Über Problem – Hamster Huey and Algebra Alex

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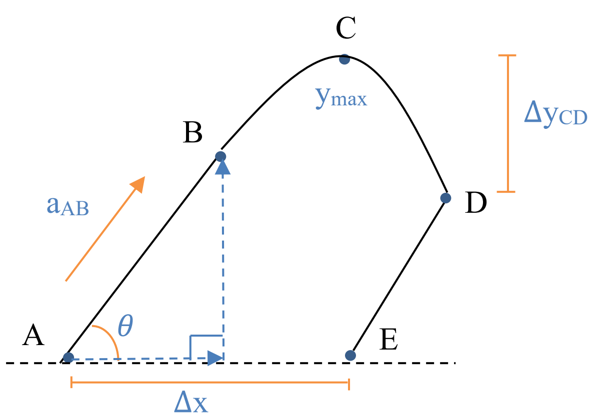
One breezy afternoon Algebra Alex decides to launch Hamster Huey into the air using a model rocket. The rocket is launched over level ground, from rest, at a specified angle above the East horizontal. The rocket engine is designed to burn for specified time while producing a constant net acceleration for the rocket. Assume the rocket travels in a straight-line path while the engine burns. After the engine stops the rocket continues in projectile motion. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens the rocket instantly changes speed and descends at a constant vertical speed. A horizontal wind blows the rocket, with parachute, from the East to West at the constant speed of the wind. Assume the wind affects the rocket only during the parachute stage.



Givens:

$\theta = 38\,°$	$\Delta y_{CD} = 76$ m
$a_{AB} = 6.7$ m/s^2	$a_{CDy} = -9.8$ m/s^2
$t_{AB} = 7$ s	$a_{CDx} = 0$ m/s^2
$a_{BCy} = -9.8$ m/s^2	$v_{DEy} = -7$ m/s
$a_{BCx} = 0$ m/s^2	$v_{DEx} = -12$ m/s

Strategy:
To find the horizontal displacement from point A to point E, the problem was broken into several stages: AB, BC, CD, and DE. Finding the total displacement in the x direction required the displacement of each individual stage. Though equations solving for final position, time, and velocity, the displacements in the x and y directions for each individual section were solved. As this problem was written in 2D, information about the y direction was relevant to solving this problem as it allowed time per section to be solved. Because time remains constant in both the x and y directions, the time of displacement in the y direction is equal to exactly that in the x direction. By building off information solved for in the previous stage of the problem, like velocity and final position, the next stage of the problem utilized and incorporated this information into its equations. Though this method, final displacement was reached though the simple method of finding time in the y direction, using that time to find displacement in the x direction, and using the previous section's displacement as initial position in the following section. After solving for x-direction displacement though this method for each stage, a final equation is reached in stage DE that gives total displacement in the x-direction.

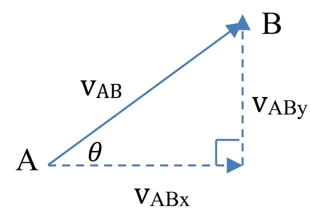
Stage AB
Step 1: Using the givens of acceleration and time, solve for net velocity.

$$a = \frac{\Delta v}{\Delta t}$$

$$6.7 = \frac{\Delta v}{7}$$

$$\underline{\Delta v = 46.9\ m/s}$$

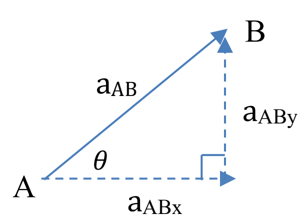
Step 2: Solve for initial velocities in the x and y directions using cosine and sine functions.



$$v_{ABx} = v_{AB} \times (cos\theta)$$
$$v_{ABX} = 46.9 \times (\cos 38) \text{ solver;}$$
$$\underline{v_{ABx} = 36.957\ m/s}$$

$$v_{ABy} = v_{AB} \times (sin\theta)$$
$$v_{ABy} = 46.9 \times (\sin 38) \text{ solver;}$$
$$\underline{v_{ABy} = 28.874\ m/s}$$

Step 3: Solve for acceleration in the x and y directions using cosine and sine functions.



$$a_{ABx} = a_{AB} \times (cos\theta)$$

$$a_{ABx} = 6.7 \times (\cos 38) \text{ solver;}$$
$$\underline{a_{ABx} = 5.279\ m/s^2}$$

$$a_{ABy} = aAB \times (sin\theta)$$
$$a_{ABy} = 6.7 \times (\sin 38) \text{ solver;}$$
$$\underline{a_{ABy} = 4.124\ m/s^2}$$

Step 4: Solve for displacment in the y-direction using equation 3, y-direction acceleration and velocity, and the givens.

$$y_{fAB} = \frac{1}{2} \times a_y \times t^2 + v_y \times t + y_i$$
$$y_{fAB} = \frac{1}{2} \times (4.124) \times (7)^2 + (0) \times (7) + 0 \text{ solver;}$$
$$\underline{y_{fAB} = 101.038\ m}$$

Step 5: Solve for displacment in the x-direction using equation 3, x-direction acceleration and velocity, and the givens.

$$x_{fAB} = \frac{1}{2} \times a_x \times t^2 + v_x \times t + x_i$$
$$x_{fAB} = \frac{1}{2} \times (5.279) \times (7)^2 + (0) \times (7) + 0 \text{ solver;}$$
$$\underline{x_{fAB} = 129.336\ m}$$

Stage BC

Step 6: Calculate meximum height in the y-direction using equation 3.

$$v_{fy}^2 = v_{iy}^2 + 2a\Delta y$$
$$0^2 = (28.8745)^2 + 2(-9.8)\Delta y \text{ solver;}$$
$$y = 42.5376\ m$$
$$y_{fBC} = 101.038 + 42.5376$$
$$\underline{y_{fBC} = 143.599\ m}$$

Step 7: Using equation 2, solve for time to maximum height. Due to the change in direction that occurs at maximum height, it can assumed that velocity in the y-direction at this postion is 0 m/s.

$$v_f = a\Delta t + v_i$$
$$0 = (-9.8)\Delta t + 28.874 \text{ solver;}$$
$$\underline{\Delta t = 2.946\ s}$$

Step 8: Using time to reach maximum height in the y-direction, calculate displacement in the x-direction using equation 3 for stage BC.

$$x_{fBC} = \frac{1}{2} \times a_x \times t^2 + v_x \times t + x_i$$
$$x_{fBC} = \frac{1}{2} \times (0) \times (2.946)^2 + (36.957) \times (2.946) + 129.336 \text{ solver;}$$
$$\underline{x_{fBC} = 238.211\ m}$$

Stage CD

Step 9: Use the given, stating that the parachute opens 76 m below maximum height, to solve for height at D.

$$y_{fD} = y_c - 76$$
$$y_{fD} = 143.558 - 76$$
$$\underline{y_{fD} = 67.558\ m}$$

Step 10: Use final height at D (y_{fD}) to solve for time using equation 3.

$$y_{fCD} = \frac{1}{2} \times a_y \times t^2 + v_y \times t + y_i$$
$$67.558 = \frac{1}{2} \times (-9.8) \times t^2 + 0 \times t + 143.558$$

$$t = \frac{\pm\sqrt{-4(-4.9)(143.558)}}{(-9.8)} \text{ solver;}$$

t = -3.9383 s or <u>t = 3.9383 s</u>

Step 11: Use time in the y-direction from CD to solve for x-direction displacement in this same stage.

$$x_{fCD} = \frac{1}{2} \times a_x \times t^2 + v_x \times t + x_i$$
$$x_{fCD} = \frac{1}{2} \times 0 \times (3.9383)^2 + 36.957 \times 3.9383 + 238.211 \text{ solver;}$$
$$\underline{x_{fCD} = 383.759}$$

Stage DE

Step 12: Using the equation 3, solve for time in the y-direction from point D to E.

$$y_{fDE} = \frac{1}{2} \times a_y \times t^2 + v_y \times t + y_i$$
$$y_{fDE} = \frac{1}{2} \times 0 \times t^2 + (-7) \times t + 67.558$$
$$y_{fDE} = -7 \times t + 67.558$$
$$-67.558 = -7 \times t \text{ solver;}$$
$$\underline{t = 9.6511\ s}$$

Step 13: Using time in the y-direction in stage DE, solve for x-direction displacment in stage DE as well by using equation 3.

$$x_{fDE} = \frac{1}{2} \times a_x \times t^2 + v_x \times t + x_i$$
$$x_{fDE} = \frac{1}{2} \times 0 \times (9.6511)^2 + (-12) \times (9.6511) + 383.759 \text{ solver;}$$
$$\underline{x_{fDE} = 267.946\ m}$$

Step 14: Final answer!

$$\boxed{\Delta x_{AE} = 267.95\ m}$$