Smita Bhogle Calculus Uber Rocket Problem Section R

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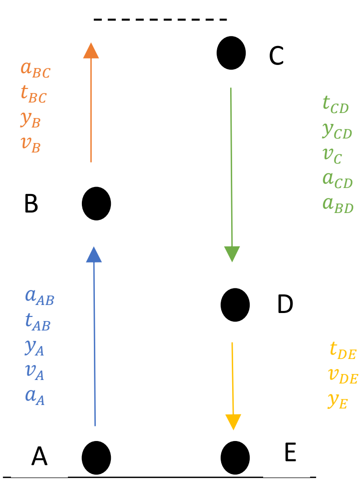
Problem:

One calm afternoon Calculus Cam decides to launch Hamster Huey into the air using a model rocket. The rocket is launched straight up off the ground, from rest. The rocket engine is designed to burn for specified time while producing non-constant net acceleration given by the equations below. After the engine stops the rocket continues upward in free-fall. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens, assume the rocket instantly stops, and then increases speed to a terminal velocity given by the equation below. Assume the air resistance affects the rocket only during the parachute stage.

Diagram:



Givens:

$a_{AB} = -0.6t^2 + 16\, m/s^2$

$t_{AB} = 6.1\, s$

$y_A = 0\, m$

$v_A = 0\, m/s$

$a_A = 0\, m/s^2$

$a_{BD} = -9.8\, m/s^2$

$v_C = 0\, m/s^2$

$\Delta y_{CD} = -77\, m$

$v_{DE} = -14\left(1 - e^{-\frac{t}{11}}\right) m/s$

$yE = 0\, m$

Approach:

The final goal of this problem is to find the total time that Hamster Huey's rocket is in the air. His journey can be split into 4 stages: AB, BC, CD, and DE. Point A represents the start of his journey, Point B is when the rocket engine dies, and Point C is the maximum height of the rocket. Point D represents when the parachute is deployed, and Point E is when the rocket lands back on Earth.

Stage AB:

One of the given equations is for acceleration for Stage AB. The time for this stage is also a given, but in order to determine the velocity at Point B, the integral of the acceleration can be taken to produce an equation for velocity:

$$\Delta v_{AB}[t] = \int_0^t (a)dt$$

$$\Delta v_{AB}[t] = \int_0^t (-0.6t^2 + 16)dt$$

$$\Delta v_{AB}[t] - v_A = -0.2t^3 + 16t$$

$$\Delta v_B[t] = -0.2t^3 + 16t$$

$$\underline{\Delta v_B[6.1] = 52.2038\, m/s}$$

Then, to find the height at Point B, I took the integral of my velocity equation which would yield an equation for y position:

$$\Delta y_{AB}[t] = \int_0^t (v)dt$$

$$\Delta y_{AB}[t] = \int_0^t (-0.2t^3 + 16t)dt$$

$$\Delta y_{AB}[t] - y_A = -0.05t^4 + 8t^2$$

$$y_B[6.1] - 0 = -0.05t^4 + 8t^2$$

$$\underline{y_B[6.1] = 228.451\, m}$$

Stage BC:

Since the rocket's engine dies at Point B, the rocket is free falling during stage BC. This means the acceleration of the rocket is gravity, $-9.8\, m/s^2$, which is also constant. A kinematic equation, Equation 3, can then be used:

$$v_C = a\Delta t + v_B$$

$$0 = (-9.8)\Delta t_{BC} + 52.2038$$

$$\underline{\Delta t_{BC} = 5.32692\, s}$$

That time can be used to find the max height at Point C:

$$y_C = \frac{1}{2}(v_C + v_B)\Delta t_{BC}$$

$$y_C = \frac{1}{2}(52.2038)(5.32692)$$

$$\underline{y_C = 139.043\, m}$$

$$y_{max} = y_B + y_C$$

$$y_{max} = 228.451 + 139.043$$

$$\underline{y_{max} = 367.494\, m}$$

Stage CD:

Then Equation 3 can be used to determine the time of Stage CD:

$\Delta y_{CD} = \frac{1}{2}at^2 + v_C t$
$-77 = \frac{1}{2}(-9.8)t^2 + 0t$
$0 = -4.9t^2 + 77, solver$
$\underline{\Delta t_{CD} = 3.96412}$

Stage DE:

At Point D, the parachute opens, and a new velocity equation is given. After taking the integral of this equation, an equation for y position is found:

$\Delta y_{DE}[t] = \int_0^t (v_{DE})dt$
$\Delta y_{DE}[t] - \Delta y_D = \int_0^t (-14\left(1 - e^{-\frac{t}{11}}\right) dt$
$y_E[t] - 290.494 = -154e^{\frac{-t}{11}} - 14t + 154$
$0 = -154e^{\frac{-t}{11}} - 14t + 444.494, solver$
$\underline{\Delta t_{DE} = 31.0986}$

Total Time:

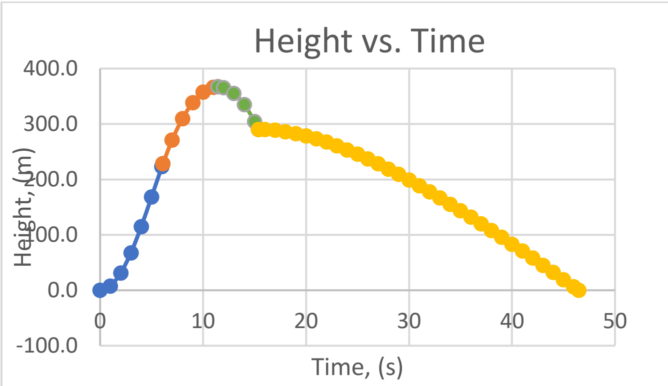
Lastly, the rocket's total flight time can be found by adding the times from each stage.

$\Delta t_{tot} = \Delta t_{AB} + \Delta t_{BC} + \Delta t_{CD} + \Delta t_{DE}$
$\Delta t_{tot} = 6.1 + 5.32692 + 3.96412 + 31.0986$
$\boxed{\Delta t_{tot} = 46.49\ s}$

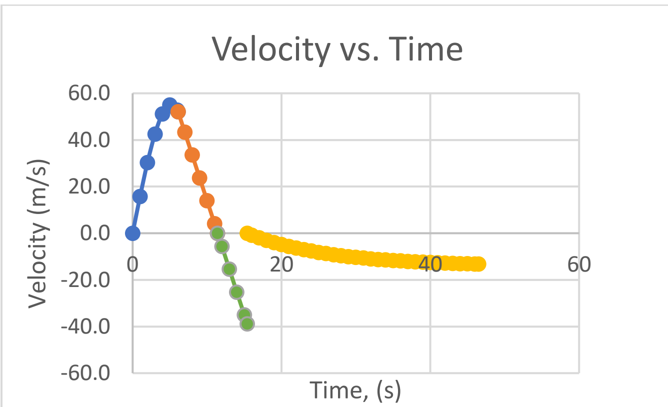
Table of Values:

t(s)	y(m)	v(m/s)	a(m/s²)
0	0.0	0.0	16.0
1	8.0	15.8	15.4
2	31.2	30.4	13.6
3	68.0	42.6	10.6
4	115.2	51.2	6.4
5	168.8	55.0	1.0
6	223.2	52.8	-5.6
6.1	228.5	52.2	-6.3
6.1	228.5	52.2	-9.8
7	271.5	43.4	-9.8
8	309.9	33.6	-9.8
9	338.6	23.8	-9.8
10	357.5	14.0	-9.8
11	366.6	4.2	-9.8
11.42	367.5	0.0	-9.8
11.42	367.5	0	-9.8
12	365.9	-5.6	-9.8
13	355.4	-15.4	-9.8
14	335.1	-25.2	-9.8
15	304.9	-35.0	-9.8
15.39	290.5	-38.8	-9.8
15.39	290.5	0.0	-1.3
16	290.3	-0.8	-1.2
17	288.9	-1.9	-1.1
18	286.5	-3.0	-1.0
19	283.0	-3.9	-0.9
20	278.7	-4.8	-0.8
21	273.5	-5.6	-0.8
22	267.5	-6.3	-0.7
23	260.9	-7.0	-0.6
24	253.6	-7.6	-0.6
25	245.7	-8.2	-0.5
26	237.3	-8.7	-0.5
27	228.4	-9.1	-0.4
28	219.0	-9.6	-0.4
29	209.3	-9.9	-0.4
30	199.2	-10.3	-0.3
31	188.7	-10.6	-0.3
32	177.9	-10.9	-0.3
33	166.9	-11.2	-0.3
34	155.6	-11.4	-0.2
35	144.1	-11.6	-0.2
36	132.3	-11.9	-0.2
37	120.4	-12.0	-0.2
38	108.2	-12.2	-0.2
39	96.0	-12.4	-0.1
40	83.5	-12.5	-0.1
41	70.9	-12.6	-0.1
42	58.2	-12.8	-0.1
43	45.4	-12.9	-0.1
44	32.5	-13.0	-0.1
45	19.5	-13.1	-0.1
46	6.4	-13.1	-0.1
46.49	0.0	-13.2	-0.1

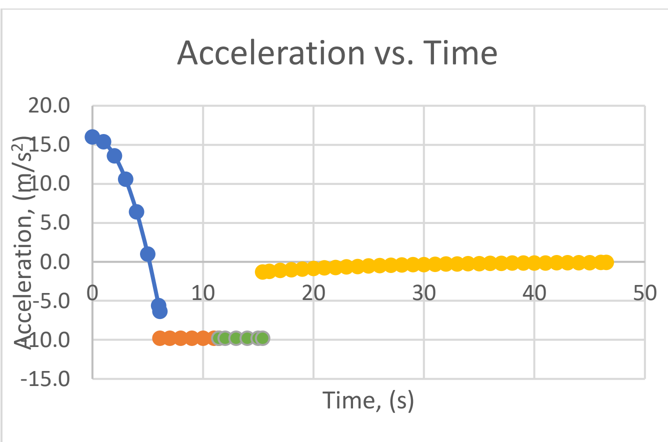
Graphs:



Height vs. Time
400.0
300.0
200.0
100.0
0.0
-100.0
Height, (m)
0
10
20
30
40
50
Time, (s)



Velocity vs. Time
60.0
40.0
20.0
0.0
-20.0
-40.0
-60.0
Velocity (m/s)
0
20
40
60
Time, (s)



Acceleration vs. Time
20.0
15.0
10.0
5.0
0.0
-5.0
-10.0
-15.0
Acceleration, (m/s²)
0
10
20
30
40
50
Time, (s)