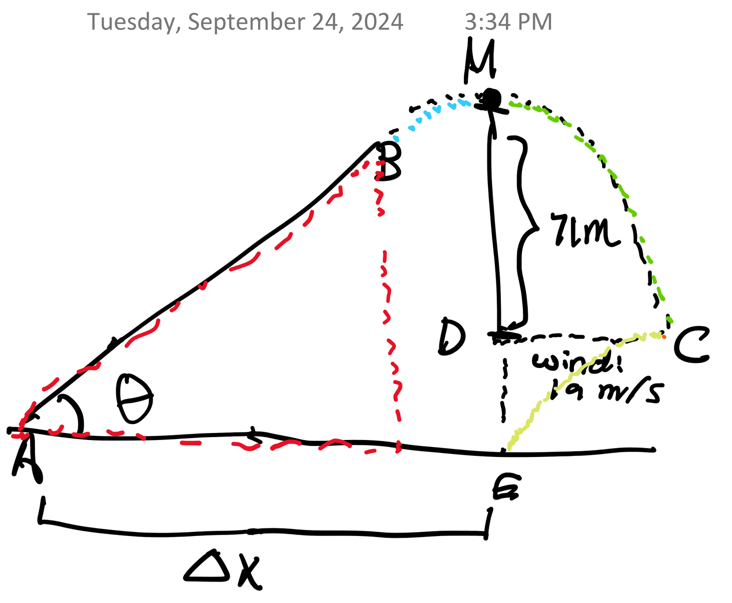
Multi-step Rocket Problem - Sharvi Pujari

Tuesday, September 24, 2024 3:34 PM



Givens:
$\theta = 44^{\circ}$
A-B time = 8.7 sec
A-B accel = 6.1 m/s²
C-E $V_x = -19$ m/s
C-E $V_{0y} = -11$ m/s

A-B: 1) Find V_f (velocity at B) by multiplying A-B accel and A-B time

$V_f = at$
$V_f = 8.7 \cdot 6.1$
$V_f = 53.07$ m/s

2) Find Δx from A-B by using $V^2 = V_0^2 + 2a\Delta x$

$(53.07)^2 = 0^2 + 2(6.1)\Delta x$
$2816.42 = 12.2\Delta x$
$\Delta x = 230.85$ m

Note right triangle formed by A, B, Z: $\overline{AB} = R$, $\overline{BZ} = R_y$, $\overline{AZ} = R_x$

2) Find values of R_x and R_y (R_x will be part of our final Δx)

$R_x = R\cos\theta \rightarrow R_x = 230.85\cos(44)$
$\boxed{R_x = 165.8\text{m}}$
$R_y = R\sin\theta \rightarrow R_y = 230.85\sin(44)$
$R_y = 160.12$ m
↳ This will be our initial height for next part

B-M: 1) Find Δx in B-M using V_f from A-B
↳ we can assume V at M is 0 because it's the vertex of the parabolic curve
↳ we can use our V_f from A-B as V_0

$V^2 = V_0^2 + 2a\Delta x$
$0^2 = (53.07)^2 + 2(-9.8)\Delta x$
$0 = 2816.4 - 19.6\Delta x$
$19.6\Delta x = 2816.4$
$\boxed{\Delta x = 143.69}$

2) Find maximum height of projectile using info from horizontal motion

H	V
$\Delta x = V_{0x}t$	$Y = Y_0 + V_{0y}t - \frac{1}{2}gt^2$
$143.69 = V_{0x}t$	$Y = 160.12 + 53.07\sin(44)(3.76) - 4.9(3.76)^2$
$143.69 = V_0\cos\theta\, t$	$Y = 160.12 + 138.6 - 69.2$
$143.69 = 53.07\cos(44)t$	$\boxed{Y = 229.52}$
$143.69 = 38.17t$	
$t = 3.76$ s	

M-C: 1) Height parachute is released from is 71 m below max. projectile height, so we can say parachute release height is $229.52 - 71 \rightarrow 158.52$m
↳ Now we need to find Δx from M-C
↳ our Δy will be 71 m
↳ V_x remains constant (same as V_x in B-M)
↳

H	V
$\Delta x = V_x t$	$Y = Y_0 + V_{0y}t - \frac{1}{2}gt$
$\Delta x = 53.07\cos(44)t$	$158.52 = 229.52 + (0)t - 4.9t^2$
$\Delta x = 38.17(3.8)$	$158.52 = 229.52 - 4.9t^2$
$\boxed{\Delta x = 145.06}$	$-71 = 4.9t^2$
	$71 = 4.9t^2$
	$t^2 = 14.48$
	$t = 3.8$ s

C-E: 1) V_{0x} is -19 m/s and V_{0y} is -11 m/s
↳ We need to find Δx, which leads to ground again
↳ Two equations: $\Delta x = V_{0x}t$ and $Y = Y_0 + V_{0y}t - \frac{1}{2}gt^2$

H	V
$\Delta x = V_x t$	$Y = Y_0 + V_{0y}t - \frac{1}{2}gt^2$
$\Delta x = -19(4.67)$	$0 = 158.52 - 11t - 4.9t^2$
$\boxed{\Delta x = -88.73}$	$t = 4.67$s

Total Δx: 165.8m + 143.69m + 145.06m − 88.73m =
= $\boxed{365.82\text{m } 0^{\circ}\text{ E}}$

Explanation:

Engine burning (A-B)
↳ Solve for final velocity given the rocket's burn time and acceleration (use $V_f = at$)
↳ Solve for change in displacement (Δx) using the $V^2 = V_0^2 + 2a\Delta x$ given our new V_f and initial V as 0 m/s
↳ Using the right triangle formed by A, B, and Z, find the values of R_y and R_x given R is Δx found before
↳ R_x is the horizontal distance travelled that we want in the final answer
↳ R_y is the initial height used in the pre-parachute projectile segment

Pre-parachute Pt. 1 (B-M)
↳ Solve for Δx using the final velocity from the previous part (use $V^2 = V_0^2 + 2a\Delta x$, where V is 0 because vertical velocity is 0 at vertex)
↳ Prep for next part and solve for maximum height of projectile
↳ First use $\Delta x = V_{0x}t$ where V_{0x} is our V_f from engine burn. Multiply V_0 by $\cos(\theta)$. Solve for time by isolating t.
↳ Substitute t value into $Y = Y_0 + V_{0y}t - \frac{1}{2}gt^2$ to find Y, where V_{0y} is $V_0\cos(\theta)$

Pre-Parachute Pt. 2 (M-C)
↳ Solve for parachute release height by taking max. projectile height and subtracting 71m
↳ Find Δx using two equations: 1) $\Delta x = V_{0x}t$ and 2) $Y = Y_0 + V_{0y}t - \frac{1}{2}gt^2$ if V_{0x} remains constant and Δy is 71
↳ Be sure to substitute value of t into Δx equation once found in Y = equation

Parachute Motion
↳ keep in mind that this Δx will be negative
↳ Solve for Δx by using the given vertical velocity and horizontal velocity to formulate $\Delta x = V_{0x}t$ and $Y = Y_0 + V_0t - \frac{1}{2}gt^2$
↳ Solve for t first in vertical motion equation
↳ Substitute t value in horizontal motion equation

Now take all of the Δx's in each segment and add them together to give you your total displacement change.