Phase Change

- Consider a closed system consisting of a unit mass of liquid water at 20°C contained within a piston-cylinder assembly.
- This state is represented by I (highlighted by the blue dot).
- Liquid states such as this, where temperature is lower than the saturation temperature corresponding to the pressure at the state, are called **compressed liquid** states.
 Pc = 22.09 MPa (3204 lbf/in.²)



Saturated Liquid

- As the system is heated at constant pressure, the temperature increases considerably while the specific volume increases slightly.
- Eventually, the system is brought to the state represented by f (highlighted by the blue dot).
- This is the saturated liquid state corresponding to the specified pressure.



Two-Phase Liquid-Vapor Mixture

- When the system is at the saturated liquid state, additional heat transfer at fixed pressure results in the formation of vapor without change in temperature but with a considerable increase in specific volume as shown by movement of the blue dot.
- With additional heating at fixed pressure, more vapor is formed and specific volume increases further as shown by additional movement of the blue dot.

Water vapor

Liquid water





Two-Phase Liquid-Vapor Mixture

- When a mixture of liquid and vapor exists in equilibrium, the liquid phase is a saturated liquid and the vapor phase is a saturated vapor.
- For a two-phase liquid-vapor mixture, the ratio of the mass of vapor present to the total mass of the mixture is its quality, x.
- The value of quality ranges from 0 to 1.
 At saturated liquid

states, x = 0.



Saturated Vapor

- If the system is heated further until the last bit of liquid has vaporized it is brought to the saturated vapor state.
- This state is represented by g (highlighted by the blue dot).
- At saturated vapor states, x = 1.





Superheated Vapor

- When the system is at the saturated vapor state, further heating at fixed pressure results in increases in both temperature and specific volume.
- This state is represented by s (highlighted by the blue dot).
- Vapor states such as this, where temperature is higher than the saturation temperature corresponding to the pressure at the state, are called superheated vapor states.





Steam Tables

- Tables of properties for different substances are frequently set up in the same general format. The tables for water, called the steam tables, provide an example of this format. The steam tables are in appendix tables A-2 through A-5.
 - Table A-4 applies to water as a superheated vapor.
 - Table A-5 applies to compressed liquid water.
 - Tables A-2 and A-3 apply to the two-phase, liquid-vapor mixture of water.

Single-Phase Regions

Example: Properties associated with superheated water vapor at 10 MPa and 400°C are found in Table A-4.

 $v = 0.02641 \text{ m}^3/\text{kg}$

▶ *u* = 2832.4 kJ/kg

► *h* = 3096.5 kJ/kg

$$> s = 6.2120 \text{ kJ/kg·K}$$

		-

т	v	и	<i>h</i>	s	v	и	<i>h</i>	s
°С	m³/kg	kJ/kg	kJ/kg	kJ/kg·K	m³/kg	kJ/kg	kJ/kg	kJ∕kg·K
	p	= 80 bar (<i>T</i> sat = 2	= 8.0 MP 95.06°C)	a	<i>p</i> =	100 bar (<i>T</i> sat = 3	= 10.0 M 1.06°C)	Pa
Sat.	0.02352	2569.8	2758.0	5.7432	0.01803	2544.4	2724.7	5.6141
320	0.02682	2662.7	2877.2	5.9489	0.01925	2588.8	2781.3	5.7103
360	0.03089	2772.7	3019.8	6.1819	0.02331	2729.1	2962.1	6.0060
400	0.03432	2863.8	3138.3	6.3634	0.02641	2832.4	3096.5	6.2120
440	0.03742	2946.7	3246.1	6.5190	0.02911	2922.1	3213.2	6.3805
480	0.04034	3025.7	3348.4	6.6586	0.03160	3005.4	3321.4	6.5282

Linear Interpolation

- When a state does not fall exactly on the grid of values provided by property tables, linear interpolation between adjacent entries is used.
- Example: Specific volume (v) associated with superheated water vapor at 10 bar and 215°C is found by linear interpolation between adjacent entries in Table A-4.

$$slope = \frac{(0.2275 - 0.2060) \text{ m}^3/\text{kg}}{(240 - 200)^{\circ}\text{C}} = \frac{(v - 0.2060) \text{ m}^3/\text{kg}}{(215 - 200)^{\circ}\text{C}} \rightarrow v = 0.2141 \text{ m}^3/\text{kg}$$



Tables A-2/A-2E

(Temperature Table) and A-3/A-3E (Pressure Table) provide

- saturated liquid (f) data
- saturated vapor (g) data



Table note: For saturated liquid specific volume, the table heading is $v_f \times 10^3$. At 8°C, $v_f \times 10^3 = 1.002 \rightarrow v_f = 1.002/10^3 = 1.002 \times 10^{-3} = 0.001002$

Table A-2		Specific Volume m³/kg		Internal Energy kJ/kg		Enthalpy kJ/kg			Entr kJ/k		
Temp °C	Press. bar	Sat. Liquid	Sat. Vapor	Sat. Liquid	Sat. Vapor	Sat. Liquid	Evap.	Sat. Vapor	Sat. Liquid	Sat. Vapor	Temp °C
01	0.00611	1,0002	206 136	0.00	2375 3	0.01	2501.3	2501.4	0,000	9 1 5 6 2	01
.01	0.00011	1.0002	200.130	0.00	2373.3	0.01	2301.3	2501.4	0.0000	9.1302	.01
4	0.00813	1.0001	157.232	16.77	2380.9	16.78	2491.9	2508.7	0.0610	9.0514	4
5	0.00872	1.0001	147.120	20.97	2382.3	20.98	2489.6	2510.6	0.0761	9.0257	5
6	0.00935	1.0001	137.734	25.19	2383.6	25.20	2487.2	2512.4	0.0912	9.0003	6
8	0.01072	1.0002	120.917	33.59	2386.4	33.60	2482.5	2516.1	0.1212	8.9501	8

- The specific volume of a two-phase liquidvapor mixture can be determined by using the saturation tables and quality, x.
- The total volume of the mixture is the sum of the volumes of the liquid and vapor phases: $V = V_{\text{lig}} + V_{\text{vap}}$



► Dividing by the total mass of the mixture, *m*, *an average* specific volume for the mixture is: $v = \frac{V}{v} = \frac{V_{\text{liq}}}{V_{\text{liq}}} + \frac{V_{\text{vap}}}{V_{\text{vap}}}$

With
$$V_{\text{liq}} = m_{\text{liq}}v_{\text{f}}$$
, $V_{\text{vap}} = m_{\text{vap}}v_{\text{g}}$, $m_{\text{vap}}/m = x$, and $m_{\text{liq}}/m = 1 - x$:

m

m

m

$$v = (1 - x)v_{f} + xv_{g} = v_{f} + x(v_{g} - v_{f})$$
 (Eq. 3.2)

- Since pressure and temperature are NOT independent properties in the two-phase liquidvapor region, they cannot be used to fix the state in this region.
- The property, quality (x), defined only in the twophase liquid-vapor region, and either temperature or pressure can be used to fix the state in this region.

$$v = (1 - x)v_{f} + xv_{g} = v_{f} + x(v_{g} - v_{f})$$
 (Eq. 3.2)

$$u = (1 - x)u_{f} + xu_{g} = u_{f} + x(u_{g} - u_{f})$$
 (Eq. 3.6)

$$h = (1 - x)h_{f} + xh_{g} = h_{f} + x(h_{g} - h_{f})$$
 (Eq. 3.7)

Example: A system consists of a two-phase liquid-vapor mixture of water at 6°C and a quality of 0.4. Determine the specific volume, in m³/kg, of the mixture.

Solution: Apply Eq. 3.2, $v = v_f + x(v_g - v_f)$

Substituting values from Table 2: $v_f = 1.001 \times 10^{-3} \text{ m}^3/\text{kg}$ and $v_g = 137.734 \text{ m}^3/\text{kg}$:

 $v = 1.001 \times 10^{-3} \text{ m}^3/\text{kg} + 0.4(137.734 - 1.001 \times 10^{-3}) \text{ m}^3/\text{kg}$ $v = 55.094 \text{ m}^3/\text{kg}$

Table A-2		Specific Volume m³/kg		Internal Energy kJ/kg			Enthalpy kJ/kg		Entr kJ/l		
Temp °C	Press. bar	Sat. Liquid v _f ×10 ³	Sat. Vapor	Sat. Liquid <i>U</i> f	Sat. Vapor	Sat. Liquid <i>h</i> t	Evap. <i>h</i> fa	Sat. Vapor <i>h</i> ₄	Sat. Liquid <i>S</i> f	Sat. Vapor <i>s</i>	Temp ℃
.01	0.00611	1.0002	206.136	0.00	2375.3	0.01	2501.3	2501.4	0.0000	9.1562	.01
4	0.00813	1.0001	157.232	16.77	2380.9	16.78	2491.9	2508.7	0.0610	9.0514	4
5	0.00872	1.0001	147.120	20.97	2382.3	20.98	2489.6	2510.6	0.0761	9.0257	5
6	0.00935	1.0001	137.734	25.19	2383.6	25.20	2487.2	2512.4	0.0912	9.0003	6
8	0.01072	1.0002	120.917	33.59	2386.4	33.60	2482.5	2516.1	0.1212	8.9501	8

Example: A piston-cylinder assembly contains 2 kg of water at 100°C and 1 bar. The water is compressed to a saturated vapor state where the pressure is 2.5 bar. During compression, there is a heat transfer of energy from the water to its surroundings having a magnitude of 250 kJ. Neglecting changes in kinetic energy and potential energy, determine the work, in kJ, for the process of the water.



Solution: An energy balance for the closed system is $\Delta K E^{0} + \Delta P E^{0} + \Delta U = Q - W$

where the kinetic and potential energy changes are neglected.

Thus

$$W = Q - m(u_2 - u_1)$$

Solution: An energy balance for the closed system is $\Delta K E^{0} + \Delta P E^{0} + \Delta U = Q - W$

where the kinetic and potential energy changes are neglected.

Thus $W = Q - m(u_2 - u_1)$

State 1 is in the superheated vapor region and is fixed by $p_1 = 1$ bar and $T_1 = 100^{\circ}$ C. From Table A-4, $u_1 = 2506.7$ kJ/kg.

State 2 is saturated vapor at $p_2 = 2.5$ bar. From Table A-3, $u_2 = u_g = 2537.2$ kJ/kg.

W = -250 kJ - (2 kg)(2537.2 - 2506.7) kJ/kg = -311 kJ

The negative sign indicates work is done **on** the system as expected for a compression process.

Specific Heats

Three properties related to specific internal energy and specific enthalpy having important applications are the specific heats c_v and c_p and the specific heat ratio k.

$$c_{v} = \left(\frac{\partial u}{\partial T}\right)_{v} \qquad c_{p} = \left(\frac{\partial h}{\partial T}\right)_{p} \qquad k = \frac{c_{p}}{c_{v}}$$
(Eq. 3.8) (Eq. 3.9) (Eq. 3.10)

- ▶ In general, c_v is a function of v and T (or p and T), and c_p depends on both p and T (or v and T).
- Specific heat data are provided in Fig 3.9 and Tables A-19 through A-21.

Property Approximations for Liquids

- Approximate values for v, u, and h at liquid states can be obtained using saturated liquid data.
- Since the values of v and u for liquids change very little with pressure at a fixed temperature, Eqs. 3.11 and 3.12 can be used to approximate their values.

 $v(T, p) \approx v_{f}(T)$ (Eq. 3.11) $u(T, p) \approx u_{f}(T)$ (Eq. 3.12)



An approximate value for *h* at liquid states can be obtained using Eqs. 3.11 and 3.12 in the definition h = u + pv: $h(T, p) \approx u_f(T) + pv_f(T)$ or alternatively $h(T, p) \approx h_f(T) + v_f(T)[p - p_{sat}(T)]$ (Eq. 3.13)

where $p_{\rm sat}$ denotes the saturation pressure at the given temperature

When the underlined term in Eq. 3.13 is small

 $h(T, p) \approx h_{\rm f}(T)$ (Eq. 3.14)

Incompressible Substance Model

► For a substance modeled as *incompressible*

 $\triangleright v = \text{constant}$ $\triangleright u = u(T)$

For a substance modeled as incompressible, c_p = c_v; the common specific heat value is represented by c.

For a substance modeled as incompressible with constant c:

> $u_2 - u_1 = c(T_2 - T_1)$ (Eq. 3.20a) $h_2 - h_1 = c(T_2 - T_1) + v(p_2 - p_1)$ (Eq. 3.20b)

In Eq. 3.20b, the contribution of the underlined term is often small enough to be ignored.

Generalized Compressibility Chart

The $p-\overline{v}-T$ relation for 10 common gases is shown in the generalized compressibility chart.



Generalized Compressibility Chart

► In this chart, the compressibility factor, Z, is plotted versus the reduced pressure, $p_{\rm R}$, and reduced temperature $T_{\rm R}$,

$$Z = \frac{pv}{\overline{R}T} \qquad p_{\rm R} = p/p_{\rm c} \qquad T_{\rm R} = T/T_{\rm c}$$
(Eq. 3.23) (Eq. 3.27) (Eq. 3.28)

\overline{R} is the universal gas constant

 $\overline{R} = \begin{cases} 8.314 \text{ kJ/kmol} \cdot \text{K} \\ 1.986 \text{ Btu/lbmol} \cdot \text{°R} \\ 1545 \text{ ft} \cdot \text{lbf/lbmol} \cdot \text{°R} \end{cases}$ (Eq. 3.22)

where

The symbols p_c and T_c denote the temperature and pressure at the critical point for the particular gas under consideration. These values are obtained from Tables A-1 and A-1E.



Studying the Generalized Compressibility Chart

- Low values of $p_{\rm R}$, where $Z \approx 1$, do not necessarily correspond to a range of low absolute pressures.
- For instance, if $p_R = 0.10$, then $p = 0.10p_c$. With p_c values from Table A-1

Water vapor	$p_{\rm c} = 220.9 \text{ bar} \rightarrow p =$	22 bar
Ammonia	$p_{\rm c}$ = 112.8 bar $\rightarrow p$ =	11.2 bar
Carbon dioxide	$p_{\rm c} = 73.9$ bar $\rightarrow p =$	7.4 bar
Air	$p_{\rm c} = 37.7 \text{ bar} \rightarrow p =$	3.8 bar

These pressure values range from 3.8 to 22 bar, which in engineering practice are not normally considered as low pressures.

To recap, the generalized compressibility chart shows that at states where the pressure p is small relative to the critical pressure p_c (where p_R is small), the compressibility factor Z is approximately 1.



At such states, it can be assumed with reasonable accuracy that Z = 1. Then

$$pv = RT \qquad (Eq. 3.32)$$

- Three alternative forms of Eq. 3.32 can be derived as follows:
 - With v = V/m, Eq. 3.32 gives

 $pV = mRT \qquad (Eq. 3.33)$

With $v = \overline{v}/M$ and R = R/M, Eq. 3.32 gives

$$p\overline{v} = \overline{R}T$$
 (Eq. 3.34)

Finally, with $\overline{v} = V/n$, Eq. 3.34 gives

$$pV = n\overline{R}T$$
 (Eq. 3.35)

Investigation of gas behavior at states where Eqs. 3.32-3.35 are applicable indicates that the specific internal energy depends primarily on temperature. Accordingly, at such states, it can be assumed with reasonable accuracy that it depends on temperature alone:

$$u = u(T)$$
 (Eq. 3.36)

With Eqs. 3.32 and 3.36, the specific enthalpy also depends on temperature alone at such states:

$$h = u + pv = u(T) + RT$$
 (Eq. 3.37)

Collecting results, a gas modeled as an ideal gas adheres to Eqs. 3.32-3.35 (pv=RT) and Eqs. 3.36 and 3.37.

- While the ideal gas model does not provide an acceptable approximations throughout, in most commonly applied engineering situations it is justified for use.
- Appropriateness of the ideal gas model can be checked by locating states under consideration on one of the generalized compressibility charts provided by appendix figures Figs. A-1 through A-3.

For a gas obeying the ideal gas model, specific internal energy depends only on temperature. Hence, the specific heat c_v, defined by Eq. 3.8, is also a function of temperature alone. That is,

$$c_v(T) = \frac{du}{dT}$$
 (ideal gas) (Eq. 3.38)

On integration,

$$u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T) dT$$
 (ideal gas) (Eq. 3.40)

Similarly, for a gas obeying the ideal gas model, specific enthalpy depends only on temperature. Hence, the specific heat c_p, defined by Eq. 3.9, is also a function of temperature alone. That is,

$$c_p(T) = \frac{dh}{dT}$$
 (ideal gas) (Eq. 3.41)

► On integration,

$$h(T_2) - h(T_1) = \int_{T_1}^{T_2} c_p(T) dT$$
 (ideal gas) (Eq. 3.43)

In applications where the specific heats are modeled as constant,

$$u(T_2) - u(T_1) = c_v[T_2 - T_1]$$
 (Eq. 3.50)
 $h(T_2) - h(T_1) = c_p[T_2 - T_1]$ (Eq. 3.51)

For several common gases, evaluation of changes in specific internal energy and enthalpy is facilitated by use of the *ideal gas tables*: Tables A-22 and A-23.

Table A-22 applies to air modeled as an ideal gas.

- **Example**: Using Table A-22, determine the change in specific enthalpy, in kJ/kg, for a process of air from an initial state where $T_1 = 300$ K, $p_1 = 1$ bar to a final state where $T_2 = 1500$ K, $p_2 = 10$ bar.
- **Solution**: $h_1 = 300.19 \text{ kJ/kg}; h_2 = 1635.97 \text{ kJ/kg}$

 $h_2 - h_1 = 1335.78 \text{ kJ/kg}$

Over such a wide temperature interval, use of $h_2 - h_1 = c_p[T_2 - T_1]$, Eq. 3.51, would not be appropriate.

IAD		-22		
Ideal	Gas	Prop	berties	of Air

TADLEA 22

	<i>T</i> (K), <i>h</i> and <i>u</i> (kJ/kg), s° (kJ/kg·K)											
				when $\Delta s = 0$						when	$\Delta s = 0$	
Т	h	и	S°	p r	ΰr	Т	h	и	S°	p r	v_{r}	
250	250.05	178.28	1.51917	0.7329	979.	1400	1515.42	1113.52	3.36200	450.5	8.919	
260	260.09	185.45	1.55848	0.8405	887.8	1420	1539.44	1131.77	3.37901	478.0	8.526	
270	270.11	192.60	1.59634	0.9590	808.0	1440	1563.51	1150.13	3.39586	506.9	8.153	
280	280.13	199.75	1.63279	1.0889	738.0	1460	1587.63	1168.49	3.41247	537.1	7.801	
285	285.14	203.33	1.65055	1.1584	706.1	1480	1611 79	1186 95	3 42892	568.8	7 468	
290	290.16	206.91	1.66802	1.2311	676.1	1500	1635.97	1205.41	3.44516	601.9	7.152	
295	295.17	210.49	1.68515	1.3068	647.9	1520	1660.23	1223.87	3.46120	636.5	6.854	
300	300.19	214.07	1.70203	1.3860	621.2	1540	1684.51	1242.43	3.47712	672.8	6.569	
305	305.22	217.67	1.71865	1.4686	596.0	1560	1708.82	1260.99	3.49276	710.5	6.301	
310	310.24	221.25	1.73498	1.5546	572.3	1580	1733.17	1279.65	3.50829	750.0	6.046	

Example: A closed, rigid tank consists of 1 kg of air at 300 K. The air is heated until its temperature becomes 1500 K. Neglecting changes in kinetic energy and potential energy and modeling air as an ideal gas, determine the heat transfer, in kJ, during the process of the air.



Solution: An energy balance for the closed system is $\Delta K E^{0} + \Delta P E^{0} + \Delta U = Q - W^{0}$

where the kinetic and potential energy changes are neglected and W = 0 because there is no work mode.

Thus $Q = m(u_2 - u_1)$

Substituting values for specific internal energy from **Table A-22**

Q = (1 kg)(1205.41 - 214.07) kJ/kg = 991.34 kJ

TABLE A-22 Ideal Gas Properties of Air

	<i>T</i> (K), <i>h</i> and <i>u</i> (kJ/kg), s° (kJ/kg·K)											
				when 🛆	s = 0					when	$\Delta s = 0$	
Т	h	и	S°	p r	ΰr	Т	h	и	S°	p r	v_{r}	
250	250.05	178.28	1.51917	0.7329	979.	1400	1515.42	1113.52	3.36200	450.5	8.919	
260	260.09	185.45	1.55848	0.8405	887.8	1420	1539.44	1131.77	3.37901	478.0	8.526	
270	270.11	192.60	1.59634	0.9590	808.0	1440	1563.51	1150.13	3.39586	506.9	8.153	
280	280.13	199.75	1.63279	1.0889	738.0	1460	1587.63	1168.49	3.41247	537.1	7.801	
285	285.14	203.33	1.65055	1.1584	706.1	1480	1611 79	1186 95	3 42892	568.8	7 468	
290	290.16	206.91	1.66802	1.2311	676.1	1500	1635.97	1205.41	3.44516	601.9	7.152	
295	295.17	210.49	1.68515	1.3068	647.9	1520	1660.23	1223.87	3.46120	636.5	6.854	
300	300.19	214.07	1.70203	1.3860	621.2	1540	1684.51	1242.43	3.47712	672.8	6.569	
305	305.22	217.67	1.71865	1.4686	596.0	1560	1708.82	1260.99	3.49276	710.5	6.301	
310	310.24	221.25	1.73498	1.5546	572.3	1580	1733.17	1279.65	3.50829	750.0	6.046	

Polytropic Process

A polytropic process is a quasiequilibrium process described by

$$pV^n = constant$$
 (Eq. 3.52)

The exponent, *n*, may take on any value from $-\infty$ to $+\infty$ depending on the particular process.

- For any gas (or liquid), when n = 0, the process is a constant-pressure (isobaric) process.
- For any gas (or liquid), when $n = \pm \infty$, the process is a constant-volume (isometric) process.

For a gas modeled as an ideal gas, when n = 1, the process is a constant-temperature (isothermal) process.