Spectral theory for Markov semigroups: New developments and option pricing

P. Patie (Cornell University) and

M. Savov (Bulgarian Academy of Sciences)

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Let $X = (X_t)_{t \geq 0}$ be a nice Markov process.

Assume that X models the dynamics of a risky asset.

If $\mathbb{P}$ stands for a risk-neutral probability measure a price at time 0 of a derivative with payoff f and maturity $t > 0$ can be expressed in terms of the functional

$$P_t f(x) = \mathbb{E}_x \left[f(X_t) \right]$$

where $\mathbb{P}_x(X_0 = x) = 1$.

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4. Feynman-Kac formula and numerical scheme for solving PDE's but limited scheme for PIDE's.
5. Spectral decomposition of the semigroup $P = (P_t)_{t\geq 0}$ ... but spectral theory available only for self-adjoint (normal) Markov semigroups.

Definition

We say that a Feller semigroup $P = (P_t)_{t\geq 0} \in \mathcal{L}$, the set of generalized Laguerre semigroups, if writing $P_t = e^{t\mathbf{G}}$, we have for f smooth on $x > 0$,

$$\mathbf{G}f(x) = \sigma^2 x f''(x) + \left(m + \sigma^2 - x\right) f'(x) + x \int_0^1 f''(xy) M(y) dy$$

where $\sigma, m \geq 0$, and $M(y) = \int_{-\ln y}^{\infty} e^{-r}\overline{\Pi}(r)dr$ with $\overline{\Pi}$ the tail of a Lévy measure, i.e. $\overline{\Pi}$ is a non-negative and non-increasing function with $\int_0^\infty (1 \wedge r)\,\overline{\Pi}(r)dr < \infty$.

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Note that by writing $\phi(u) = \sigma^2 u + m + \int_0^{\infty}(1 - e^{-uy})\overline{\Pi}(y)dy$, one has with $p_n(x) = x^n, n \geq 1$,

$$\mathbf{G}p_n(x) = n\phi(n)p_{n-1}(x) - np_n(x).$$

Hence $\mathcal{L}$ is a subclass of the polynomials semigroups introduced by Cuchiero, Keller-Ressel and Teichmann (12).

We denote by $\mathcal{N}$ the set of functions ϕ of the form, for any $u > 0$,

$$\phi(u) = \sigma^2 u + m + \int_0^\infty (1 - e^{-uy})\overline{\Pi}(y)dy,$$

where $\sigma, m \geq 0$, $\overline{\Pi}$ is the tail of a Lévy measure.

Proposition

- There exists a bijection between the sets $\mathcal{L}$ and $\mathcal{N}$.
- Any $P \in \mathcal{L}$ admits a unique invariant probability measure $\mathcal{V}$ which is absolutely continuous with a density $\nu > 0$ on its support $(0, \phi(\infty) = \infty\mathbb{I}_{\{\sigma^2>0\}} + \overline{\overline{\Pi}} + m)$, where $\overline{\overline{\Pi}} = ||\overline{\Pi}||_{L^1(\mathbb{R}^+)} \leq \infty$.
- P extends to a strongly continuous contraction semigroup on the weighted Hilbert space $\mathrm{L}^2(\nu)$ with $||f||_\nu = \int_0^\infty f^2(x)\nu(x)dx$.
- If $\overline{\Pi} \neq 0$, then P is non-self-adjoint, i.e. $P^* \neq P$ where P^* is the $\mathrm{L}^2(\nu)$-adjoint of P, i.e.

$$\langle P_t f, g\rangle_\nu = \langle f, P_t^* g\rangle_\nu.$$

The Laguerre semigroup: $\sigma = 1$, $m = 0$, $\overline{\Pi} \equiv 0$

- Its semigroup Q is self-adjoint in $\mathrm{L}^2(\varepsilon)$ where $\varepsilon(x) = e^{-x}$ and

$$\mathbf{G}_0 f(x) = x f''(x) + (1-x) f'(x).$$

- Moreover, $\forall f \in \mathrm{L}^2(\varepsilon)$, $t > 0$,

$$Q_t f(x) = Q_t \sum_{n=0}^{\infty} \langle f, \mathcal{L}_n \rangle_\varepsilon \, \mathcal{L}_n(x) = \sum_{n=0}^{\infty} e^{-nt} \langle f, \mathcal{L}_n \rangle_\varepsilon \, \mathcal{L}_n(x),$$

  where $Q_t \mathcal{L}_n = e^{-nt} \mathcal{L}_n$ and the $\mathcal{L}_n$'s are the Laguerre polynomials

$$\mathcal{L}_n(x) = \sum_{k=0}^{n} \binom{n}{k} \frac{(-1)^k}{k!} x^k = \frac{(x^n e^{-x})^{(n)}}{e^{-x} n!} = \mathcal{R}^{(n)} \varepsilon(x),$$

  with $f^{(n)}(x) = \frac{d^n}{dx^n} f(x)$ and $\mathcal{R}^{(n)}$ the Rodrigues operator.

- For any $f \in \mathrm{L}^2(\varepsilon)$ and $t > 0$,

$$||Q_t f - \varepsilon f||_\varepsilon \leq \underbrace{e^{-t}}_{\text{spectral gap}} ||f - \varepsilon f||_\varepsilon.$$

McKean(56), Getoor(58), Karlin and McGregor(66).

We aim at developing the spectral decomposition of the class $\mathcal{L}$ of non-local and non self-adjoint (NSA) semigroups.

Motivation

1. Explicit representation of $P_t f(x) = \mathbb{E}_x [f(X_t)]$ solution to the associated Cauchy problem

$$\frac{d}{dt} u(t,x) = \mathbf{G} u(t,x), \quad u(0,x) = f(x).$$

and of the heat kernels.

2. Smoothness of the mappings $(t,x) \mapsto P_t f(x)$ and $(t,x,y) \mapsto p_t(x,y)$. Note that Malliavin calculus and P(I)DE techniques are not applicable, see Picard (99), Fournier (12), Caffarelli and Silvestre (09), Silvestre (14) ...

3. Speed of convergence to stationarity, i.e. can we find a constant $C_f > 0$ such that

$$||P_t f - \nu f||_\nu \leq C_f \, \gamma(t)$$

where the rate function γ is such that $\lim_{t\to\infty} \gamma(t) = 0$ and is independent of f.

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Unlike for self-adjoint operators, there is no spectral theorem available.

The recent developments for NSA (differential) operators compromise a collection of methods, each of which is useful for some class of such operators (no unified theory),
Cialenco (00), E.B. Davies (02), Davies and Kujilaars (07), Sj$\ddot{o}$strand (09).

Step 1: NSA $\leftrightarrow$ SA via intertwining

Let us introduce the Markov operator

$$\Lambda f(x) = \mathbb{E}\left[f(xV_\phi)\right]$$

where $V_\phi = \int_0^\infty e^{-\xi_t} dt$ and $(\xi_t)_{t\geq 0}$ is a subordinator with Laplace exponent ϕ.

Theorem

For any $\phi \in \mathcal{N}$, we have on $\mathrm{L}^2(\varepsilon)$,

$$P_t\Lambda = \Lambda Q_t \tag{1}$$

where $\Lambda \in \mathbf{B}(\mathrm{L}^2(\varepsilon), \mathrm{L}^2(\nu))$, the set of bounded linear operators from $\mathrm{L}^2(\varepsilon)$ into $\mathrm{L}^2(\nu)$.

Dynkin (65), Rogers-Williams (91), Diaconis-Fill (90), Biane (95), Carmona, Petit and Yor (98) and P. and Simon (12), Pal and Shkolnikov (13).

In functional analysis, this relation is called similarity, transmutation or transplantation (Delsart and Lions (57)) but the kernel is in general an isomorphism.

We recall that the generator of Q takes the form

$$\mathbf{G}_0 f(x) = x f''(x) + (1-x) f'(x) \qquad \text{(classical Laguerre).}$$

1. From SA to SA : $\phi(u) = \sigma^2 u + m$

$$\mathbf{G} f(x) = \sigma^2 x f''(x) + \left(m + \sigma^2 - x\right) f'(x).$$

2. Perturbation: $\phi(u) = u + \int_0^\infty (1 - e^{-uy}) \overline{\Pi}(y) dy$ where $\overline{\Pi}(y) = \int_y^\infty \Pi(dr)$.

$$\mathbf{G} f(x) = \mathbf{G}_0 f(x) + \int_0^\infty \left(f(e^{-y}x) - f(x) - yxf'(x)\right) \frac{\Pi(dy)}{x}$$

3. Beyond perturbation: $\phi(u) = \int_0^\infty (1 - e^{-uy}) \overline{\Pi}(y) dy$

$$\mathbf{G} f(x) = -x f'(x) + \int_0^\infty \left(f(e^{-y}x) - f(x) - yxf'(x)\right) \frac{\Pi(dy)}{x}.$$

Proposition

Let $\phi \in \mathcal{N}$, *then for any* $n \in \mathbb{N}$,

$$P_t \mathcal{P}_n(x) = e^{-nt} \mathcal{P}_n(x)$$

where

$$\mathcal{P}_n(x) = \Lambda \mathcal{L}_n(x) = \sum_{k=0}^{n} \binom{n}{k} \frac{(-1)^k}{k!} \frac{k!}{\prod_{j=1}^{k} \phi(j)} x^k.$$

As ν *is moment determinate, we have* $\overline{Span}\{\mathcal{P}_n, n \geq 0\} = \mathrm{L}^2(\nu)$.
Moreover, for any $p \in \mathbb{N}$, $x > 0$ *and large* n

$$|\mathcal{P}_n^{(p)}(x)| = \mathrm{O}\left(e^{\frac{1}{\phi(\infty)}(nx)^{\frac{1}{1+\underline{\phi}}}} \right)$$

where $\underline{\phi} = \liminf_{u \to \infty} \frac{\ln \phi(u)}{\ln u} \in \left[0, \frac{1}{2}\right]$.

Step 3: A first expansion via the concept of Frames

From the intertwining identity (1) and the expansion of Q_t, we deduce

$$P_t \Lambda f(x) = \Lambda \sum_{n=0}^{\infty} e^{-nt} \langle f, \mathcal{L}_n \rangle_\varepsilon \mathcal{L}_n(x) \stackrel{?}{=} \sum_{n=0}^{\infty} e^{-nt} \langle f, \mathcal{L}_n \rangle_\varepsilon \mathcal{P}_n(x).$$

For $\stackrel{?}{=}$, we resort to the concept of Frames introduce by Duffin and Schaeffer (52).
$(P_n)_{n\geq 0}$ is a frame in the Hilbert space $\mathrm{L}^2(\nu)$ if $\exists A, B > 0$ s.t.

$$B||f||_\nu^2 \leq \sum_{n=0}^{\infty} |\langle f, P_n \rangle_\nu|^2 \leq A||f||_\nu^2 \quad \forall f \in \mathrm{L}^2(\nu).$$

If in addition $(P_n)_{n\geq 0}$ is minimal then it is a Riesz basis. If only the upper frame bound holds, then $(P_n)_{n\geq 0}$ is a Bessel sequence and the synthesis operator

$$\begin{aligned} S : \ell^2 &\to \mathrm{L}^2(\nu) \\ S((c_n)_{n\geq 0}) &= \sum_{n=0}^{\infty} c_n P_n \quad \text{is bounded.} \end{aligned}$$

We have $\Lambda \in \mathbf{B}(\mathrm{L}^2(\varepsilon), \mathrm{L}^2(\nu))$ with a dense range but is not onto since

$$||\Lambda f||_{\mathrm{L}^2(\nu)} \geq c||f||_{\mathrm{L}^2(\varepsilon)}$$

for some $c > 0$ fails even on monomials.

Proposition

$(\mathcal{P}_n)_{n\geq 0}$ is a complete Bessel sequence in $\mathrm{L}^2(\nu)$ but it is not a Riesz basis.

Moreover, $\overline{\mathrm{Ran}}\Lambda = \mathrm{L}^2(\nu)$ *and for any* $f \in \mathrm{Ran}\Lambda$, *i.e.* $f = \Lambda\mathfrak{f}$ *with* $\mathfrak{f} \in \mathrm{L}^2(\varepsilon)$, *we have for all* $t > 0$,

$$P_t f(x) = \sum_{n=0}^{\infty} e^{-nt} \langle \mathfrak{f}, \mathcal{L}_n \rangle_{\varepsilon} \mathcal{P}_n(x) \quad \text{in } \mathrm{L}^2(\nu),$$

$$(t, x) \mapsto P_t f(x) \in C^{\infty}\left(\mathbb{R}^+ \times (0, \phi(\infty)t)\right),$$

and

$$||P_t f - \nu f||_{\nu} \leq \underbrace{C_f \quad e^{-t}}_{\textit{perturbed spectral gap}} \quad ||f - \nu f||_{\nu}$$

where $C_f \geq 1$.

Step 4: Existence of co-eigenfunctions

Let Λ^* the $\mathrm{L}^2(\nu)$-adjoint of Λ, we get the adjoint intertwining relationship

$$\Lambda^* P_t^* = Q_t \Lambda^* \quad \text{on } \mathrm{L}^2(\nu).$$

Thus, if for some $n \geq 0$, the equation

$$\Lambda^* g = \mathcal{L}_n \text{ has a solution } g = \mathcal{V}_n \text{ in } \mathrm{L}^2(\nu),$$

then $\mathcal{V}_n$ is an eigenfunction for P_t^*, that is

$$P_t^* \mathcal{V}_n = e^{-nt} \mathcal{V}_n.$$

One shows that $(\mathcal{V}_n, \mathcal{P}_n)_{n \geq 0}$ forms a biorthogonal sequence in $\mathrm{L}^2(\nu)$

$$\langle \mathcal{P}_m, \mathcal{V}_n \rangle_\nu = \delta_{nm}.$$

However they are not a Riesz basis of $\mathrm{L}^2(\nu)$.

By means of Mellin transform techniques, we get in the sense of distributions,

$$\mathcal{V}_n(x) = \frac{(x^n \nu(x))^{(n)}}{\nu(x) n!} = \mathcal{R}^{(n)} \nu(x).$$

Questions: (A) : $\nu \in C^\infty(\mathbb{R}^+)$? and (B) : $\mathcal{V}_n \in \mathrm{L}^2(\nu)$?

Proposition

(A) $\nu \in C^K(\mathbb{R}^+) \setminus C^{K+1<\infty}(0, \phi(\infty))$ *where* $K = \infty$ *if* $\sigma > 0$ *or* $\overline{\Pi}(0^+) = \infty$ *and* $K = \left[\frac{\overline{\Pi}(0^+)}{\phi(\infty)}\right] - 1$ *otherwise.*

(B) *Assume that* $\sigma^2 > 0$ *or* $\overline{\overline{\overline{\Pi}}} = \infty$, *then for any* $n \in \mathbb{N}$ *and large* x

$$\nu^{(n)}(x) \sim C x^{n+1} \varphi^n(x) \sqrt{\varphi'(x)} e^{-\int_m^x \frac{\varphi(r)}{r} dr},$$

where φ *is the inverse function of* ϕ *and* $C > 0$.

Consequently, if $\sigma^2 > 0$ *or* $\overline{\Pi}(0^+) = \infty$ *then* $\mathcal{V}_n \in \mathrm{L}^2(\nu)$.
Otherwise, for $n > K$, $\mathcal{V}_n \notin \mathrm{L}^2(\nu)$.

Step 5: Extension of the spectral operator

Hence, if $\sigma^2 > 0$ or $\overline{\Pi}(0^+) = \infty$ then we have with $f = \Lambda\mathfrak{f}$,

$$\langle \mathfrak{f}, \mathcal{L}_n \rangle_\varepsilon = \langle \mathfrak{f}, \Lambda^* \mathcal{V}_n \rangle_\nu = \langle f, \mathcal{V}_n \rangle_\nu,$$

that is for any $f \in \mathrm{Ran}\Lambda$,

$$P_t f = S_t f := \underbrace{\sum_{n=0}^{\infty} e^{-nt} \langle f, \mathcal{V}_n \rangle_\nu \mathcal{P}_n}_{\text{Spectral expansion operator}}$$

Questions:

- For which (normed) linear space $\mathrm{L} \subseteq \mathrm{L}^2(\nu)$,

  $$(e^{-nT} \langle f, \mathcal{V}_n \rangle_\nu)_{n \geq 0} \in \ell^2$$

  for some $T > 0$ and any $f \in \mathrm{L}$? As $(\mathcal{P}_n)_{n \geq 0}$ is a Bessel sequence we have $S_t f \in \mathrm{L}^2(\nu)$.
- When $P_t f = S_t f$?

Expansions in $\mathrm{L} = \mathrm{L}^2(\mathbb{R}^+)$, $\left|e^{-nT}\langle f, \mathcal{V}_n\rangle_\nu\right| \leq e^{-nT}||f||\,||\mathcal{V}_n\nu||$

One shows that

$$\mathcal{M}_{\mathcal{V}_n\nu}(s) = \int_0^\infty x^{s-1}\mathcal{V}_n(x)\nu(x)dx = (-1)^n \frac{\Gamma(s)}{\Gamma(s-n)\Gamma(n)}\mathcal{M}_\nu(s)$$

Fix $a > 0$. Then, by Mellin inversion for any $x > 0$,

$$|\mathcal{V}_n(x)\nu(x)| \leq Cx^{-a}\int_{-\infty}^{\infty}\left|\frac{\Gamma(a+ib)}{\Gamma(a+ib-n)\Gamma(n)}\right| |\mathcal{M}_\nu(a+ib)|db$$

where $\mathcal{M}_\nu$ is the solution to

$$\mathcal{M}_\nu(s+1) = \phi(s)\mathcal{M}_\nu(s), \; \mathfrak{Re}(s) > 0.$$

We express $\mathcal{M}_\nu(s) = \frac{e^{-\gamma_\phi s}}{\phi(s)}\prod_{k=1}^\infty \frac{\phi(k)}{\phi(k+s)}e^{s\frac{\phi'(k)}{\phi(k)}}$ and most importantly

$$|\mathcal{M}_\nu(a+ib)| \overset{|b|\to\infty}{\asymp} e^{-\left(\frac{1}{|b|}\int_0^{|b|}\arg\phi(a+ir)dr\right)|b|}.$$

Theorem

We have $\Theta = \liminf_{|b|\to\infty} \frac{\int_0^{|b|} \arg\phi(a+ir)dr}{|b|} \in \left[0, \frac{\pi}{2}\right]$.
Assume that $\Theta > 0$, *then for any* $t > T_\Theta = -\ln\sin\Theta$ *and* $f \in \mathrm{L}^2(\mathbb{R}^+)$

$$P_t f = S_t f = \sum_{n=0}^{\infty} e^{-nt}\langle f, \mathcal{V}_n\rangle_\nu \mathcal{P}_n \qquad (2)$$

and there exist $C > 0$ *and an integer* $k \geq 0$ *such that*

$$||P_t f - \nu f||_\nu \leq C\sqrt{\left(\frac{1}{e^{2(t-T_\Theta)} - 1}\right)^{(k)}}||f - \nu f||_\nu$$

If in addition ν *is holomorphic on* $\mathbb{C}_+ = \{z \in \mathbb{C}; \mathfrak{Re}(z) > 0\}$, *the transition density is computed as*

$$p_t(x, y) = \sum_{n=0}^{\infty} e^{-nt}\mathcal{P}_n(x)\mathcal{V}_n(y)\nu(y)$$

where the series is uniformly convergent in $t, x, y > 0$.

Theorem

Let $\sigma^2 > 0$. *Then* $\overline{Span}\{\mathcal{V}_n, n \geq 0\} = \mathrm{L}^2(\nu)$, $||\mathcal{V}_n||_\nu = o\left(e^{\epsilon n}\right), \forall \epsilon > 0$, *and hence for any* $f \in \mathrm{L}^2(\nu)$ *and* $t > \epsilon > 0$

$$P_t f = S_t f = \sum_{n=0}^{\infty} e^{-nt} \langle f, \mathcal{V}_n \rangle_\nu \mathcal{P}_n,$$

$$||P_t f - \nu f||_\nu \leq C_\epsilon \sqrt{\frac{1}{e^{2t} - 1}} ||f - \nu f||_\nu$$

where $C_\epsilon > 0$.

If in addition $\overline{m} = \frac{m + \overline{\overline{\overline{\Pi}}}}{\sigma^2} < \infty$ *then the hypocoercivity phenomena holds:*

$$||P_t f - \nu f||_\nu \leq \sqrt{\frac{\overline{m} + 1}{\underline{d} + 1}} e^{-t} ||f - \nu f||_\nu,$$

where $\overline{m} > \overline{d} = \sup_{d \geq 0}\{\int_1^\infty e^{dr} \overline{\Pi}(r) dr < \infty\}$ and for large n

$$||\mathcal{V}_n||_\nu = 0(n^{\overline{m}}) \text{ and } ||\mathcal{P}_n||_\nu = 0(n^{-\underline{d}}),$$

see Desvillettes and Villani (01), Eckmann and Hairer (03), Villani (09), Baudoin (13), Gadat and Miclo (13), Mischler and Mouhot (15) ...

Ideas of Proof: The concept of reference semigroups.

Find a parametric family of gL semigroups $(\overline{P}^{\alpha})_{0<\alpha<1}$, such that:

1. There exists $T_\alpha > 0$ such that $\lim_{\alpha\to 1} T_\alpha = 0$ and
$$||\overline{\mathcal{V}}_n||_{\overline{\nu}} = 0(e^{T_\alpha n}),$$
2. There exists a (class of) gl semigroup P and Markov kernel Λ_α such that
$$\overline{P}_t^{\alpha}\Lambda_\alpha = \Lambda_\alpha P_t.$$

Indeed in such case $||\mathcal{V}_n||_{\nu} = ||\Lambda_\alpha^* \overline{\mathcal{V}}_n||_{\overline{\nu}} = 0(e^{T_\alpha n})$.

References

- Patie, P. and Savov, M., Cauchy problem of the non-self-adjoint Gauss-Laguerre semigroups and uniform bounds of generalized Laguerre polynomials, J. Spect. Theory, 2016.
- Patie P. and Savov M., Spectral expansions of non-self-adjoint generalized Laguerre semigroups, submitted.
- Chazal M., Loeffen R. and Patie P., Smoothness of continuous state branching with immigration semigroups, submitted.
- Chazal, M. Loeffen, R. and Patie, P., Option pricing in one dimensional affine term structure model via spectral representation, submitted.
- Patie P. and Zhao Y., Spectral decomposition of fractional operators and reflected stable semigroups, submitted.
- Patie P. and Savov, M., Positive self-similar semigroups, working paper.
- Patie P. and Zhao Y., Reflected generalized Laguerre semigroups and Krein theory of strings, working paper.