POW #1 Write-Up

Sumanth Sura, Luke Pepin, Joshua Schnee

I. Problem Statement

What is the day of the week that a given date falls on, assuming that the date falls between the years 1900 and 2100? [NOTE: We were given a calendar for the month of October 2022.]

II. Process

Since we know that the days of the week cycle rotates modulo 7, we decided to give the days of the week a number, each representing one of the possible numbers in mod 7. We made Sunday equal 0, Monday equal 1, …, Saturday = 6. We also decided to start from an origin day and add days, rather than subtracting days from the current date as our general method for two reasons: one, because the current day changes, and two, because having negative numbers makes it slightly more cumbersome to work in modular arithmetic (and may not be feasible for the user).

We decided that the origin date, or the date that the user starts from to count up to their birthday, should be December 31, 1899, which was a Sunday. This date works well because each following day is the 1st, 2nd, 3rd, etc. day in the 1900s. We calculated this day by counting backwards from the current date. Today's date is Friday, October 7, 2022. We want to get to December 31, 2022 and then find out how the date changes when moving 123 (2022 - 1899 = 123) years backwards. Note that 365 is congruent to 1 mod 7, so going back one year equates to moving back a day (for example, October 7, 2021 was a Thursday).

To get to December 31, 2022, we first move forward 24 days, or 3 in mod 7 from Friday, October 7, to get to Monday, October 31, 2022. Then, we know that November and December have 30 and 31 days, respectively. These equate to 2 and 3 mod 7. So moving forward from October 31, 2022 to December 31, 2022 would equate to moving forward 5 days. So, December 31, 2022 was a Saturday. Using our rule from before about moving back a year equating to moving back a day, we must move back 123 days.

However, we also have to account for leap years. To calculate leap years between the current year and another, the formula would be (current year - past year)/4. Ignore the remainder. This formula is inclusive of the current year and exclusive of the past year. The number of leap years between 2022 and 1900 (going to the end of 1899) is (2022 - 1900)/4 = 30 remainder 2. The fact that this formula is exclusive of the past year is convenient because 1900 is not a leap year. This means we have to move back 30 more days. We move back a total of 153 days (123 + 30 = 153). -153 is congruent to 1 mod 7. We have to move forward 1 weekday from Saturday, December 31, 2022 to get to December 31, 1899. Therefore, December 31, 1899 was a Sunday.

Our idea to bring the user to any given date from Jan. 1 1901 to Dec. 31 2099 reverses this process:

- Move the user to December 31 of the year before they were born.
- Move the user to the last day of the month before they were born.
- Add their birth date to get to their birthday.

To move to the year previous to the given year, you would add 1 for every year between 1899 and the year previous. This is the same as subtracting 1900 from their year. Then, add an extra for every leap year (The same method as calculated above).

Then, to move to the end of the previous month, you must add the mod 7 number of the total number of days of each month leading up to the given month. For instance, January has 31 (3 mod 7) days, so moving from December 31 to January 31 would move you forward 3 weekdays. February has 28* (0 mod 7) days, so moving to the end of February would also be +3. The results of these calculations are shown in the table in Section III, with each month corresponding to the mod 7 number of days leading up to it.

Lastly, the day of the month must be added to the total to move your "position" from the last day of the previous month to the exact date.

*February has 29 days during leap years. However, this additional day is accounted for because the given year is included when adding 1 for every leap year. However, any day prior to March 1 on a leap year would not require adding that extra day. Therefore, for any dates landing on leap years in January or February, 1 must be subtracted.

III. Solutions

1. Calculate the difference between your birth year and 1900. (For example, if you were born in 1989, the difference would be 89). Keep that number in mind.
2. Divide the number by 4, and ignore the remainder. (For example 89 / 4 = 22 with a remainder of 1). Add this to your original number.
3. Find your birth month in the following table and add the value to your running total.

Birth Month	*k*
January	0
February	3
March	3
April	6
May	1
June	4
July	6
August	2
September	5

October	0
November	3
December	5

4. Add the numerical day of your birthday to your running total. (For example, if you were born on August 4th, this number would be 4)
5. If your birthday falls on a leap year, but before March 1, subtract one from your number. (Note that leap years are the years divisible by 4 evenly. E.g 1964 but not 1987.) (For example, you would subtract 1 if your birthday is January 14th, 1988, but not August 4th, 1988.)
6. Divide your total by 7 and find the remainder. (For example, if your total is 138, 138 / 7 = 19 with a remainder of 5.) The remainder is your new number.
7. Your resulting number corresponds with a day of the week! 0 = Sunday, 1 = Monday, and so on until 6 = Saturday. That day is the day you were born!

IV. Extensions

How can the day of the week be found if the given date is after 1 CE?

- Find the day of Dec 31, 1 BCE
- Divide the given year by 100, subtract the quotient (years divisible by 100 are not leap years)
- Divide the given year by 400, add the quotient (years divisible by 400 ignore the previous rule)
- Repeat the same steps as the previous solution.
    - A necessary modification to include is that rather than adding the number of years, you would want to add one less than the number of years. This is because 0 CE does not exist, and while our original method starts counting in a year ending in 0, this one cannot possibly.

Find the date if given the day of the week, month, and year (For example, the second Wednesday of March 1980)

- Find the last day of the month before they were born
- Find the first day of the month that falls on the same day of the week they were born
- Add 7 until you get to the date.