Ch04 Forces - Über Problem - Leaping Larry's Luge Launcher (Algebra version)

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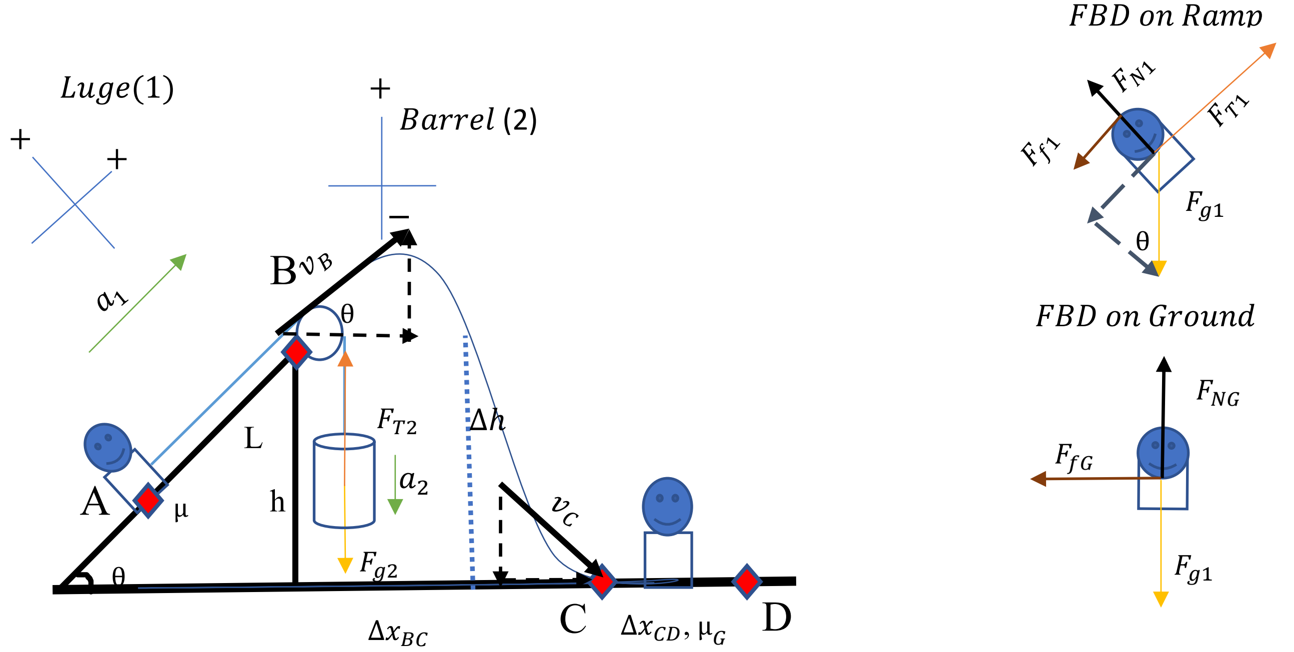
Leaping Larry decided to make 9.1 meter tall ($h = 9.1m$) laborious launcher for his luxury luge using a pulley and ramp system (see diagram). His method was to attach one end of a massless stretchless rope to a barrel of rocks ($m_B = 48kg$) and to hold the other end of the rope. He placed the rope over a massless frictionless pulley, and then walked down the ramp far as down possible to point A (where L = h). When he sat in the luge ($m_l = 38kg$) he accelerated up the ramp, despite friction between his luge and ramp,($\mu = 0.20$) to point B. He then launched off the top at the same angle ($\theta = 25\ deg$) as the ramp (all while releasing the rope and avoiding the pulley). He flew through the air as a projectile to point C, smoothly transitioning all of his (net) speed into the horizontal direction, and eventually slid to a stop at point D having traveled a distance of 50m from B to D ($\Delta x_{BD} = 50m$). Note: Ignore any height differences between luge height, barrel height, and size of the pulley, and the diagram is not drawn to scale.



- Strikethrough means term = 0
- Used bold to indicate where I am substituting as underlining underlines whole thing.

Assume

$$F_{T1} = F_{T2}$$
$$a_1 = -a_2$$

Stage AB

First step is to find the force of friction. To do this find the normal force that the ramp exerts on the luge and Larry and multiply that by μ.

$$\sum F_{j1}: F_{N1} - F_g \cos\theta = \cancel{m_l a_{1j}}$$
$$\underline{F_{N1} = m_l g \cos\theta}$$
$$F_{f1} = \mu \mathbf{F_{N1}}$$
$$\underline{F_{f1} = \mu \boldsymbol{m_l g} \cos\theta}$$

Then, find the sum of forces in the i-direction for the luge and Larry, including the previously found friction.

$$\sum F_{i1}: F_{T1} - \boldsymbol{F_{f1}} - F_{g1}\sin\theta = m_l a_1$$
$$F_{T1} = m_l a_1 + m_L g \sin\theta + \boldsymbol{\mu m_l g \cos\theta}$$

Now, find the sum of forces in the y-direction for the barrel and then use the assumption that the forces of tension of larry + luge and the barrel of rocks are equal to set the two equations equal.

$$\sum F_{y2}: F_{T2} - F_{g2} = m_B a_2$$
$$F_{T2} = m_B a_2 + m_B g$$

Set equal, use other assumption that accelerations are same magnitude but negations of each other to solve for one of the accelerations.

$$m_l \boldsymbol{a_1} + m_l g \sin\theta + \mu m_l g \cos\theta = m_B a_2 + m_B g$$
$$m_l \boldsymbol{a_1} - m_B(-a_1) = -m_l g \sin\theta + m_B g - \mu m_l g \cos\theta$$
$$38a_1 + 48a_1 = -38(9.8)(\sin 25) + 48(9.8) - 0.2(38)(9.8)(\cos 25)$$
$$86a_1 + 67.5018 = -157.383 + 470.4$$
$$86a_1 = 245.51516$$
$$\underline{a_1 = 2.854827 \frac{m}{s^2}}$$

Use found acceleration to find what the velocity at point B will be.

$$v_B^2 = \cancel{v_A^2} + 2\boldsymbol{a_1} L$$
$$v_B^2 = 2(\mathbf{2.854827})(9.1)$$
$$v_B^2 = 51.957859 \text{ , pos makes sense here}$$
$$\underline{v_B = 7.20818 \frac{m}{s}}$$

Set velocity equations in x and y direction up for stage BC.

$$v_{Bx} = v_B \cos\theta$$
$$v_{By} = v_B \sin\theta$$

Stage BC(y-dir)

The luge and Larry start at a height of 9.1m and when it reaches the vertex, the velocity in the y-dir will be 0. With this information, the time it takes to reach the vertex can be found as we also assume that accerlation due to gravity is $-9.8\frac{m}{s^2}$.

$$v_v = gt_v + \boldsymbol{v_B}\sin\theta$$
$$0 = -9.8(t_v) + \mathbf{7.20818} * \sin 25$$
$$3.0463 = 9.8(t_v)$$
$$\underline{t_v = 0.3108478s}$$

Since parabolas are symmetric, he will reach his starting height of 9.1m from the ground in another t_v seconds. At that point, he will also have a negative v_{By} at that point as he would be falling. From there, we can find the time it takes for him to fall that distance of 9.1m and reach point C.

$$\Delta h = \frac{1}{2}gt_f^2 + (-\boldsymbol{v_B}\sin\theta)$$
$$-9.1 = \frac{1}{2}(-9.8)t_f^2 - \mathbf{7.20818} * \sin 25 * t_f^2$$
$$0 = -4.9t_f^2 - 3.0463t_f^2 + 9.1, \text{ solver}$$

$t_f = -1.7086197$ *or* 1.0869258, will use positive in this case. Negative = to total time but opposite sign. Not sure why.

$$\underline{t_f = 1.0869258s}$$

Now, his total time in the air is his time spent falling 9.1m 2* the time it takes him to reach the vertex, as the time it takes him to reach his starting height of 9.1m is the same as the time it takes him to reach the vertex.

$$t_{tot} = \boldsymbol{t_f} + 2\boldsymbol{t_v}$$
$$t_{tot} = \mathbf{1.0869258} + 2 * \mathbf{0.3108478}$$
$$\underline{t_{tot} = 1.7086214\text{s}}$$

Now we can use the time he spends falling after the vertex at a height of 9.1m (not his total time)to find his velocity in the y-dir at point C. We know his velocity at this point because parabolas are symmetrical

$$v_{Cy} = g\boldsymbol{t_f} + (-\boldsymbol{v_B}\sin\theta)$$
$$v_{Cy} = -9.8 * \mathbf{1.0869258} - \mathbf{7.20818} * \sin 25$$
$$\underline{v_{Cy} = -13.698181\frac{m}{s}}$$

Stage BC(x-dir)

Now that we have the time it takes him to reach point C, we can use that time to find how far he travels while he is in the air.

$$\Delta x_{BC} = \frac{1}{2}(v_{Bx} + v_{Cx})\boldsymbol{t_{tot}}$$
$$\Delta x_{BC} = \boldsymbol{v_B}\cos\theta\ \boldsymbol{t_{tot}}$$
$$\Delta x_{BC} = \mathbf{7.20818} * \cos 25 * \mathbf{1.7086214}$$
$$\underline{\Delta x_{BC} = 11.162133m}$$

Since we assume no air resistance, his velocity in the x-direction at point B is equal to that at point C.

$$v_{Cx} = \boldsymbol{v_B}\cos\theta$$

Using this in conjunction witht the previously found velocity at C in the y-direction yields the net veloicty at point C which is then transferred into horizontal velocity in stage CD.

$$v_{Cy}^2 + (v_B cos\theta)^2 = v_{Ctot}^2$$
$$\underline{v_{Ctot} = 15.176233\frac{m}{s}}$$

Stage CD

Using the velocity at point C as the initial velocity, the given information that Larry travels 50m from BD, and the calculated distance he travels in the air from BC, we can find his acceleration during stage CD.

$$\cancel{v_D^2} = \boldsymbol{v_{Ctot}^2} + 2a_{CD}(\Delta x_{BD} - \Delta\boldsymbol{x_{BC}})$$
$$0 = \mathbf{15.176233}^2 + 2a_{CD}(50 - \mathbf{11.162133})$$
$$-230.311 = 77.675734a_{CD}$$
$$\underline{a_{CD} = -2.9651221\frac{m}{s^2}}$$

Since Larry is on a horizontal surface with no action forces affecting his normal force, the force of friction is simply equal to $\mu_G F_{g1}$or:

$$\underline{F_{fG} = \mu_G m_l g}$$

Since friction is the only force acting on him in the horizontal direction, the equation to find what μ_G is equal to relies on having calcuated the accerlation for stage CD.

$$\sum F_{xG} : -\boldsymbol{F_{fG}} = m_l a_{CD}$$
$$-\boldsymbol{\mu_G m_l g} = m_l\boldsymbol{a_{CD}}$$
$$-\mu_G * 38 * 9.8 = 38 * (-\mathbf{2.9651221})$$
$$\mu_G * 9.8 = \mathbf{2.9651221}$$
$$\boxed{\mu_G = 0.3026}$$