Chapter 4

Control Volume Analysis Using Energy

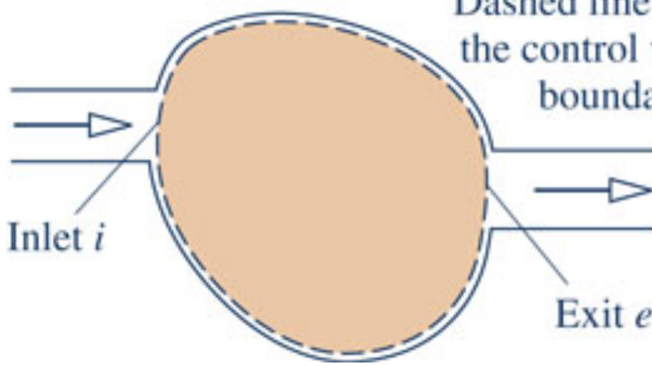
Learning Outcomes

- ►Demonstrate understanding of key concepts related to control volume analysis including distinguishing between steady-state and transient analysis, distinguishing between mass flow rate and volumetric flow rate, and the meanings of one-dimensional flow and flow work.
- ►Apply mass and energy balances to control volumes.

Learning Outcomes, cont.

- ►Develop appropriate engineering models for control volumes, with particular attention to analyzing components commonly encountered in engineering practice such as nozzles, diffusers, turbines, compressors, heat exchangers, throttling devices, and integrated systems that incorporate two or more components.
- ►Use property data in control volume analysis appropriately.

Mass Rate Balance



$$\begin{bmatrix} \text{time } \textit{rate of change} \text{ of} \\ \text{mass contained within the} \\ \text{control volume } \textit{at time t} \end{bmatrix} = \begin{bmatrix} \text{time } \textit{rate of flow} \text{ of} \\ \text{mass } \textit{in} \text{ across} \\ \text{inlet } \textit{i at time t} \end{bmatrix} - \begin{bmatrix} \text{time } \textit{rate of flow} \\ \text{of mass } \textit{out} \text{ across} \\ \text{exit } \textit{e at time t} \end{bmatrix}$$

$$\frac{dm_{\mathrm{cv}}}{dt} = \dot{m}_i - \dot{m}_e \qquad \textbf{(Eq. 4.1)}$$

Mass Rate Balance

In practice there may be several locations on the boundary through which mass enters or exits. **Multiple inlets** and **exits** are accounted for by **introducing summations**:

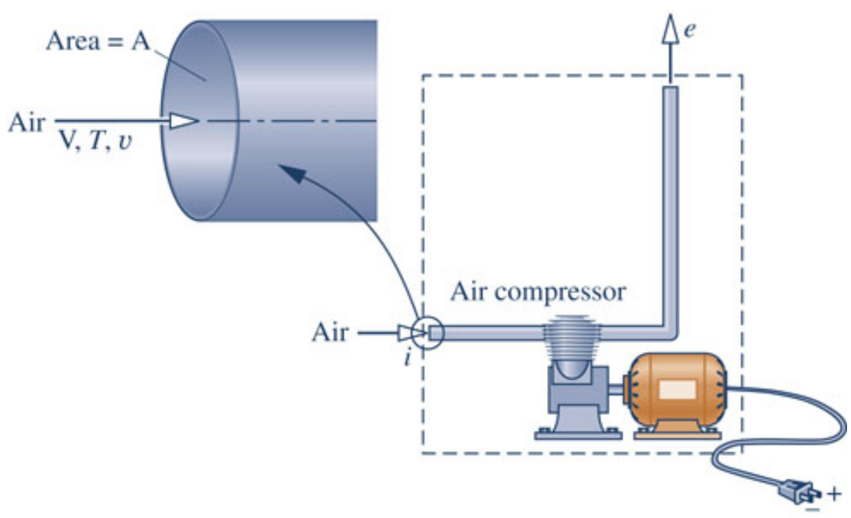
$$\frac{dm_{\text{cv}}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e \qquad \textbf{(Eq. 4.2)}$$

Eq. 4.2 is the **mass rate balance** for control volumes with **several inlets and exits**.

Mass Flow Rate

(One-Dimensional Flow)

- ►**Flow is normal** to the boundary at locations where mass enters or exits the control volume.
- ►***All*** intensive properties are ***uniform with position*** over each inlet or exit area (**A**) through which matter flows.



$$\dot{m} = \frac{AV}{v}$$ **(Eq. 4.4b)**

where

V is velocity

v is specific volume

Mass Rate Balance

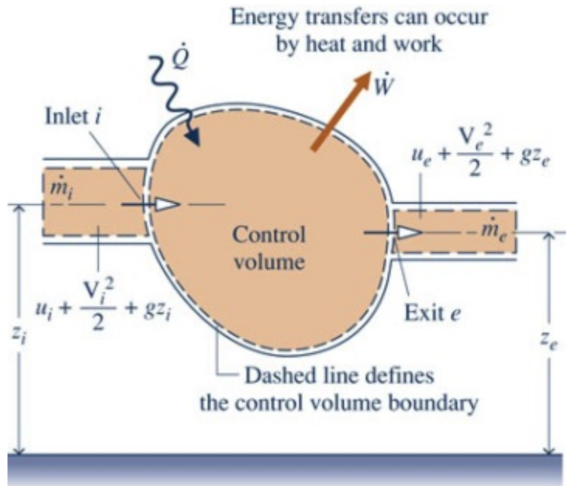
(Steady-State Form)

- **Steady-state**: all properties are unchanging in time.
- For steady-state control volume, $dm_{cv}/dt = 0$.

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e \qquad \text{(Eq. 4.6)}$$

(mass rate in) **(mass rate out)**

Energy Rate Balance



[time ***rate of change*** of the energy contained within the control volume ***at time t***] = [***net rate*** at which energy is being transferred in by heat transfer ***at time t***] − [***net rate*** at which energy is being transferred out by work ***at time t***] + [***net rate*** of energy transfer ***into*** the control volume accompanying mass flow]

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \dot{m}_i\left(u_i + \frac{V_i^2}{2} + gz_i\right) - \dot{m}_e\left(u_e + \frac{V_e^2}{2} + gz_e\right) \quad \textbf{(Eq. 4.9)}$$

Evaluating Work for a Control Volume

The expression for work is

$$\dot{W} = \dot{W}_{\mathrm{cv}} + \dot{m}_e(p_e v_e) - \dot{m}_i(p_i v_i) \quad \textbf{(Eq. 4.12)}$$

where

- $\dot{W}_{\mathrm{cv}}$ accounts for work associated with rotating shafts, displacement of the boundary, and electrical effects.
- $\dot{m}_e(p_e v_e)$ is the flow work at exit e.
- $\dot{m}_i(p_i v_i)$ is the flow work at inlet i.

Control Volume Energy Rate Balance

(One-Dimensional Flow Form)

Using Eq. 4.12 in Eq. 4.9

$$\frac{dE_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m}_i\left(\underline{u_i + p_i v_i} + \frac{V_i^2}{2} + gz_i\right) - \dot{m}_e\left(\underline{u_e + p_e v_e} + \frac{V_e^2}{2} + gz_e\right)$$

(Eq. 4.13)

For convenience substitute enthalpy, ***h* = *u* + *pv***

$$\frac{dE_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

(Eq. 4.14)

Control Volume Energy Rate Balance

(One-Dimensional Flow Form)

In practice there may be several locations on the boundary through which mass enters or exits. **Multiple inlets** and **exits** are accounted for by **introducing summations**:

$$\frac{dE_{\mathrm{cv}}}{dt} = \dot{Q}_{\mathrm{cv}} - \dot{W}_{\mathrm{cv}} + \sum_i \dot{m}_i\left(h_i + \frac{\mathrm{V}_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{\mathrm{V}_e^2}{2} + gz_e\right)$$

(Eq. 4.15)

Eq. 4.15 is the **accounting balance** for the energy of the control volume.

Control Volume Energy Rate Balance

(Steady-State Form)

► **Steady-state**: all properties are unchanging in time.

► For steady-state control volume, ***dE*$_{cv}$*/dt*** **= 0**.

$$0 = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

(Eq. 4.18)

Control Volume Energy Rate Balance

(Steady-State Form, One-Inlet, One-Exit)

- Many important applications involve one-inlet, one-exit control volumes at steady state.
- The mass rate balance reduces to $\dot{m}_1 = \dot{m}_2 = \dot{m}$.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

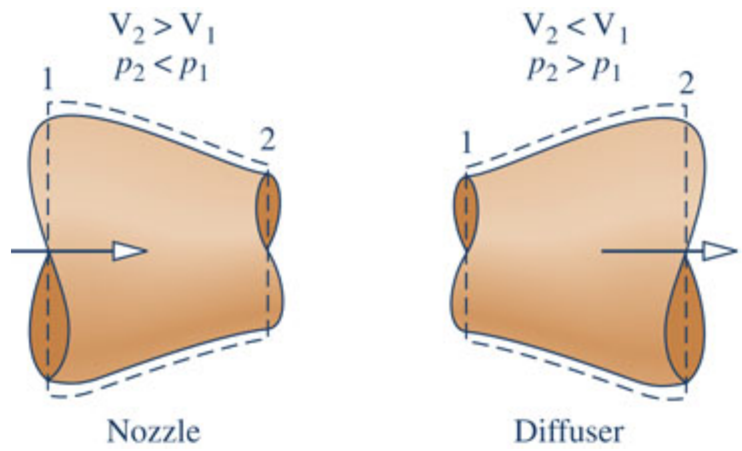
Eq. 4.20a

or dividing by mass flow rate

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)$$

Eq. 4.20b

Nozzles and Diffusers



►**Nozzle**: a flow passage of varying cross-sectional area in which the velocity of a gas or liquid increases in the direction of flow.

►**Diffuser**: a flow passage of varying cross-sectional area in which the velocity of a gas or liquid decreases in the direction of flow.

Nozzle and Diffuser Modeling

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

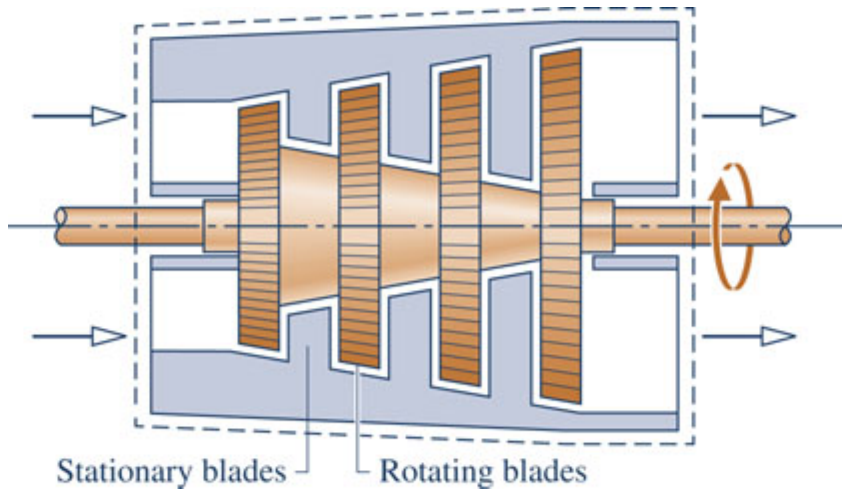
Eq. 4.20a

- ► $\dot{W}_{cv} = 0.$
- ► If the change in potential energy from inlet to exit is negligible, $g(z_1 - z_2)$ drops out.
- ► If the heat transfer with surroundings is negligible, $\dot{Q}_{cv}$ drops out.

$$0 = (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

(Eq. 4.21)

Turbines



► **Turbine**: a device in which power is developed as a result of a gas or liquid passing through a set of blades attached to a shaft free to rotate.

Turbine Modeling

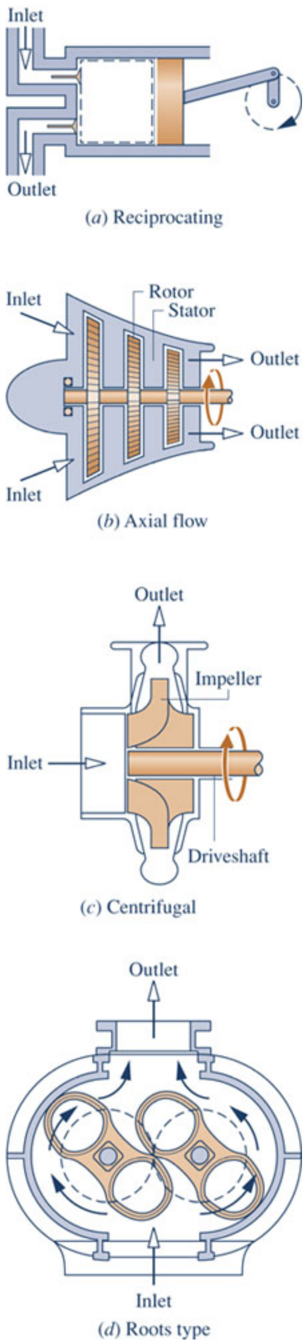
$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

Eq. 4.20a

- If the change in kinetic energy of flowing matter is negligible, $\frac{1}{2}(V_1^2 - V_2^2)$ drops out.
- If the change in potential energy of flowing matter is negligible, $g(z_1 - z_2)$ drops out.
- If the heat transfer with surroundings is negligible, $\dot{Q}_{cv}$ drops out.

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

Compressors and Pumps



- **Compressors and Pumps**: devices in which work is done on the substance flowing through them to change the state of the substance, typically to increase the pressure and/or elevation.
- *Compressor*: substance is gas
- *Pump*: substance is liquid

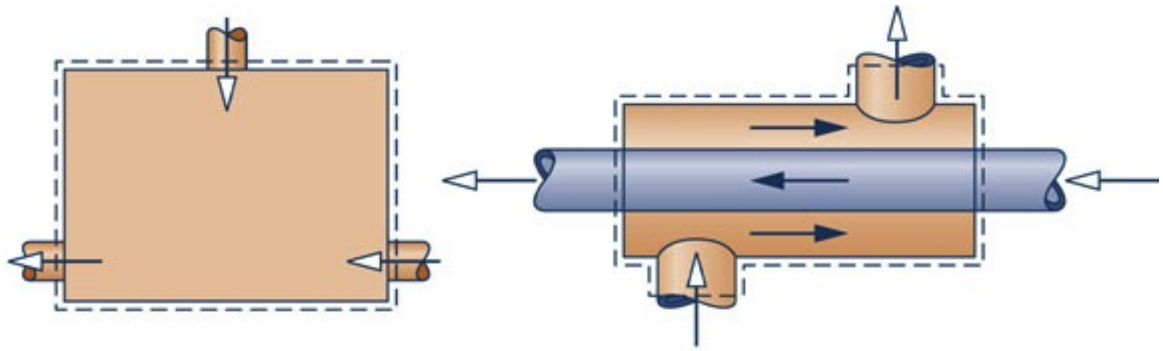
Compressor and Pump Modeling

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$ **Eq. 4.20a**

- If the change in kinetic energy of flowing matter is negligible, $\frac{1}{2}(V_1^2 - V_2^2)$ drops out.
- If the change in potential energy of flowing matter is negligible, $g(z_1 - z_2)$ drops out.
- If the heat transfer with surroundings is negligible, $\dot{Q}_{cv}$ drops out.

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

Heat Exchangers



(a) (b)

- ► **Direct contact:** A mixing chamber in which hot and cold streams are mixed directly.
- ► **Tube-within-a-tube counterflow:** A gas or liquid stream is *separated* from another gas or liquid by a wall through which energy is conducted. Heat transfer occurs from the hot stream to the cold stream as the streams flow in opposite directions.

Heat Exchanger Modeling

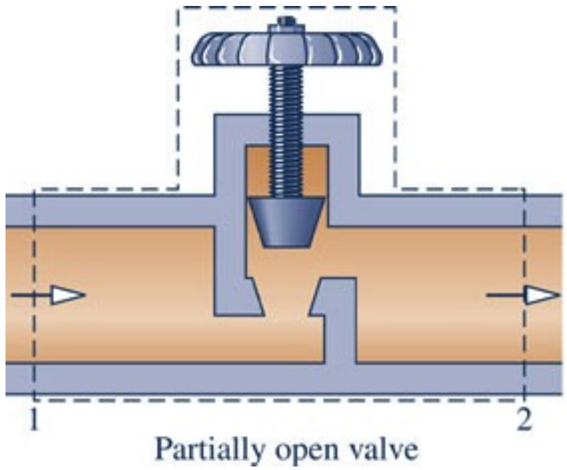
$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

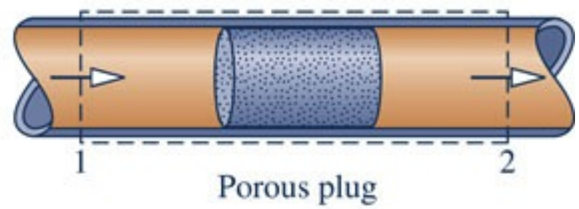
(Eq. 4.18)

- ► $\dot{W}_{cv} = 0.$
- ► If the kinetic energies of the flowing streams are negligible, $\dot{m}_i(V_i^2/2)$ and $\dot{m}_e(V_e^2/2)$ drop out.
- ► If the potential energies of the flowing streams are negligible, $\dot{m}_i g z_i$ and $\dot{m}_e g z_e$ drop out.
- ► If the heat transfer with surroundings is negligible, $\dot{Q}_{cv}$ drops out.

$$0 = \sum_i \dot{m}_i h_i - \sum_e \dot{m}_e h_e$$

Throttling Devices





- ►**Throttling Device**: a device that achieves a significant reduction in pressure by introducing a restriction into a line through which a gas or liquid flows. Means to introduce the restriction include a partially opened valve or a porous plug.

Throttling Device Modeling

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

Eq. 4.20a

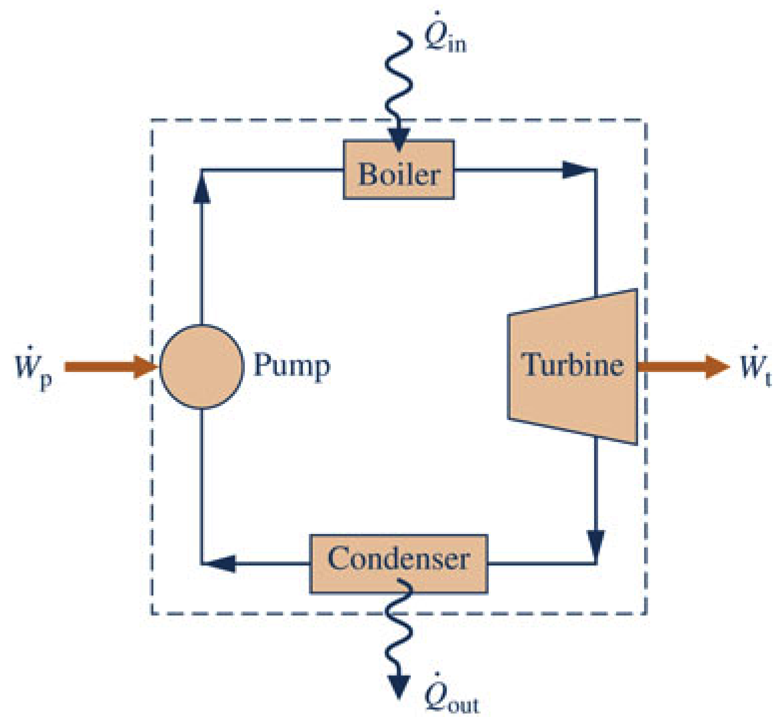
- $\dot{W}_{cv} = 0$.
- If the change in kinetic energy of flowing matter upstream and downstream of the restriction is negligible, $\frac{1}{2}(V_1^2 - V_2^2)$ drops out.
- If the change in potential energy of flowing matter is negligible, $g(z_1 - z_2)$ drops out.
- If the heat transfer with surroundings is negligible, $\dot{Q}_{cv}$ drops out.

$$h_2 = h_1$$

(Eq. 4.22)

System Integration

- Engineers creatively combine components to achieve some overall objective, subject to constraints such as minimum total cost. This engineering activity is called **system integration**.
- The simple vapor power plant of **Fig 4.16** provides an illustration.



The Mass Balance

(Transient Analysis)

► **Transient**: state changes with time.

► **Integrate** mass rate balance (**Eq. 4.2**) from time **0** to a final time ***t***.

$$\int_0^t \left(\frac{dm_{\text{cv}}}{dt} \right) dt = \int_0^t \left(\sum_i \dot{m}_i \right) dt - \int_0^t \left(\sum_e \dot{m}_e \right) dt$$

This becomes

$$m_{\text{cv}}(t) - m_{\text{cv}}(0) = \sum_i m_i - \sum_e m_e \qquad \textbf{(Eq. 4.23)}$$

where

•m_i is amount of mass entering the control volume through inlet i, from time **0** to t.

•m_e is amount of mass exiting the control volume through exit e, from time **0** to t.

The Energy Balance

(Transient Analysis)

►**Integrate** energy rate balance (**Eq. 4.15**), ignoring the effects of kinetic and potential energy, from time **0** to a final time ***t***.

$$\int_0^t \left(\frac{dU_{cv}}{dt} \right) dt = \int_0^t \dot{Q}_{cv} dt - \int_0^t \dot{W}_{cv} dt + \int_0^t \left(\sum_i \dot{m}_i h_i \right) dt - \int_0^t \left(\sum_e \dot{m}_e h_e \right) dt$$

When the specific enthalpies at inlets and exits are constant with time, this becomes

$$U_{cv}(t) - U_{cv}(0) = Q_{cv} - W_{cv} + \sum_i m_i h_i - \sum_e m_e h_e \qquad \textbf{(Eq. 4.25)}$$