Network Uncertainty and Systemic Loss

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Systemic Risk in Financial Networks

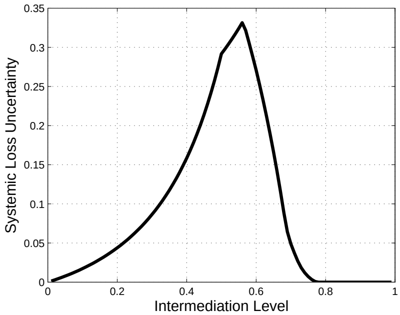
- Interbanking system consists of financial institutions linked to each other via unsecured debt contracts.
- Each institution holds external assets and claims on other institutions in the network.
- If an institution cannot meet its liabilities in full, it defaults and repays its creditors on a pro rata basis.
- Failure of an institution to repay its debt may impair the ability of its creditors to repay their own creditors: systemic risk.

Related Literature

- Most of the studies assume the values of pairwise interbank exposures to be known with certainty:
  - Foundational work : Eisenberg and Noe (2001)
  - Enhancement with bankruptcy cost: Glasserman and Young (2015), Rogers and Veraart (2012)
  - Impact of the network architecture on systemic loss: Acemoglu et al. (2015), Capponi et al. (2015)
- However, the pairwise interbank exposures are NOT publicly revealed and unknown even to regulators.
- Several works have tried to estimate the interbank liability matrix from balance sheet data, e.g. Upper and Worms (2004), Anand et al. (2013), and Gandy and Veraart (2015).

Overview of Main Contributions

- Introduce a financial network model which accounts for uncertainty in the matrix of interbank liabilities.
- Investigate how the level of intermediation in the financial system impacts systemic loss uncertainty.



Interbank Contracts with Uncertainty

- The actual (unobserved) interbank liability matrix belongs to the set of matrices

$$\mathcal{L} := \left\{ \mathbf{L} \middle| \hat{\mathbf{L}} \circ (\mathbf{1} - \mathbf{\Delta}) \leq \mathbf{L} \leq \hat{\mathbf{L}} \circ (\mathbf{1} + \mathbf{\Delta}), \sum_{j=1}^{n} l_{ij} = \underbrace{\ell_i}_{\text{known}} \right\}$$

$\hat{\mathbf{L}}$: best estimate of the actual interbank liability matrix
$\mathbf{\Delta}$: matrix of uncertainty

- Correspondingly, the actual relative liability matrix varies in the set

$$\mathcal{U} := \{ \mathbf{\Pi} | \mathbf{L} \in \mathcal{L} \}, \quad \pi_{ij} = \begin{cases} \frac{l_{ij}}{\ell_i} & \text{if } \ell_i > 0 \\ 0 & \text{if } \ell_i = 0 \end{cases}$$

Minimum and Maximum Systemic Losses

Consider a financial system $(\mathcal{U}, \ell, \kappa)$,

- For each $\mathbf{\Pi} \in \mathcal{U}$, the shortfall vector is defined as

$$s(\mathbf{\Pi}, \ell, \kappa) := \ell - p(\mathbf{\Pi}, \ell, \kappa)$$

  where p is the clearing payment (Eisenberg and Noe (2001)).

- Minimum and maximum systemic losses are given by

$$s^m(\mathcal{U}, \ell, \kappa) = \min_{\mathbf{\Pi} \in \mathcal{U}} \| s(\mathbf{\Pi}, \ell, \kappa) \|, \quad s^M(\mathcal{U}, \ell, \kappa) = \max_{\mathbf{\Pi} \in \mathcal{U}} \| s(\mathbf{\Pi}, \ell, \kappa) \|.$$

Systemic Loss Uncertainty

Definition

The *systemic loss uncertainty* of a financial system $(\mathcal{U}, \ell, \kappa)$ is defined as

$$\epsilon(\mathcal{U}, \ell, \kappa) := \frac{s^M(\mathcal{U}, \ell, \kappa) - s^m(\mathcal{U}, \ell, \kappa)}{\|\ell\|}$$

Intermediation Level

Definition

The intermediation level of a bank i in the financial system $(\mathcal{U}, \ell, \kappa)$, denoted by μ_i, is defined as

$$\mu_i := \frac{\min\left\{\sum_{j=1}^{n} \hat{\pi}_{ji}\ell_j, \ell_i\right\}}{\max\left\{\sum_{j=1}^{n} \hat{\pi}_{ji}\ell_j, \ell_i\right\}}$$

where $\sum_{j=1}^{n} \hat{\pi}_{ji}\ell_j$ is the best estimate of the interbanking assets of bank i.

- $0 \le \mu_i \le 1$ for $i = 1, \ldots, n$
- Consistent with finance literature, e.g. Cocco et al. (2009)

Network Class - Emprical Evidence

- Real interbank networks consist of core nodes acting as intermediaries and periphery nodes which lend to them. See Craig and Von Peter (2014), Fricke and Lux (2013), Cocco et al. (2009) and Furfine (1999).
- To resemble these activities, we consider a class of financial networks consisting of intermediaries and creditors, denoted by $\mathcal{I} := \{1, \ldots, n_I\}$ and $\mathcal{C} := \{n_I + 1, \ldots, n\}$ respectively.
- Assume the set of intermediaries to be homogeneous, i.e with identical liabilities, outside assets, and contract uncertainty.
- Creditors are lenders and hence do not have any liabilities to the system, i.e., $\ell_i = 0, i \in \mathcal{C}$.

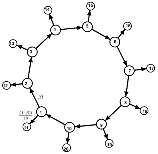
Two Network Architectures

All intermediaries-creditors networks are generated by a convex combination of two network classes:

$$\hat{\mathbf{\Pi}}^{\alpha} = \alpha \hat{\mathbf{\Pi}}^{1} + (1-\alpha)\hat{\mathbf{\Pi}}^{0}.$$

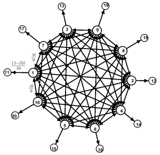
Ring:

$$\hat{\pi}^{1}_{ij} = \begin{cases} \beta & \text{if } i \in \mathcal{I} \setminus \{n_I\}, j = i+1 \text{ or } i = n_I, j = 1 \\ \frac{1-\beta}{n-n_I} & \text{if } i \in \mathcal{I}, j \in \mathcal{C} \\ 0 & \text{else} \end{cases}$$



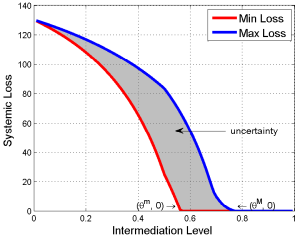
Complete:

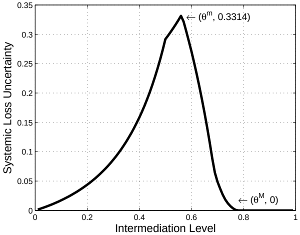
$$\hat{\pi}^{0}_{ij} = \begin{cases} \frac{\beta}{n_I-1} & \text{if } i,j \in \mathcal{I}, i \neq j \\ \frac{1-\beta}{n-n_I} & \text{if } i \in \mathcal{I}, j \in \mathcal{C} \\ 0 & \text{else} \end{cases}$$



- β is the proportion of liabilities from an intermediary to all others.
- The intermediation level of $\hat{\mathbf{\Pi}}^{\alpha}$ is β.

Systemic Losses v.s. Intermediation Level





$$\underbrace{s^m(\mathcal{U}^\alpha(\beta), \ell, \kappa)}_{\downarrow\beta} = \begin{cases} n_l\left(l - \frac{k}{1-\overline{\eta}(\beta)}\right) & \text{if } \beta < \theta^m \\ 0 & \text{else} \end{cases}$$

$$\underbrace{s^M(\mathcal{U}^\alpha(\beta), \ell, \kappa)}_{\downarrow\beta} = \begin{cases} \sum_{i\in\mathcal{D}}(l_i - (\sum_{j=1}^{|\mathcal{D}|}\pi_{ji}^M p_j + \sum_{j=|\mathcal{D}|+1}^{n_l}\pi_{ji}^M l + k_i)) & \text{if } \beta < \theta^M \\ 0 & \text{else} \end{cases}$$

$$\epsilon(\mathcal{U}^\alpha(\beta+\Delta\beta), \ell, \kappa) - \epsilon(\mathcal{U}^\alpha(\beta), \ell, \kappa) \begin{cases} \geq 0 & \text{if } \beta + \Delta\beta \leq \theta^m \\ \leq 0 & \text{if } \theta^m \leq \beta \leq \beta + \Delta\beta \leq \theta^M \\ = 0 & \text{if } \beta \geq \theta^M \end{cases}$$

Policy Implication

- When the intermediation level is higher than θ^m:
  - Increasing it reduces both the maximum systemic loss and the systemic loss uncertainty.
  - Encouraging higher intermediation activity is always beneficial.
- When the intermediation level is lower than θ^m:
  - Increasing it reduces both minimum and maximum systemic losses but also results in higher systemic loss uncertainty.
  - Trade off between reduced losses in the extreme scenarios and higher systemic loss uncertainty.

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Minimal Loss Network I

Lemma

It holds that $\mathbf{\Pi}^m$ *defined by*

$$\pi_{ij}^m := \begin{cases} \left(\frac{\overline{\eta}}{\beta}\right)\hat{\pi}_{ij}^{\alpha} & \text{if } i,j \in \mathcal{I} \\ \left(\frac{1-\overline{\eta}}{1-\beta}\right)\hat{\pi}_{ij}^{\alpha} & \text{if } i \in \mathcal{I}, j \in \mathcal{C} \\ 0 & \text{else} \end{cases}$$

belongs to $\underset{\mathbf{\Pi} \in \mathcal{U}}{\operatorname{argmin}} \| s(\mathbf{\Pi}, \ell, \kappa) \|$. $\overline{\eta}$ *denotes the maximum value of liabilities owed by an intermediary i to all other intermediaries.*

Minimal Loss Network II

Example

Let $(\mathcal{U}^0, \ell, \kappa)$ be a financial system. $n_I = 4, \delta = 0.5, I = 25, k = 15, \beta = 0.6$.

$$\mathbf{\Pi}^m = \begin{pmatrix} 0 & 0.8/3 & 0.8/3 & 0.8/3 & 0.1 & 0.1 \\ 0.8/3 & 0 & 0.8/3 & 0.8/3 & 0.1 & 0.1 \\ 0.8/3 & 0.8/3 & 0 & 0.8/3 & 0.1 & 0.1 \\ 0.8/3 & 0.8/3 & 0.8/3 & 0 & 0.1 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad s(\mathbf{\Pi}^m, \ell, \kappa) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

- Proportion of liabilities from every intermediary to other intermediaries in the system is maximized ($\overline{\eta} = 0.8$).
- All intermediaries evenly bear the losses coming from unfulfilled liabilities

Maximal Loss Network I

Lemma

It holds that $\mathbf{\Pi}^M$ *defined by*

$$\pi_{ij}^M := \begin{cases} \max\left\{(1-\delta)\hat{\pi}_{ij}^{\alpha}, \underline{\eta} - \sum_{w=1}^{j-1} \pi_{iw}^M - \sum_{w=j+1}^{n_I}(1+\delta)\hat{\pi}_{iw}^{\alpha}\right\} & \text{if } i,j \in \mathcal{I}, i \neq j \\ \left(\frac{1-\underline{\eta}}{1-\beta}\right)\hat{\pi}_{ij}^{\alpha} & \text{if } i \in \mathcal{I}, j \in \mathcal{C} \\ 0 & \text{else} \end{cases}$$

belongs to $\underset{\mathbf{\Pi}\in\mathcal{U}}{\operatorname{argmax}} \|s(\mathbf{\Pi}, \ell, \kappa)\|$. $\underline{\eta}$ *denotes the minimum value of liabilities owed by an intermediary* i *to all other intermediaries.*

Maximal Loss Network II

Example

Let $(\mathcal{U}^0, \ell, \kappa)$ be a financial system. $n_I = 4, \delta = 0.5, l = 25, k = 15, \beta = 0.6$.

$$\mathbf{\Pi}^M = \begin{pmatrix} 0 & 0.1 & 0.1 & 0.2 & 0.3 & 0.3 \\ 0.1 & 0 & 0.1 & 0.2 & 0.3 & 0.3 \\ 0.1 & 0.1 & 0 & 0.2 & 0.3 & 0.3 \\ 0,1 & 0.1 & 0.2 & 0 & 0.3 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad s(\mathbf{\Pi}^M, \ell, \kappa) = \begin{pmatrix} 2.84 \\ 2.84 \\ 0.57 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

- Proportion of liabilities from an intermediary to any other is minimized ($\underline{\eta} = 0.4$).
- Contagion leads to higher systemic loss: default of 1 may lead to default of 2, and further trigger sequential defaults of intermediary $3, 4, \ldots$.