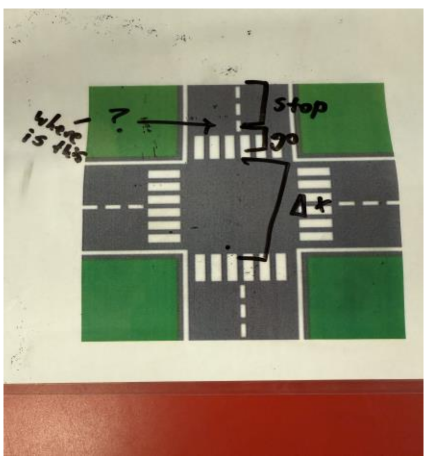
Physics POW Traffic Light Design

Problem Statement:

How long should a traffic light stay yellow considering factors like the reaction time, deceleration time, speed limit, distance of the intersection, and the weather/road conditions? In this problem we assumed that the cars approaching the intersection are traveling at the speed limit; cars proceeding through an intersection must not exceed the speed limit in order to cross before the light turns red; and a driver that makes the decision to stop must be able to safely stop before the intersection's stop line. We also assumed that humans would account for differences in deceleration time that might occur due to the weight/length of the car and the road or weather conditions, and that everyone on the road is qualified to drive. Some of the terminology we used was: GO Zone, which is the area where it is safe for the car to go through the light at the speed limit, and STOP Zone, which is the region where the cars can safely stop while going on the speed limit while not entering the GO Zone. The length of the intersection is labeled by Δx in the diagram below.

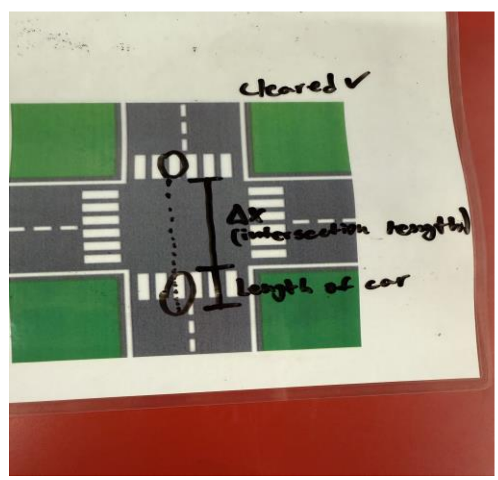


Process:

When approaching this problem, we first tried to consider what exactly the problem was asking us and determined that we needed to separate the problem into 2 parts. The first was to figure out how much distance the car would need to stop safely. The second part was the time we needed to pass through the intersection from the start of the go zone. At the start of this process, we identified some factors of the problem along with their units, which were: reaction time (s), deceleration time (m/s^2), speed limit (m/s), intersection length (m), and car length (m). We also made the conclusion that the go zone must account for how long it takes for the car to slow down.

In the first part, we determined that the go zone would be the deceleration distance of the car plus the distance covered during the reaction time. Therefore, we decided to use $v^2 = v_0^2 + 2a\Delta x$ to determine the length of the go zone. We then translated the variables so that v_0 = speed limit, v = 0m/s because the car ends up stopping, x_0 = start of the go zone, x is the start of the intersection, and a = how fast the car can decelerate safely. Taking the general equation, we rearranged it to solve for x. This gives us $x = -v_0^2/2a$. This means the go zone length is equal to the negative of the speed limit squared divided by 2 times the deceleration of the vehicle. After this we realized we needed to factor in the distance covered in the reaction time. We found this using the equation $v = x/t$, which rearranges to $x_{reaction}=v_o * t_{reaction}$, where v is the speed limit and t is the reaction time. Therefore, our final equation for the go zone length is $-v_0^2/2a + x_{reaction}$.

In the second part, we use the calculation for the start of the go zone to calculate the time it takes to go through the entire GO zone and the intersection. To do this, we can insert our equation for the length of the GO zone into a larger equation. For this, we used the time = distance/velocity equation and modified it, since we are calculating the amount of time it takes to cross the intersection. The velocity would be the speed limit while the distance is the go zone length, intersection length, and car length. Simplifying this so that reaction time is set separately, we are given ((-(speed limit)^2 /2*deceleration) + intersection length + car length)/speed limit + average human reaction time to get the length of time the yellow light needs to stay on. Another way to put this is ($-v_0^2/2a$ + intersection length (m) + car (m))/ v_0 + avg reaction time(s), to get the ideal time for the yellow light.

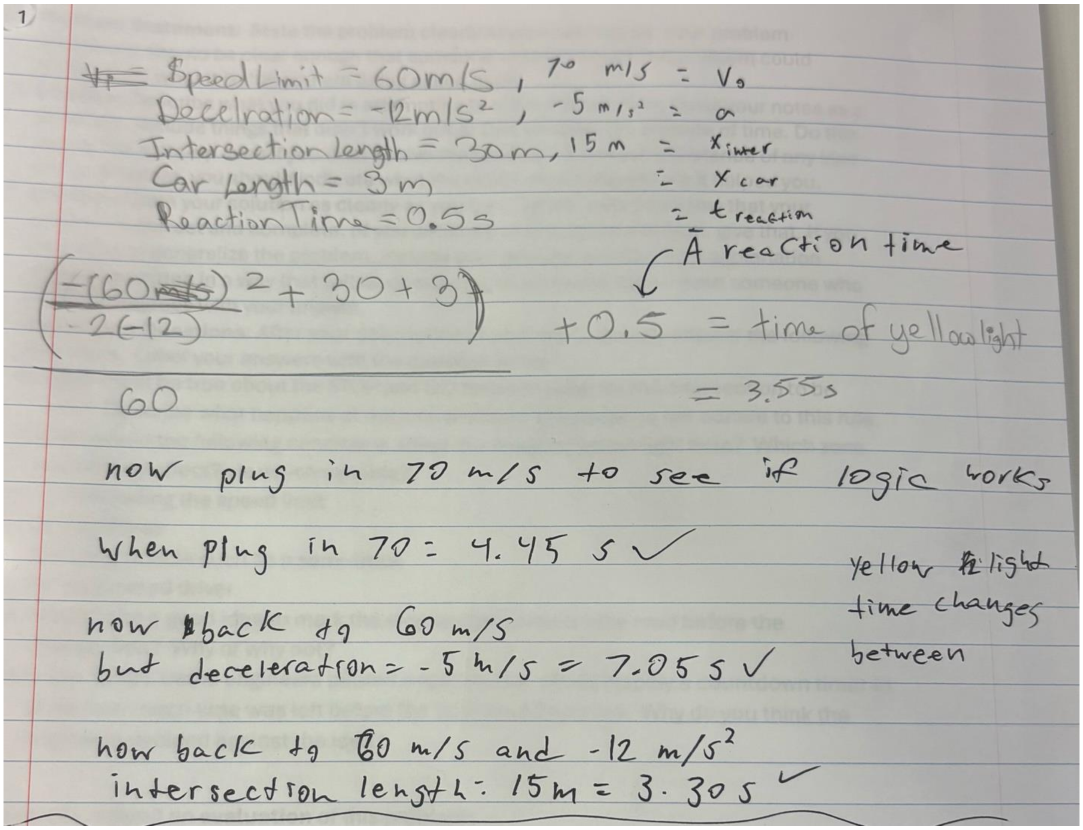


After this, we realized that the ideal yellow light time would change depending on the person driving and the type of vehicle passing through the intersection. Additionally, road and weather conditions could affect the deceleration and reaction times. With a deeper analysis of this, we realized that humans already account for these factors themselves. In wet, icy, or foggy conditions, humans drive slower and shorten their go zone, eliminating the effect on reaction time and deceleration. For large vehicles, such as semi-trucks, drivers are specifically taught how to go on regular streets, and that includes stopping and going through stop lights. They would shorten their go zone in accordance with their training. For reaction time based on drivers, it is a very minimal amount of change, and any extreme outliers would not be able to get a license. Therefore, despite some variations based on encounters, this formula would continue to work.

Solution:

To figure out how long a traffic light should stay yellow, we created the equation:

(((-(speed limit)^2)/(2*deceleration) + intersection length + car length)/speed limit) + avg reaction time = time of the yellow light. Another way to put this is $(((-(v_0^2)/2a + x_{intersection} + x_{car})/v_0) + t_{reaction} = t_{yellow\ light}$. We know that this is correct because we checked it with a few sets of variables, with minor changes made to each one, and recalculated to affirm that the variables are working logically. In the example below we used 60m/s for the speed limit, -12m/s^2 for the deceleration, 30m for the intersection length, 3 m for the car length and 0.5 seconds for the reaction time. This gave us a yellow light time of 3.55 seconds, which is reasonable for the given values of the variables. We changed the speed limit to 70m/s and kept the other variables the same, and the $t_{yellow\ light}$ changed to equal 4.45 seconds. This logically makes sense because if a car is going faster, it will take longer to stop, increasing the go zone and time for the yellow light. Then we went back to the original variable values but lowered the deceleration to 5 m/s^2. This increased the time to 7.05 sec which also makes sense because if a car cannot slow down fast, it will need more time to stop, increasing the yellow light time. Then, we used the same original values but decreased the intersection length to 15m which decreased the time to 3.3sec. This is in line with logical reasoning because if the car has less area to drive across, it will need less time. While we cannot account for every variable, such as exact reaction times, we assume that everyone on the road is qualified to drive and therefore knows how to account for this.



Speed Limit = 60 m/s , 70 m/s = V_0

Decelration = $-12\ m/s^2$, $-5\ m/s^2$ = a

Intersection length = 30 m, 15 m = x_{inter}

Car Length = 3 m = x_{car}

Reaction Time = 0.5 s = $t_{reaction}$

Ā reaction time

$$\frac{\left(\frac{-(60\ m/s)^2}{2(-12)} + 30 + 3\right)}{60} + 0.5 = \text{time of yellow light} = 3.55\ s$$

now plug in 70 m/s to see if logic works

when plug in 70 = 4.45 s ✓

now back to 60 m/s but deceleration = -5 m/s = 7.05 s ✓

now back to 60 m/s and $-12\ m/s^2$ intersection length = 15 m = 3.30 s ✓

yellow light time changes between

Discussion Questions:

a. For the intersection to be safe, the go and stop zones must connect or overlap with the start of the go zone before or equal to the end of the stop zone. This is because when the car is in the stop zone, it can safely decelerate before the intersection, while in the go zone, it can make it through before the light turns. If these two zones do not connect or overlap, then there is the possibility that a car will be stuck in the area between them, where it can neither slow safely nor go through the intersection on time, which is dangerous. Therefore, there must not be any gap between the go and stop zone.

b.

1. Exceeding Speed Limit:
   Exceeding the speed limit would increase the initial velocity. In turn, the calculated overall time for the yellow light would increase. This change would make stop zone shorter, as a faster velocity would take more time to stop. However, it would also make the go zone larger, because the car vehicle would be moving at a faster velocity, and would cover more distance in the time given.

2. Bald Tires:

   Bald tires would decrease the absolute value acceleration. As in, a car with good tires might slow down at $-12 m/s^2$ while bald tires might slow down at only $-7 m/s^2$. Even though the acceleration is technically increasing, the absolute value acceleration would be decreasing (towards 0). With a decrease in absolute value acceleration, the calculated yellow light time would increase. Also, the stop distance would decrease similar to part B1. Different, however, is how the go zone does not increase in this example, as the car is not moving faster but the deceleration distance is longer. This creates an issue where there would be a “gap” between the where the stop and go zones meet. This does create an issue in our problem. However, in real life, a driver with bald tires would most likely travel below the speed limit, so they can stop in a reasonable amount of time when needed. This would be the same situation with slippery roads. While the deceleration distance is longer, making a gap between the go and stop zones, drivers adjust accordingly by driving under the speed limit, canceling the longer deceleration period with a slower speed to balance the equation.

3. Long Vehicles such as a semi-truck:
   Long vehicles would increase the car length, x_{car}, which in turn would increase the time of the yellow light. This makes sense because a longer car would take longer to get through the intersection. It would take a larger distance for these vehicles to reach a complete stop, which would mean that their stop zone increases because drivers need to brake earlier than those in a smaller vehicle. Therefore, their go zone decreases because cars need to be closer to the intersection to make it all the way through.

4. Distracted Driver:
   Having a distracted driver would increase the reaction time, $t_{reaction}$. This would also increase the time of the yellow light. This would also shrink the stop zone because the driver would cover more distance before realizing that they must stop, shrinking the area they have to brake. It doesn’t affect the go zone because if the driver is going at the speed limit and realizes after that they’re in the go zone, they merely just continue going through the intersection. Therefore, the go zone does not shift at all.

c. Funnily enough, the go zone and stop zone are already on the road! They are a real thing in multi-lane traffic. Observe below Gold Star BLVD in Worcester MA. The short white dashed lines when driving down the road represent the “stop” zone. If the light turns yellow while you are still in the dashed zone, you should stop because you will not make it through the light before it turns red. Meanwhile, if you are in in the long solid white line zone, it means you are in the “go” zone. If the light turns yellow while you are in this area (and you are going the speed limit) then you can safely make it through the intersection before the light turns red. Physically putting these lines on the roadway allows drivers to make a more educated decision when a light turns yellow. Instead of speeding up to make the light or slamming on the breaks, the driver can know immediately whether they should slow down and stop or continue through the light safely.

Images provided by Google Maps 2025

d. Having a countdown timer to show how much time was left before the traffic light turned red would be a bad idea because it would influence people to speed up to beat the yellow light. Just like how people run across a crosswalk to beat the pedestrian crosswalk signal, cars would similarly feel encouraged to race across the road to make it in time. A countdown timer can also distract drivers from focusing on the road and could potentially increase the number of car crashes. This is probably why the engineers decided not to add a countdown in 1960.