

## Multi-Step Rocket Problem

A rocket is launched over level ground, from rest, at a specified angle above the East horizontal. The rocket engine is designed to burn for specified time while producing a constant net acceleration for the rocket. Assume the rocket travels in a straight-line path while the engine burns. After the engine stops the rocket continues in projectile motion. A parachute opens after the rocket falls a specified distance from its maximum height. When the parachute opens the rocket instantly changes speed and descends at a constant vertical speed. A horizontal wind blows the rocket, with parachute, from the East to West at the constant speed of the wind. Assume the wind affects the rocket only during the parachute stage.

Givens for your individualized problem:

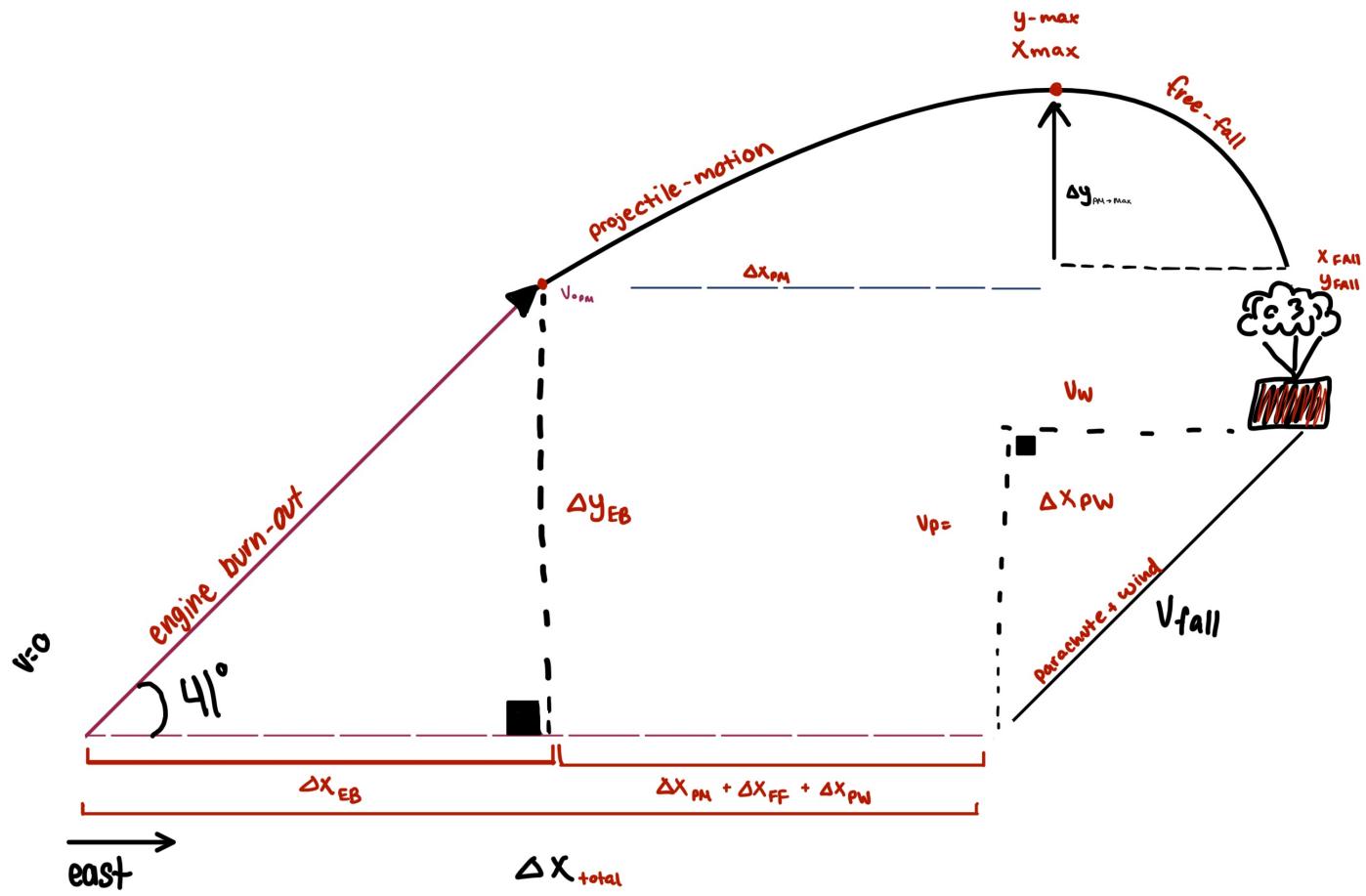
Launch angle	41 deg
Engine burn time	7.3 sec
Net acceleration of rocket while engine burns	6.2 m/s <sup>2</sup>
Vertical distance rocket falls from max height before parachute opens	60 m
Rocket with parachute constant vertical speed	7 m/s
Wind and rocket with parachute constant horizontal speed	13 m/s

Calculate the x-displacement of where the rocket lands relative to the initial x-position.

Your written work, which may be hand-written, must include:

- Your given values including units, and the variable names you've assigned them
- A clear diagram with each **variable** labeled (no values). Note that there are likely multiple y and v values that you will use for the different stages. You should subscript your variables to help tell them apart. The intent of the diagram is to help indicate the meaning of each variable.
- Your work shown for each part of the solution including:
  - the equations before any substitutions
  - a clear substitution step
  - the work shown to get a numerical solution
- Your circled final answer to 4 sig figs with units. Include the magnitude and direction of the displacement.

The assignment is worth 8 points. It will be graded for correctness and work shown.



$y$  - max = max height of path

$x$  - max = horizontal position at max-height

$\Delta x_{EB}$  - change in position for engine burn out stage

$\Delta x_{PM}$  - change in position for projectile motion stage

$\Delta x_{FF}$  - change in position for free-fall stage

$\Delta x_{PW}$  - change in position for parachute+wind stage

$v_w$  = velocity of wind

$v_p$  = velocity of parachute

$v_{0,PM}$  = initial velocity for projectile motion stage

$\Delta y_{EB}$  = change in height for engine-burn stage

$\Delta y_{PM \rightarrow MAX}$  = change in height from end of engine burn out to max-height

$x_{fall}$  = position when parachute opens

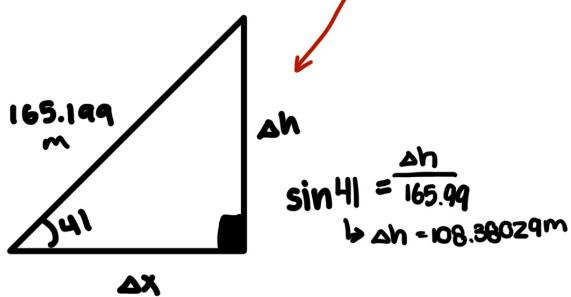
$y_{fall}$  = height when parachute opens

## Variables

# Engine Burn Out:

x	y
$v_0 = 0 \text{ m/s}$	$v = 45.26 \sin 41 \rightarrow 29.6932$
$a = 6.2 \text{ m/s}^2$	$t = 7.36$
$t = 7.36$	$y = v_0 t + \frac{1}{2} a t^2$
$x = x_0 + v_0 t + \frac{1}{2} a t^2$	$x = 0 + 0 + \frac{1}{2} (6.2) (7.3)^2$
$x = 165.199 \text{ m}$	$x = 165.199 \text{ m}$
$v = v_0 + a t$	
$v = 0 + (6.2)(7.3)$	
$v_{0PM} = 45.26 \text{ m/s}$	

Solving the right triangle for  $\Delta x_{EB}$  and  $\Delta y_{EB}$



$$\tan 41 = \frac{\Delta h}{\Delta x}$$

$$\Delta x_{EB} = 124.67726 \text{ m}$$

# Projectile Motion:

x	y
$x_0 = 165.199 \text{ m}$	$v_0 = 45.26 \sin 41$
$x_f = ?$	$a_y = 45.26 \sin 41 + 2(-9.8) a_y$
$v = 45.26 \cos 41$	$v^2 = v_0^2 + 2 a y$
$\Delta x_{PM} = v t$	$\Delta y_{PM \rightarrow \text{max}} = 44.4687 \text{ m} \rightarrow \text{Final } a_y = 108.3$
$\Delta x_{PM} = 45.26 \cos 41 \cdot 3.0299216$	$y = y_0 + v_0 t + \frac{1}{2} a t^2$
$\Delta x_{PM} = 103.496533$	$y = -4.9 t^2 + 45.26 \sin(41)t + 108.3$
$\Delta x_{PM} = \frac{\Delta x_{EB}}{124.677} + \frac{\Delta x_{PM}}{103.496533}$	$\hookrightarrow \text{Vertex} = \frac{-b}{2a} \rightarrow \frac{-45.26 \sin(41)}{2(-4.9)} = \frac{\text{time to reach } y_{\text{max}}}{3.0299216 \text{ s}}$
$x_{\text{max}} = 228.17379468 \text{ m}$	$\rightarrow \text{Plug that into the equation:}$
	$\hookrightarrow -4.9(3.02...)^2 + 42.26 \sin 41 (3.02...) + 108.3$
	$y_{\text{max}} = 153.3643719 \text{ m}$
	$\hookrightarrow \text{could also be found by adding } 108.3... + 44.46...$

$$\Delta y = v_0 t + \frac{1}{2} a t^2$$

$$-60 = \frac{1}{2}(-9.8)t^2$$

$$t = 3.49275 \text{ s} \rightarrow \text{time to drop 60 meters}$$

$$\text{Total Time: } 3.02... + 3.49... = 6.5291926 \text{ s}$$

$$\Delta x = 45.26 \sin 41 \cdot 6.529... = 223.025178 \text{ m}$$

↙ x right before element opens

# Parachute:

x	y
$v_0 = -13 \text{ m/s}$	$\Delta y_{PM} = \text{max height} - 60 \text{ m}$
$\Delta x = v \cdot t$	$153.3643719 - 60 = 93.3643719 \text{ m} = y_{\text{fall}}$
$t = 13.33776 \text{ s}$	$v_w = 7 \text{ m/s West}$
$x_f = ?$	$\rightarrow \text{constant speed, so } t = \frac{q}{v} \text{ or } t = \frac{93.36}{7} = 13.33776 \text{ s}$
$\Delta x = -13 \cdot 13.33776$	
$\Delta x_{PM} = -173.39088$	

$\Delta x$  Total:

$$174.67726 + 773.025178 - 173.39088 = 174.311558 \text{m}$$

↳ 174.3m east