time $t$.

\[
\int_0^t \left( \frac{dm_{cv}}{dt} \right) dt = \int_0^t \left( \sum_i \dot{m}_i \right) dt - \int_0^t \left( \sum_e \dot{m}_e \right) dt
\]

This becomes

\[
m_{cv}(t) - m_{cv}(0) = \sum_i m_i - \sum_e m_e \quad \text{(Eq. 4.23)}
\]

where

• $m_i$ is amount of mass entering the control volume through inlet $i$, from time 0 to $t$.

• $m_e$ is amount of mass exiting the control volume through exit $e$, from time 0 to $t$. 
**The Energy Balance**

**(Transient Analysis)**

**Integrate** energy rate balance (Eq. 4.15), ignoring the effects of kinetic and potential energy, from time $0$ to a final time $t$.

\[
\int_0^t \left( \frac{dE_{cv}}{dt} \right) dt = \int_0^t \dot{Q}_{cv} dt - \int_0^t \dot{W}_{cv} dt + \sum_{i} \int_0^t \left( \dot{m}_i h_i \right) dt - \sum_{e} \int_0^t \left( \dot{m}_e h_e \right) dt
\]

When the **specific enthalpies at inlets and exits are constant with time**, this becomes

**Eq. 4.25**

\[
E_{cv}(t) - E_{cv}(0) = Q_{cv} - W_{cv} + \sum_{i} \dot{m}_i h_i - \sum_{e} \dot{m}_e h_e
\]
Consider a typical garden hose

Assume the pressure in the hose (state 1) is 30 psig at a temperature of 70 °F with a velocity of 5 ft/sec. The child receives the water at 65 °F. What is the exit velocity?
Consider the figure below of a perfect gas situation. One kilogram of nitrogen fills the cylinder of a piston-cylinder assembly. There is no friction between the piston and the cylinder walls, and the surroundings are at 1 atm. The initial volume and pressure in the cylinder are 1 m$^3$ and 1 atm, respectively. Heat transfer to the nitrogen occurs until the volume is doubled. (1 atm = 1.01325 bar; 1 bar = $10^5$ N/m$^2$)

a) Determine the work for the process, in kJ.

b) Determine the heat transfer for the process, in kJ, assuming the specific heat (0.742 kJ/(kgK)) is constant. Recall: $R = 8.314$ kJ/(kmol K); $M_{N_2} = 28.01$ kg/kmol
Molecular weight of N\textsubscript{2} gas is \(M_{N2} = 28.01\text{kg/kmol}\)

\[R = \frac{R(\text{universal constant})}{M \text{ (Molecular weight)}} = \frac{8.314}{28.01} = 0.2968 \text{ kJ/(kg K)}\]

\[PV = mRT \text{ (Know } P_1, V_1, m_1, R) \text{ Solve for } T_1 = 341.4K\]

\[PV = mRT \text{ (Know } P_2, V_2, m_2, R) \text{ Solve for } T_2 \text{ (But } P_2 = P_1, V_2 = 2V_1), \text{ therefore } T_2 = 2*T_1 = 682.8K\]

\[Q = m(u_2-u_1) + W\]

\[
\frac{du}{dT} = c_v \text{ Or } (u_2-u_1) = c_v (T_2-T_1) = 0.742(341.4K) = 253.32 \text{ kJ/kg}
\]

\[Q = 1\text{kg}(253.32\text{kJ/kg}) + 1.01325 \times 10^5 \text{ N/m}^2(2\text{m}^3-1\text{m}^3)(1\text{kJ/(10}^3 \text{ N/m}^2)) = 354.6 \text{ kJ}\]
Two kilograms of water at 25°C are placed in a piston cylinder device under 100 kPa absolute pressure as shown in the diagram (State(1)). Heat is added to the water at constant pressure until the piston reaches the stops at a total volume of 0.4 m$^3$ (State (2)). More heat is then added at constant volume until the temperature of the water reaches 300°C (State (3)).

i) Draw a P-v and a T-v diagram of the states and processes of the problem and include all the relevant information on the diagram. In this case there are three states and two processes (state 1 to state 2 and state 2 to state 3). The diagrams do not have to be drawn to scale.

ii) Determine the quality (x) of the fluid and the mass of the vapor at state (2). (Schematic is not necessarily to scale.) Calculate the specific volume, specific internal energy, and specific enthalpy of the state.
i) Draw a P-v and a T-v diagram of the states and processes of the problem and include all the relevant information on the diagram. In this case there are three states and two processes (state 1 to state 2 and state 2 to state 3). The diagrams do not have to be drawn to scale.

It is not known where quality x2 is, just somewhere on the horizontal line in the wet region. $P_{CR}$ line is not necessary, nor are the blue lines necessary.
i) Determine the quality (x) of the fluid and the mass of the vapor at state (2). (Schematic is not necessarily to scale.) Calculate the specific volume, specific internal energy, and specific enthalpy of the state.

\[ v = v_f + x(v_g - v_f) = \frac{V}{m} = \frac{0.4}{2} = 0.2 \text{ kg} \]

\[ x = \frac{v - v_f}{(v_g - v_f)} = \frac{0.2 - 0.0010432}{(1.694 - 0.0010432)} = 0.117 \]

\[ u = u_f + x(u_g - u_f) = 417.36 + 0.117(2506.1 - 417.36) = 661.74 \]

\[ h = h_f + x(h_g - h_f) = 417.46 + 0.117(2675.5 - 417.46) = 681.65 \]

\[ x = \frac{v - v_f}{(v_g - v_f)} = \frac{0.2 - 0.0010432}{(1.694 - 0.0010432)} = 0.117 \]

mass of vapor = \( x \) (mass) = 0.117 \times 2 \text{ kg} = 0.234 \text{ kg}