Chapter 4

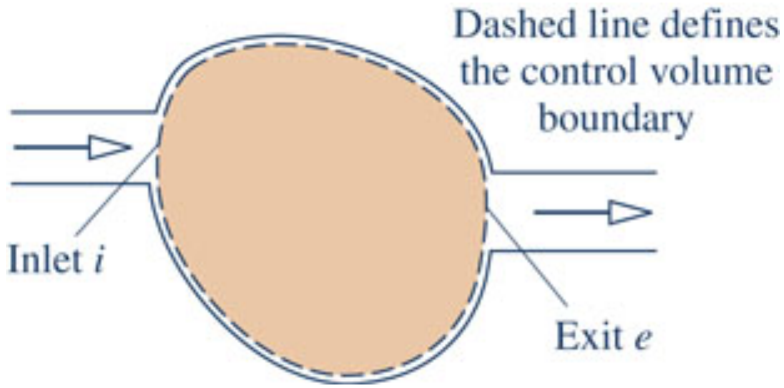
Control Volume Analysis Using Energy

(continued)

Learning Outcomes

- ►Distinguish between steady-state and transient analysis,
- ►Distinguishing between mass flow rate and volumetric flow rate.
- ►Apply mass and energy balances to control volumes.
- ►Develop appropriate engineering models to analyze nozzles, turbines, compressors, heat exchangers, throttling devices.
- ►Use property data in control volume analysis appropriately.

Mass Rate Balance



$$\left[\begin{array}{c}\text{time } \textit{rate of change} \text{ of} \\ \text{mass contained within the} \\ \text{control volume } \textit{at time t}\end{array}\right] = \left[\begin{array}{c}\text{time } \textit{rate of flow} \text{ of} \\ \text{mass } \textit{in} \text{ across} \\ \text{inlet } \textit{i at time t}\end{array}\right] - \left[\begin{array}{c}\text{time } \textit{rate of flow} \\ \text{of mass } \textit{out} \text{ across} \\ \text{exit } \textit{e at time t}\end{array}\right]$$

$$\frac{dm_{\mathrm{cv}}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e \qquad \textbf{(Eq. 4.2)}$$

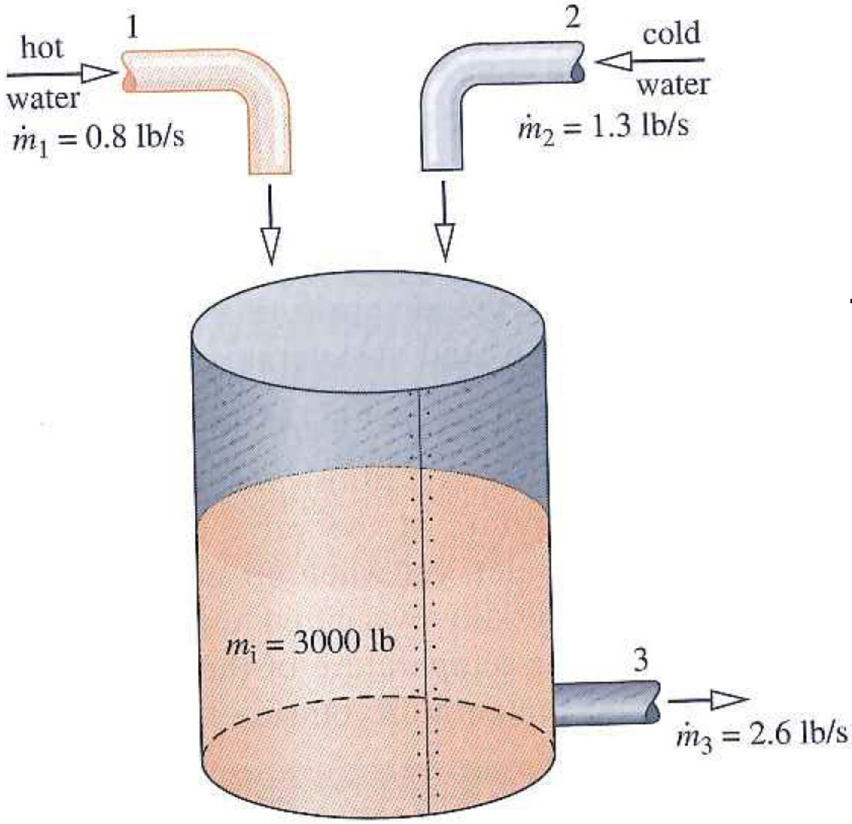
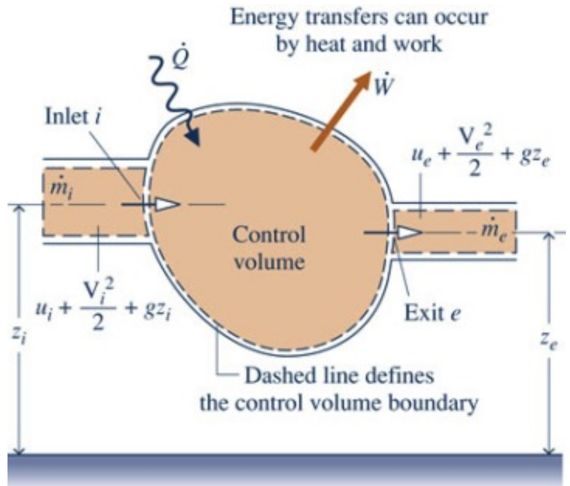


Fig. P4.6

Determine the amount of water
In tank after 1 hour

$$\frac{dm_{\text{cv}}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

Energy Rate Balance



[time ***rate of change*** of the energy contained within the control volume ***at time t***] = [***net rate*** at which energy is being transferred in by heat transfer ***at time t***] − [***net rate*** at which energy is being transferred out by work ***at time t***] + [***net rate*** of energy transfer ***into*** the control volume accompanying mass flow]

$$\frac{dE_{\mathrm{cv}}}{dt} = \dot{Q}_{cv} - \dot{W} + \dot{m}_i\left(u_i + \frac{\mathrm{V}_i^2}{2} + gz_i\right) - \dot{m}_e\left(u_e + \frac{\mathrm{V}_e^2}{2} + gz_e\right)$$

(Eq. 4.9)

Evaluating Work for a Control Volume

The expression for work is

$$\dot{W} = \dot{W}_{\text{cv}} + \dot{m}_e(p_e v_e) - \dot{m}_i(p_i v_i) \quad \textbf{(Eq. 4.12)}$$

where

- $\dot{W}_{\text{cv}}$ accounts for work associated with rotating shafts, displacement of the boundary, and electrical effects.
- $\dot{m}_e(p_e v_e)$ is the flow work at exit e.
- $\dot{m}_i(p_i v_i)$ is the flow work at inlet i.

Control Volume Energy Rate Balance

(One-Dimensional Flow Form)

Using Eq. 4.12 in Eq. 4.9

$$\frac{dE_{\mathrm{cv}}}{dt} = \dot{Q}_{\mathrm{cv}} - \dot{W}_{\mathrm{cv}} + \dot{m}_i\left(\underline{u_i + p_i v_i} + \frac{\mathrm{V}_i^2}{2} + g z_i\right) - \dot{m}_e\left(\underline{u_e + p_e v_e} + \frac{\mathrm{V}_e^2}{2} + g z_e\right)$$

(Eq. 4.13)

For convenience substitute enthalpy, $\boldsymbol{h = u + pv}$

$$\frac{dE_{\mathrm{cv}}}{dt} = \dot{Q}_{\mathrm{cv}} - \dot{W}_{\mathrm{cv}} + \dot{m}_i\left(h_i + \frac{\mathrm{V}_i^2}{2} + g z_i\right) - \dot{m}_e\left(h_e + \frac{\mathrm{V}_e^2}{2} + g z_e\right)$$

(Eq. 4.14)

Control Volume Energy Rate Balance

(One-Dimensional Flow Form)

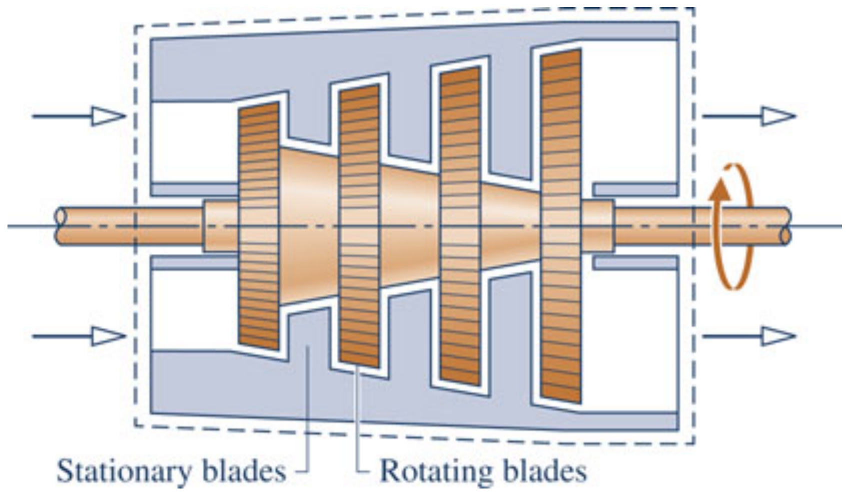
In practice there may be several locations on the boundary through which mass enters or exits. **Multiple inlets** and **exits** are accounted for by **introducing summations**:

$$\frac{dE_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \sum_i \dot{m}_i \left(h_i + \frac{\text{V}_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e \left(h_e + \frac{\text{V}_e^2}{2} + gz_e\right)$$

(Eq. 4.15)

Eq. 4.15 is the **accounting balance** for the energy of the control volume.

Turbines



► **Turbine**: a device in which power is developed as a result of a gas or liquid passing through a set of blades attached to a shaft free to rotate.

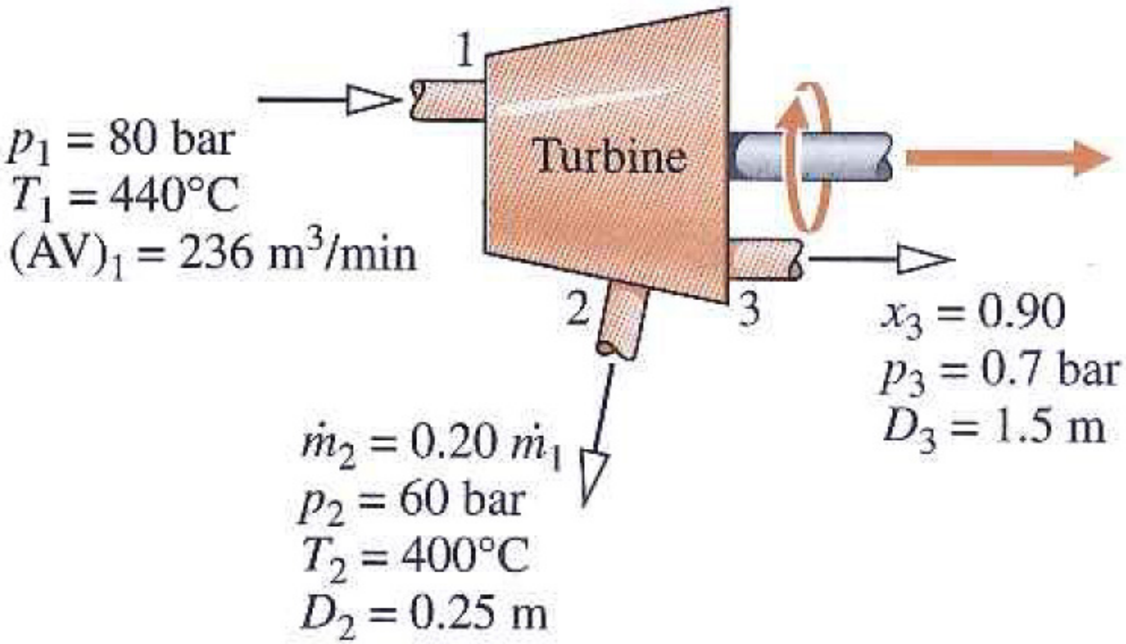


Fig. P4.13

Determine the Velocity
At each exit duct

$$\dot{m} = \frac{AV}{v}$$

where

V is velocity

v is specific volume

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

Turbine Modeling

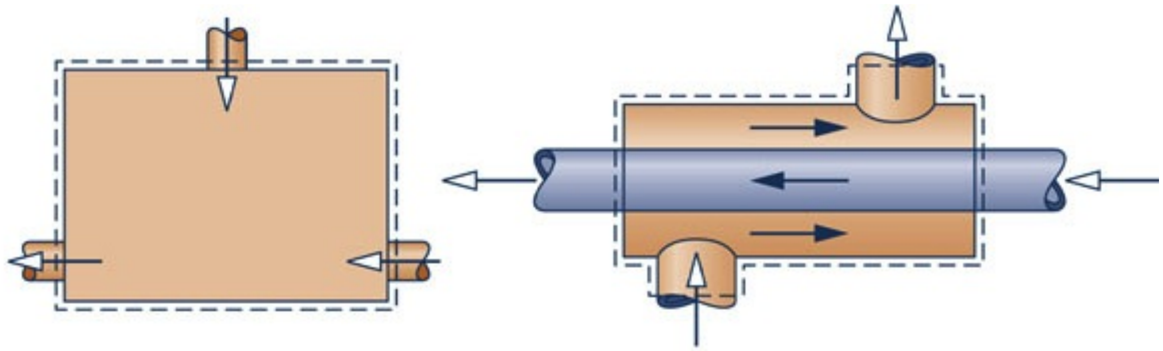
$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

Eq. 4.20a

- ► If the change in kinetic energy of flowing matter is negligible, $\frac{1}{2}(V_1^2 - V_2^2)$ drops out.
- ► If the change in potential energy of flowing matter is negligible, $g(z_1 - z_2)$ drops out.
- ► If the heat transfer with surroundings is negligible, $\dot{Q}_{cv}$ drops out.

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

Heat Exchangers



(*a*) (*b*)

- **Direct contact:** A mixing chamber in which hot and cold streams are mixed directly.
- **Tube-within-a-tube counterflow:** A gas or liquid stream is *separated* from another gas or liquid by a wall through which energy is conducted. Heat transfer occurs from the hot stream to the cold stream as the streams flow in opposite directions.

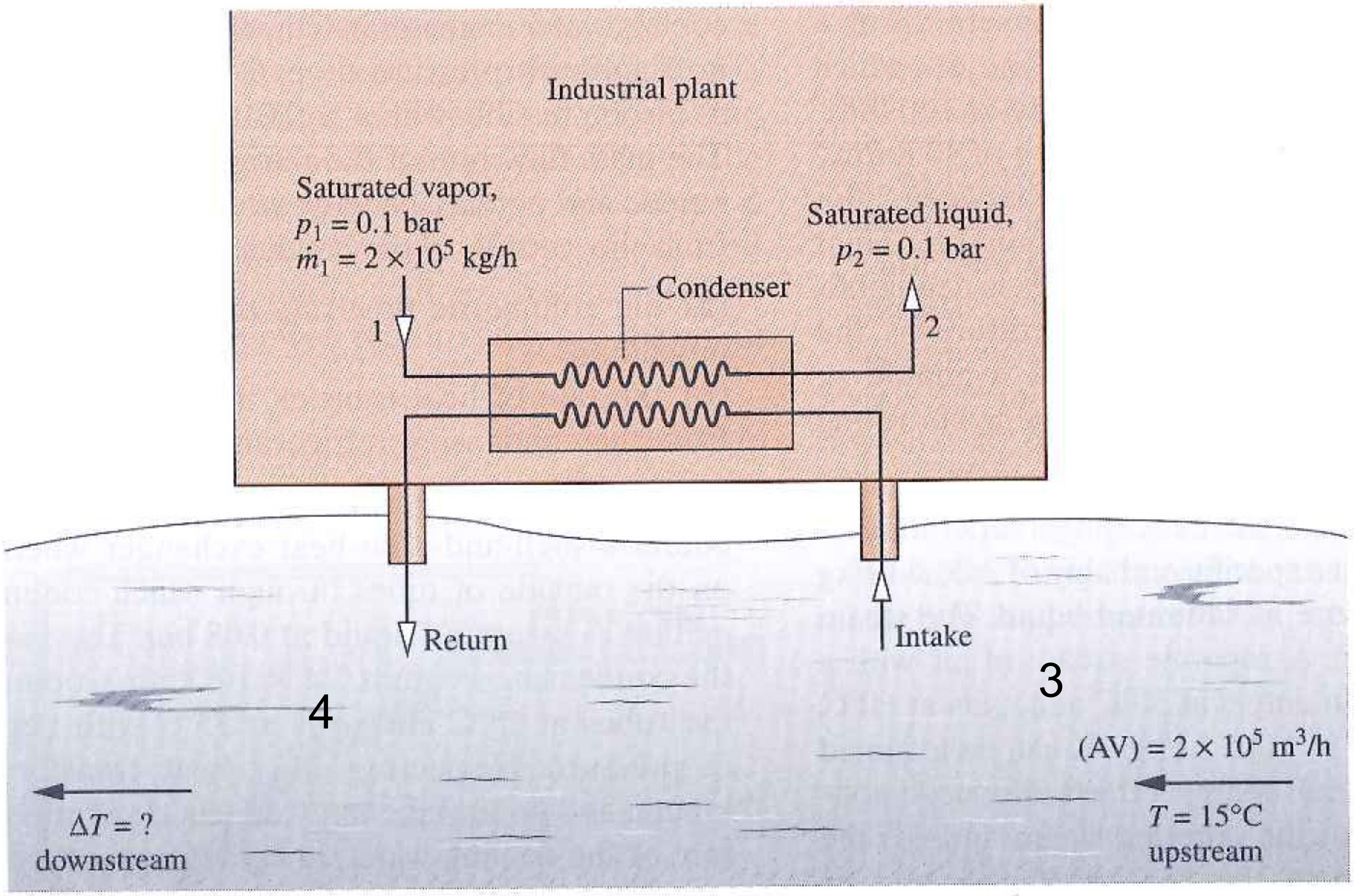
Heat Exchanger Modeling

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

(Eq. 4.18)

- ► $\dot{W}_{cv} = 0.$
- ► If the kinetic energies of the flowing streams are negligible, $\dot{m}_i(V_i^2/2)$ and $\dot{m}_e(V_e^2/2)$ drop out.
- ► If the potential energies of the flowing streams are negligible, $\dot{m}_i g z_i$ and $\dot{m}_e g z_e$ drop out.
- ► If the heat transfer with surroundings is negligible, $\dot{Q}_{cv}$ drops out.

$$0 = \sum_i \dot{m}_i h_i - \sum_e \dot{m}_e h_e$$



$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

$$\dot{m}_3(h_4 - h_3) = \dot{m}_1(h_1 - h_2) \qquad h_4 = h_3 + \frac{\dot{m}_1}{\dot{m}_3}(h_1 - h_2)$$