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Math Modeling

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Preliminary/Raw Work

Problem Statement

At the Pythagoras High School, the football team raises money by selling season tickets that allows an individual to attend all the football games at the high school for one year. The captains of the football team are trying to decide what would be the best price for a full season ticket price would be in order to make the most income. Unfortunately, due to CTE, they are having a hard time deciding on that price. In order to gauge what the families of the Pythagoras High School would be willing to pay, a survey was sent out to 914 families. The data from the survey can be used as accurate information because the captains assume that the parents want the sale to be a success. The question on the form was “What is the most that you would be willing to pay for a football season ticket for the school year?” This data is found in the table below.

In[∘]:=
```
TableForm[{ticketPrice, surveyResponses},
 TableHeadings → {{"Ticket Price ($)", "Expected ticket sales"}, None}]
```

Out[∘]//TableForm=

Ticket Price ($)	55	80	95	100	120	135	155	180
Expected ticket sales	140	85	45	90	115	80	65	155

Using this data, a method was developed to determine the best possible ticket price.

Assumptions

1. The greatest number of people who will buy a ticket is the total number of people who answered the survey.
~The survey was sent out to 914 families, but the football team only received 775 individual responses. People who do not want to buy a season pass would not have filled out the form.
2. Individuals will definitely buy a ticket if it is lower than the price they stated.
~This is because individuals would be happy to pay a price lower than what they originally stated.
For clarity for my write up:
a = ticket price
b = max price the individual is willing to pay
If b ≥ a, the individual will definitely buy the ticket.
3. There are some people out there who are willing to pay greater than what they stated on the form; individuals will change their mind once the sale happens.
~People go over their budget all the time. Additionally, with football season tickets, individuals are likely to just pay the one time fee to be able to go to as many games as they want. The families would also be willing to pay the higher price because they want to support the football team.
4.The prices listed on the survey are the only possible prices that the tickets could be sold at.
~The survey that was sent out was created by the individuals selling the tickets and they would set up the form in a way that only listed possible ticket prices. If they were able to sell tickets at any price, they would've added at least a couple more options. Additionally, there is no consistency for how the possible ticket prices relates to one another. (The increments between the prices are 25, 15, 5, 20, 15, 20, and 25)

Process/Solution

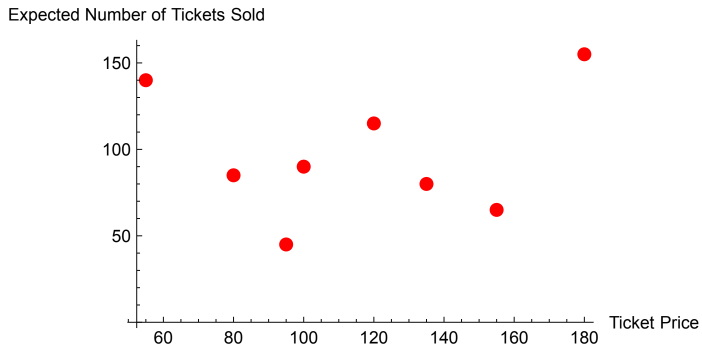
```
In[∘]:= ClearAll
Out[∘]= ClearAll
```

First, I reorganized the data into separate lists so I could manipulate the data easier in Mathematica. I put ticket prices into one list, and survey responses in another. I then manipulated with the data a bit to see if I could find any patterns. I graphed the expected ticket sales vs. the ticket price to see if I could find any visual patterns in the data. I also multiplied the expected number of tickets sold by the ticket price to find how much income would be made with only the given data and graphed that income against ticket price to see if there were any patterns there. I used ticket price as the x-value for these points because the number of tickets sold and income relies on ticket price. After analyzing the graphs I was not able to conclude anything.

```
In[∘]:= surveyResponses = {140, 85, 45, 90, 115, 80, 65, 155}
In[∘]:= ticketPrice = {55, 80, 95, 100, 120, 135, 155, 180}
```

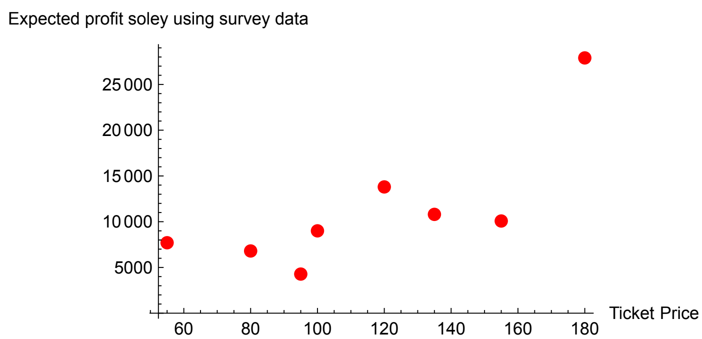
In[ ]:=
```
ListPlot[givenData, PlotStyle → {Red, PointSize[0.03]},
 AxesLabel → {"Ticket Price", "Expected Number of Tickets Sold"}]
```

Out[ ]=



In[ ]:=
```
ListPlot[data1, PlotStyle → {Red, PointSize[0.03]},
 AxesLabel → {"Ticket Price", "Expected profit soley using survey data"}]
```

Out[ ]=



I then took a different approach and added up all the survey responses to see how many total people were willing to buy a ticket. The calculated sum is where Assumption #1 came from.

In[ ]:=
```
Total[surveyResponses]
```

Out[ ]=

```
775
```

After this, I started thinking of the number of tickets sold and income as a function of ticket price.

expected income = expected number of tickets sold * ticket price

At this point I realized that I could calculate an expected income if I came with a method to find out how many tickets would be sold at a given price. The ticket price that resulted in the largest income would be the ticket price that the football team should choose since the football team wants the greatest possible income.

I then tried to see if could come up with an equation that would output the number of tickets sold if a ticket price was inserted. At this point, I had to start making assumptions in order to get an equation. This is where Assumption #2 and Assumption #3 come into play. At any given ticket price, the survey responses are split into two groups: individuals who stated that they would pay an amount greater than or equal to that ticket price, and individuals who stated that they would pay less than that price. For example: at a ticket price of $80, 635 people stated that they would pay for a ticker price greater than or equal to $80, but 140 people stated that the max ticket price they would pay is $55. I believed that the data provided by the survey was accurate, but I also believed that some people would change their mind (Assumption #3). This means that the total number of tickets sold is = number of people who would definitely buy a ticket + number of people who would change their mind

I then began the process to find what the number of people from these two groups. I started with number of people who would definitely buy a ticket since that was simpler.

Based off of Assumption #2, I can find the number of people who will definitely buy a ticket at a given price if I add up the number of people who said that they would pay greater than or equal to that ticket price. I added up these values manually and then inserted them into Mathematica in a list that corresponds with the ticket price list I made earlier.

In[ ]:=
```
peopleWhoWouldDefinitelyBuy = {775, 635, 550, 505, 415, 300, 220, 155}
```

Then I started by finding the number of people who would buy a ticket anyways by finding out the total number of people who stated that their max ticket price is lower than the given ticket price. Again, I added up these values manually and then inserted them into Mathematica in a corresponding list.

In[ ]:=
```
peopleWhoSaidTheyWouldPayLess = {0, 140, 225, 270, 360, 475, 555, 620}
```

Out[ ]=
```
{0, 140, 225, 270, 360, 475, 555, 620}
```

From here, I knew that the number of people who changed their mind would be some percent of the total number people who stated their max ticket price is lower; therefore to find the number of people who changed their mind, I would need a calculate/find the percent of people who would change their mind. To find this rate, I saw that at a price of $55, 100% of the people surveyed would buy a ticket at that price; I also knew that the percent of people who would change their mind relies on the ticket price. Somehow, 55 related to 100%, or 1. With these facts, I quickly realized that 55/55 = 1. After that realization, I had an equation to find the percent of people who would change their mind.
x = ticket price
x/55 = percent of people who would change their mind.
I then manually calculated these percents, rounded to two decimal places, and created another list.

In[ ]:=
```
percentOfPeopleWhoWouldBuyAnyways = {1, 0.69, 0.58, 0.55, 0.46, 0.41, 0.35, 0.3}
```

Out[ ]=
```
{1, 0.69, 0.58, 0.55, 0.46, 0.41, 0.35, 0.3}
```

Since I now had the number of people who said their max ticket price is lower, and the percent of people who would change their mind, I could multiply the two together to see how many people would change their mind at a given price point. The product was rounded down to the nearest whole number since there cannot be half of a person who changed their mind.

In[ ]:=
```
peopleWhoWouldBuyTicketAnyways =
 Floor[peopleWhoSaidTheyWouldPayLess * percentOfPeopleWhoWouldBuyAnyways]
```
Out[ ]=
```
{0, 96, 130, 148, 165, 194, 194, 186}
```

I then found the total number of people who would buy a ticket at a given price by adding the number of people who would definitely buy a ticket to number of people who would change their mind.

In[ ]:=
```
totalNumberOfPeopleWhoWouldBuyTicket =
 peopleWhoWouldDefinitelyBuy + peopleWhoWouldBuyTicketAnyways
```
Out[ ]=
```
{775, 731, 680, 653, 580, 494, 414, 341}
```

Since I now had the expected number of tickets sold, and the ticket prices I could plug the corresponding values into my original equation: expected income = expected number of tickets sold * ticket price, to solve for expected income.

In[ ]:=
```
expectedIncome = totalNumberOfPeopleWhoWouldBuyTicket * ticketPrice
```
Out[ ]=
```
{42 625, 58 480, 64 600, 65 300, 69 600, 66 690, 64 170, 61 380}
```

In order to visualize the steps I took, I put the results of each step I completed into a table with the given price as the column name.

In[ ]:=
```
TableForm[{peopleWhoWouldDefinitelyBuy,
  peopleWhoSaidTheyWouldPayLess, peopleWhoWouldBuyTicketAnyways,
  totalNumberOfPeopleWhoWouldBuyTicket, expectedIncome},
 TableHeadings → {{"Number of people who would definitely buy at this price",
    "Number of people who said they would pay < than this amount",
    "Expected number of people who would change their mind",
    "Expected total number of people who would buy a ticket at this price",
    "Expected income ($)"},
   {"$55", "$80", "$95", "$100", "$120", "$135", "$155", "$180"}}]
```
Out[ ]//TableForm=

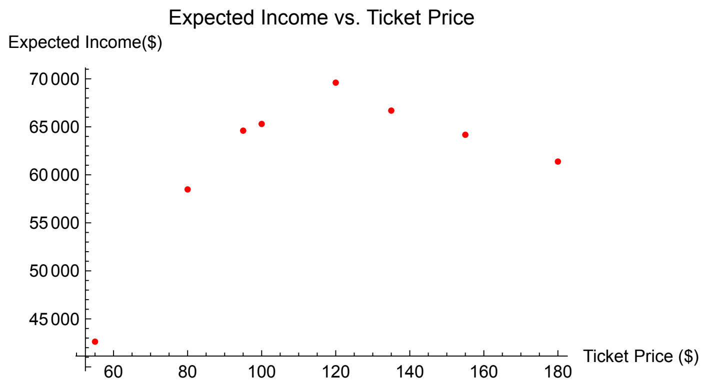
	$55	$80
Number of people who would definitely buy at this price	775	635
Number of people who said they would pay < than this amount	0	140
Expected number of people who would change their mind	0	96
Expected total number of people who would buy a ticket at this price	775	731
Expected income ($)	42 625	58

I also graphed the expected income vs. ticket price in order to see how the expected income changes as the ticket price changes. As you can see below, the data is graphed in a parabola-like shape.

In[ ]:=
```
muffin = Transpose @ {ticketPrice, expectedIncome}
```
Out[ ]=
```
{{55, 42 625}, {80, 58 480}, {95, 64 600}, {100, 65 300},
 {120, 69 600}, {135, 66 690}, {155, 64 170}, {180, 61 380}}
```

In[∘]:=
```
ListPlot[{muffin}, PlotLabel → HoldForm["Expected Income vs. Ticket Price"],
 AxesLabel → {"Ticket Price ($)", "Expected Income($)"},
 PlotStyle → {Red, PointSize → 0.03}]
```

Out[∘]=



In order to have the largest expected income of $69,600, the full season football ticket should have a price of $120.

I also verified my result by plugging in different values for the percents of people who would change their mind and saw that the expected income for a ticket price of $120 was still the largest for these new percents. (One example of arbitrary percents I chose were: 1, 0.6, 0.55, 0.5, 0.45, 0.4, 0.35, 0,3).