Time Consistency of Dynamic Risk and Performance Measures

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Joint work with Tomasz R. Bielecki and Marcin Pitera

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Thanks to

the Organizers and WPI

ECMF - let's all support this great initiative ...

"The dynamic consistency axiom turns out to be the heart of the matter."

A. Jobert and L. C. G. Rogers

Valuations and dynamic convex risk measures,
Math Fin 18(1), 2008, 1-22.

- The decisions made at a given point of time have impact on the future resolution of uncertainty;
- Decisions are made through time so that today's decisions account on their temporal impact;
- Assessment of preferences should be done in such a way that future preferences are **consistent** with the present ones.

Main Goal:

to develop a **unified theory for studying time consistency** for dynamic risk and dynamic performance measures.

1. T.R. Bielecki, Ig. Cialenco, M. Pitera, *A unified approach to time consistency of dynamic risk measures and dynamic performance measures in discrete time*, Preprint, 2015.
2. T.R. Bielecki, Ig. Cialenco, M. Pitera, *Time consistency of risk and performance measures: a survey*, Preprint, 2016.

Before the Theory of Risk Measures

- Koopmans [Koo60] - Stationary ordinal utility and impatience
- Kreps and Porteus [KP78] - Temporal resolution of uncertainty and dynamic choice theory
- Duffie and Epstein [DE92] - Stochastic differential utility
- Sarin and Wakker [SW98] - dynamic consistency for non-Expected utilities in a decision theoretic framework

Illustrative Examples

Dynamic Entopic Risk Measure (measure of risk)

$$\rho_t(X) = \mathsf{dEnt}_t(X) = \frac{1}{\gamma}\log \mathbb{E}[e^{-\gamma X} \mid \mathcal{F}_t], \quad t = 0, t, \ldots, T.$$

with its appropriate time consistency

$$\rho_{t+1}(X) = \rho_{t+1}(Y) \quad \Longrightarrow \quad \rho_t(X) = \rho_t(Y). \tag{1}$$

Dynamic Gain-Loss Ratio (measure of performance)

$$\alpha_t(X) = \mathsf{dGLR}_t(X) = \frac{\mathbb{E}[X \mid \mathcal{F}_t]}{\mathbb{E}[X^- \mid \mathcal{F}_t]},$$

with its appropriate time consistency

$$m_t \le \alpha_{t+1}(X) \le n_t \quad \Longrightarrow \quad m_t \le \alpha_t(X) \le n_t.$$

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$$m_t \le \alpha_{t+1}(X) \le n_t \quad \Longrightarrow \quad m_t \le \alpha_t(X) \le n_t.$$

Fact: dGLR_t does not satisfy (1)

Basic notation

- T-fixed time horizon; Discrete time setup $\mathcal{T} := \{0, 1, \ldots, T\}$;
- $(\Omega, \mathcal{F}_T, \mathbb{F} = (\mathcal{F}_t)_{t\in\mathcal{T}}, \mathbb{P})$ - the underlying filtered probability space;
- $L^p(\mathcal{F}_t) := L^p(\Omega, \mathcal{F}_t, \mathbb{P};\ \mathbb{R})$; $\bar{L}^p_t := L^p(\Omega, \mathcal{F}_t, \mathbb{P};\ \bar{\mathbb{R}})$; $p \in \{0, 1, \infty\}$.
  $X \in L^p(\mathcal{F}_T)$ is interpreted as terminal payoff.
- $\mathbb{V}^p := \{(V_t)_{t\in\mathbb{T}} : V_t \in L^p_t\}$; $V \in \mathbb{V}^p$ is interpreted as a cash flow with dividend paymetnts V_t at $t \in \mathcal{T}$.
- $\mathcal{X}$ will denote either $L^p(\mathcal{F}_T)$ (case of random variables) or $\mathbb{V}^p$ (case of stochastic processes). In this talk, mostly focus on $\mathcal{X} = L^p(\mathcal{F}_T)$.
- $\infty - \infty = -\infty \quad \text{and} \quad 0 \cdot \infty = 0$
  $E[X|\mathcal{F}_t] := \lim_{n\to\infty} E[(X^+ \wedge n)|\mathcal{F}_t] - \lim_{n\to\infty} E[(X^- \wedge n)|\mathcal{F}_t]$,

Dynamic LM-measure

Definition

A **dynamic LM-measures** is a family $\{f_t\}_{t\in\mathbb{T}}$ of maps $f_t : \mathcal{X} \to \bar{L}^0_t$, such that for any $t \in \mathbb{T}$, f_t is

- **Local:** $1_A f_t(X) = 1_A f_t(1_A \cdot X)$, for any $X, Y \in \mathcal{X}$, $A \in \mathcal{F}_t$
- **Monotone:** $X \geq Y \Rightarrow f_t(X) \geq f_t(Y)$, for any $X, Y \in \mathcal{X}$.

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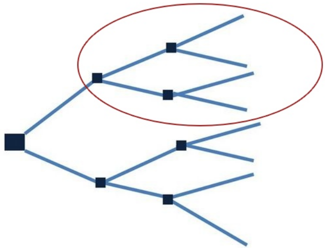

Examples of dynamic LM-mesures; $\mathcal{X} = L^p_T$

- **Dynamic monetary utility measures**
  $f_t : \mathcal{X} \to \bar{L}^0_t$, Local, Monotone and Cash-Additivity

$$f_t(X + m_t) = f_t(X) + m_t, \qquad X \in \mathcal{X},\ m \in L^p_t.$$

  Dynamic monetary risk measures $\rho_t = -f_t$;
  V@R, TV@R, Expected Shortfall, Entropic, etc.

- **Dynamic acceptability indices**
  $f_t : \mathcal{X} \to \bar{L}^0_t$, Local, Monotone and Scale-Invariant

$$f_t(a_t X) = f_t(X), \qquad X \in \mathcal{X},\ a_t \in L^p_t,\ a > 0.$$

  dGLR, Sortino Ratio, RAROC, etc.

Plus some additional properties to capture diversification, such as concavity, quasi-concavity.

Generic Approach to time consistency

Families of benchmark sets, initiated by Tutsch [Tut08].
$\mathcal{Y} = \{\mathcal{Y}_t\}_{t\in\mathbb{T}}$ is a **benchmark family** if $\mathcal{Y}_t \subseteq \mathcal{X}$, $0 \in \mathcal{Y}_t$, and $\mathcal{Y}_t + \mathbb{R} = \mathcal{Y}_t$.

Definition

A dynamic LM-measure φ is **acceptance time consistent** with respect to the benchmark family $\mathcal{Y}$, if

$$\varphi_s(X) \geq \varphi_s(Y) \quad \Longrightarrow \quad \varphi_t(X) \geq \varphi_t(Y),$$

for all $s \geq t$, $X \in \mathcal{X}$ and $Y \in \mathcal{Y}_s$.
Respectively, **rejection time consistent**, if

$$\varphi_s(X) \leq \varphi_s(Y) \quad \Longrightarrow \quad \varphi_t(X) \leq \varphi_t(Y).$$

The larger is $\{\mathcal{Y}_s\}_{s\in\mathbb{T}}$ the stronger the degree of time consistency.

Selected benchmark sets; $\mathcal{X} = L^p_T$

- $\mathcal{Y}_s = L^p_T$, for $s \in \mathbb{T}$, then **strong** accept/reject time consistency.
- $\mathcal{Y}_s = L^p_s$, for $s \in \mathbb{T}$, then **middle** accept/reject time consistency.
- If $\mathcal{Y}_s = \mathbb{R}$, for $s \in \mathbb{T}$, then **weak** accept/reject time consistency.

Equivalent conditions on L^∞ for dynamic risk measures

For $\mathcal{X} = L^\infty$, and φ a monetary utility measure (or $\rho = -\varphi$ is a monetary risk measure), then

- strong time consistent iff $\varphi_t(X) = \varphi_t(\varphi_s(X))$

  Dynamic Programming Principle!
- middle acceptance time consistent iff $\varphi_t(X) \geq \varphi_t(\varphi_s(X))$.
- weakly acceptance time consistent iff $\varphi_s(X) \geq 0 \Rightarrow \varphi_t(X) \geq 0$.

for any $X \in \mathcal{X}$, $s \geq t$.

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for any $X \in \mathcal{X}$, $s \geq t$.

Fact: Performance measures do not fit in this framework.

Idiosyncratic Approaches for dynamic monetary utility measure

A dynamic monetary utility measure φ is *representable*, if

$$\varphi_t(X) = \operatorname*{ess\,inf}_{Q\in\mathcal{M}(\mathbb{P})} \left(\mathbb{E}_{\mathbb{Q}}[X \mid \mathcal{F}_t] + \alpha_t^{\min}(\mathbb{Q})\right), \quad X \in \mathcal{X},$$

where $\alpha_t^{\min}(\mathbb{Q})$ is the minimal penalty function.
Describe/define time consistency in terms of

- *conditional acceptance sets* $\mathcal{A}_{t,s} := \{X \in L^p \cap \bar{L}_s^0 :\ \varphi_t(X) \geq 0\}$.
- *minimal penalty functions* $\alpha_{t,s}^{\min}(\mathbb{Q}) := -\operatorname{ess\,inf}_{X\in\mathcal{A}_{t,s}} \mathbb{E}_{\mathbb{Q}}[X \mid \mathcal{F}_t]$.
  $\alpha_t := \alpha_{t,T}$, $\mathcal{A}_t = \mathcal{A}_{t,T}$

Ex: strong time consistency: $\varphi_t(X) = \varphi_t(\varphi_s(X))$ iff $\mathcal{A}_t = \mathcal{A}_{t,s} + \mathcal{A}_s$ iff $\alpha_t^{\min}(\mathbb{Q}) = \alpha_{t,s}^{\min}(\mathbb{Q}) + \mathbb{E}_{\mathbb{Q}}[\alpha_s^{\min}(\mathbb{Q}) \mid \mathcal{F}_t]$.

Ex: middle acceptance time consistency: $\varphi_t(X) \geq \varphi_t(\varphi_s(X))$ iff $\mathcal{A}_t \supseteq \mathcal{A}_{t,t+1} + \mathcal{A}_{t+1}$ iff $\alpha_t^{\min}(\mathbb{Q}) \geq \alpha_{t,t+1}^{\min}(\mathbb{Q}) + \mathbb{E}_{\mathbb{Q}}[\alpha_{t+1}^{\min}(\mathbb{Q}) | \mathcal{F}_t]$.

- Cocycle condition, g-expectation, dynamics of penalty function, recursive construction, rectangular property, prudence, etc

Update Rules Approach; $\mathcal{X} = L^p_T$

Definition

A family $\mu = \{\mu_{t,s} : t, s \in \mathbb{T},\ t < s\}$ of maps $\mu_{t,s} : \bar{L}^0_s \to \bar{L}^0_t$ is called an **update rule** if μ satisfies the following conditions:

1) (Local) $\mathbb{1}_A \mu_{t,s}(m) = \mathbb{1}_A \mu_{t,s}(\mathbb{1}_A m)$;

2) (Monotone) if $m \geq m'$, then $\mu_{t,s}(m) \geq \mu_{t,s}(m')$;

for any $s > t$, $A \in \mathcal{F}_t$ and $m, m' \in \bar{L}^0_s$.

Definition

Dynamic LM-measure φ is **μ-acceptance/rejection** time consistent if

$$\varphi_s(X) \geq m_s \quad (\text{resp. } \leq) \quad \Longrightarrow \quad \varphi_t(X) \geq \mu_{t,s}(m_s) \quad (\text{resp. } \leq),$$

for all $s > t$, $s, t \in \mathbb{T}$, $X \in \mathcal{X}$, and $m_s \in \bar{L}^0_s$.

If satisfied for $s = t + 1$, then we say that φ is *one-step μ-accept/reject time consistent*.

Proposition

The LM-measure φ is μ-acceptance time consistent if and only if

$$\varphi_t(X) \geq \mu_{t,s}(\varphi_s(X)),$$

for any $X \in \mathcal{X}$ and $s > t$.

The update rule μ is called

- s-**invariant** if $\mu_{t,s}(m) = \mu_t(m)$, for $s \geq t$, $m \in \bar{L}^0_s$.
- **projective** if it is s-invariant and $\mu_t(m_t) = m_t$, for $t \in \mathbb{T}$, $m_t \in \bar{L}^0_t$.

Example

The families $\mu^1 = \{\mu^1_t\}_{t\in\mathbb{T}}$ and $\mu^2 = \{\mu^2_t\}_{t\in\mathbb{T}}$ given by

$$\mu^1_t(m) = \mathbb{E}[m|\mathcal{F}_t], \quad \text{and} \quad \mu^2_t(m) = \operatorname{ess\,inf}_t m, \quad m \in \bar{L}^0,$$

are projective update rules.

Connection between Benchmark Sets and Update Rules

Proposition

For any LM-measure φ and for any benchmark family $\mathcal{Y}$, there exists an update rule μ such that φ is time consistent with respect to $\mathcal{Y}$ if and only if it is μ-time consistent.

Proof. Put

$\mu_{t,s}(m_s) := \operatorname{ess\,sup}_{A\in\mathcal{F}_t}\Big[\mathbb{1}_A \operatorname{ess\,sup}_{Y\in\mathcal{Y}^-_{A,s}(m_s)}\varphi_t(Y) + \mathbb{1}_{A^c}(-\infty)\Big]$,

$\mathcal{Y}^-_{A,s}(m_s) := \{Y\in\widehat{\mathcal{Y}}_s : \mathbb{1}_A m_s \geq \mathbb{1}_A\varphi_s(Y)\}$ and

$\widehat{\mathcal{Y}}_t := \{Y\in\mathcal{X} : Y = \mathbb{1}_A Y_1 + \mathbb{1}_{A^c}Y_2,\ \text{for some } Y_1, Y_2\in\mathcal{Y}_t \text{ and } A\in\mathcal{F}_t\}$.

Remark

The converse implication does not hold true, in general. For example, time consistency for dynamic acceptability index cannot be expressed in terms of a single benchmark family.

Proposition (Update Rule $\mu_t(m) = \operatorname{ess\,inf}_t m$)

Let φ be a dynamic LM-measure on L^p_T. The following are equivalent:

1) *φ is is* **weakly acceptance** *time consistent, i.e.*

$$\varphi_t(X) \geq \operatorname{ess\,inf}_t \varphi_s(X).$$

2) *φ is μ-acceptance time consistent, i.e. for any $X \in L^p_T$, $s > t$,*

$$\varphi_s(X) \geq m_s \implies \varphi_t(X) \geq \operatorname{ess\,inf}_t m_s.$$

3) *For any $X \in L^p_T$, $s > t$, with $\mathcal{M}_t(\mathbb{P}) := \{\mathbb{Q} \in \mathcal{M}(\mathbb{P}) : \mathbb{Q}|_{\mathcal{F}_t} = \mathbb{P}|_{\mathcal{F}_t}\}$,*

$$\varphi_t(X) \geq \operatorname*{ess\,inf}_{\mathbb{Q} \in \mathcal{M}_t(\mathbb{P})} \mathbb{E}_{\mathbb{Q}}[\varphi_s(X)|\mathcal{F}_t].$$

4) *For any $X \in L^p_T$, $s > t$, and $m_t \in \bar{L}^0_t$,*

$$\varphi_s(X) \geq m_t \Rightarrow \varphi_t(X) \geq m_t.$$

Most of the known examples are weakly acceptance or rejection time consistent.

Proposition

Let φ be a dynamic LM-measure on L^p_T, and let μ be any projective update rule. If φ is μ-acceptance time consistent, then φ is weakly acceptance time consistent.

A dynamic LM-measure φ is **strongly time consistent** if

$$\varphi_s(X) = \varphi_s(Y) \quad \Longrightarrow \quad \varphi_t(X) = \varphi_t(Y),$$

for any $X, Y \in L^p$ and $s > t$.

Proposition

1) *φ is strongly time consistent.*
2) *There exist an update rule μ such that φ is both μ-acceptance and μ-rejection time consistent.*
3) *φ is acceptance time consistent with respect to $\{\mathcal{Y}_t = L^p_T\}_{t\in\mathbb{T}}$.*
4) *There exists an update rule μ such that*
$$\mu_{t,s}(\varphi_s(X)) = \varphi_t(X), \quad X \in L^p_T,\ s > t.$$
5) *There exists a one-step update rule μ such that for any*
$$\mu_{t,t+1}(\varphi_{t+1}(X)) = \varphi_t(X), \quad X \in L^p,\ t < T.$$

Time consistency induced by LM-measures

Definition

Let φ be a dynamic LM-measure on L^p. A family $\widehat{\varphi} = \{\widehat{\varphi}_t\}_{t\in\mathbb{T}}$ of maps $\widehat{\varphi}_t : \bar{L}^0 \to \bar{L}^0_t$ is **an LM-extension of φ**, if for any $t \in \mathbb{T}$, $\widehat{\varphi}_t|_{\mathcal{X}} \equiv \varphi_t$, and $\widehat{\varphi}_t$ is local and monotone on $\bar{L}_0$.

Define the collection of functions $\varphi^{\pm} = \{\varphi^{\pm}_t\}_{t\in\mathbb{T}}$, where $\varphi^{\pm}_t : \bar{L}^0 \to \bar{L}^0_t$ and

$$\varphi^{+}_t(X) := \operatorname*{ess\,inf}_{A\in\mathcal{F}_t} \Big[\mathbb{1}_A \operatorname*{ess\,inf}_{Y\in\mathcal{Y}^+_A(X)} \varphi_t(Y) + \mathbb{1}_{A^c}(+\infty) \Big],$$

$$\varphi^{-}_t(X) := \operatorname*{ess\,sup}_{A\in\mathcal{F}_t} \Big[\mathbb{1}_A \operatorname*{ess\,sup}_{Y\in\mathcal{Y}^-_A(X)} \varphi_t(Y) + \mathbb{1}_{A^c}(-\infty) \Big],$$

is called the *upper/lower LM-extension of* φ, where $\mathcal{Y}^+_A(X) := \{Y \in \mathcal{X} \mid \mathbb{1}_A Y \geq \mathbb{1}_A X\}$, $\quad \mathcal{Y}^-_A(X) := \{Y \in \mathcal{X} \mid \mathbb{1}_A Y \leq \mathbb{1}_A X\}$.

Proposition

The functions φ^- and φ^+ are LM-extensions of φ. Moreover, for any $\widehat{\varphi}$ LM-extension of φ we have

$$\varphi_t^-(X) \leq \widehat{\varphi}_t(X) \leq \varphi_t^+(X).$$

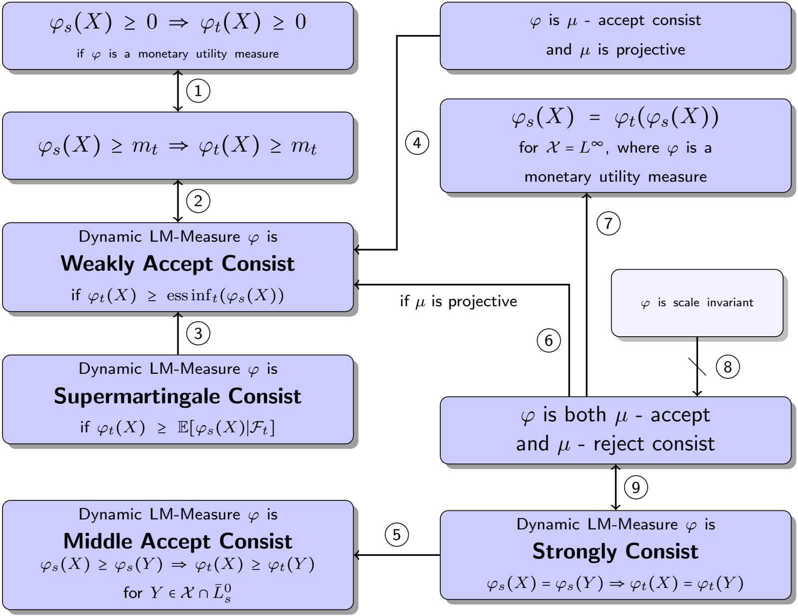
Proposition

Any LM-extension $\widehat{\varphi}$ of a dynamic LM–measure φ is an s-invariant update rule. Moreover, $\widehat{\varphi}$ is projective if and only if $\varphi_t(X) = X$, for $t \in \mathbb{T}$ and $X \in L^p \cap \bar{L}_t^0$.

Definition

φ is **strongly* time consistent**, if there exists an LM-extension $\hat{\varphi}$, of φ, such that φ is both $\hat{\varphi}$-acceptance and $\hat{\varphi}$-rejection time consistent, or equivalently $\varphi_t(X) = \hat{\varphi}_t(\varphi_s(X)), \quad X \in \mathcal{X},\ s,t \in \mathbb{T},\ s > t.$

Figure: *Summary of results for acceptance time consistency for random variables*



Time consistency for stochastic processes

If $\mathcal{X} = \mathbb{V}$, the space of adapted stochastic processes, then the update rule may/should depend on the intermediate values of the underlying process.

Definition

A family of maps $\mu_{t,s} : \bar{L}^0_s \times \mathcal{X} \to \bar{L}^0_t$ is an **update rule** if

- (locality) $1_A \mu_{t,s}(m, V) = 1_A \mu_{t,s}(1_A m, V)$;
- (monotonicity) $m \geq m' \Rightarrow \mu_{t,s}(m, V) \geq \mu_{t,s}(m', V)$;

for $V \in \mathcal{X}$, $A \in \mathcal{F}_t$ and $m, m' \in \bar{L}^0_s$, $s > t$.

Definition

As before, we say that a dynamic LM-measure $f = \{f_t\}_{t\in\mathbb{T}}$ is *μ-acceptance time consistent* if

$$f_s(V) \geq m_s \quad \Longrightarrow \quad f_t(V) \geq \mu_{t,s}(m_s, V),$$

for all $s > t$, $V \in \mathcal{X}$ and $m_s \in \bar{L}^0_s$.

We deal only with one-step update rules $s = t + 1$.

Selected update rules

- $\mu_{t,t+1}(V) = \operatorname{ess\,inf}_t(V_{t+1}) + V_t$.
  Weak acceptance time consistency for dynamic risk measures for stochastic processes: $\varphi_{t+1}(V) \geq m_t \Rightarrow \varphi_t(V) \geq m_t + V_t$.

- $\mu_{t,t+1}(V) = E(V_{t+1}|\mathcal{F}_t)] + V_t$.
  Supermartingale property for stochastic processes.

- $\mu_{t,t+1}(V) = 1_{\{V_t \geq 0\}} \operatorname{ess\,inf}_t(V_{t+1}) + 1_{\{V_t<0\}}(-\infty)$.
  Semi-weak acceptance time consistency (suitable for scale invariant measures)

$$\varphi_{t+1}(V) \geq m_t, \text{ and } V_t \geq 0 \quad \Longrightarrow \quad \varphi_t(V) \geq m_t.$$

Theorem

Let $\{\varphi_t^x\}_{t\in\mathbb{T}}, x \in \mathbb{R}_+$ be a decreasing family of weakly acceptance/rejection time consistent dynamic LM-measures. Then $\alpha_t : \mathcal{X} \to \bar{L}_t^0$ defined by

$$\alpha_t(V) = \sup\{x \in \mathbb{R}_+ : \varphi_t^x(V) \geq 0\},$$

is a semi-weakly acceptance/rejection time consistent dynamic LM-measure.

Theorem

Let $\{\alpha_t\}_{t\in\mathbb{T}}$ be a semi-weakly acceptance/rejection time consistent dynamic LM-measure, which is independent of the past and translation invariant. Then for any $x \in \mathbb{R}_+$ the family $\{\varphi_t^x\}_{t\in\mathbb{T}}$ defined by

$$\varphi_t^x(V) = \inf\{c \in \mathbb{R} : \alpha_t(V - c1_{\{t\}}) \leq x\},$$

is a weakly rejection/acceptance time consistent dynamic LM-measure.

Examples

	$\mathcal{X}$	WA	WR	sWA	sWR	MA	MR	STR	Sub	Sup
Cond. WV@R	L^p	✓		✓						✓
TV@R AI	$\mathbb{V}^p$			✓						
RAROC	$\mathbb{V}^p$			✓						
dGLR	$\mathbb{V}^p$			✓	✓					
dEnt $\gamma \geq 0$	L^p	✓	✓	✓	✓	✓	✓	✓		✓
dEnt $\gamma \leq 0$	L^p	✓	✓	✓	✓	✓	✓	✓	✓	
dEnt+ $\gamma_t \downarrow$	L^p	✓		✓		✓				✓*
dEnt+ $\gamma_t \uparrow$	L^p		✓		✓		✓		✓**	
RSC	L^p	✓	✓	✓	✓	✓	✓	✓		

*if $\gamma_t \geq 0$, **if $\gamma_t \leq 0$

RSC = Risk Sensitive Criterion

Thank You !

The end of the talk ...
but not of the story

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