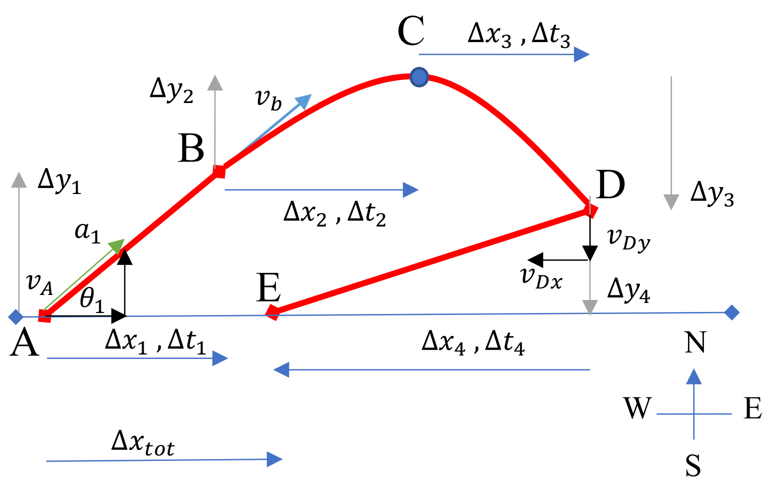
One breezy afternoon Algebra Alex decides to launch Hamster Huey into the air using a model rocket. The rocket is launched over level ground, from rest, at a specified angle above the East horizontal. The rocket engine is designed to burn for specified time while producing a constant net acceleration for the rocket. Assume the rocket travels in a straight-line path while the engine burns. After the engine stops the rocket continues in projectile motion. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens the rocket instantly changes speed and descends at a constant vertical speed. A horizontal wind blows the rocket, with parachute, from the East to West at the constant speed of the wind. Assume the wind affects the rocket only during the parachute stage.

<u>Question: What is the x-displacement of where the rocket lands from the initial x-position?</u>



Givens

$a_1 = 5.8\frac{m}{s^2}$ at	$v_{Ay} = 0\frac{m}{s}$
$\theta = 57°\ N\ of\ E$	$v_{Ax} = 0\frac{m}{s}$
$\Delta t_1 = 8.9s$	$v_{Cy} = 0\frac{m}{s}$
$\Delta y_3 = 71m$ S	$Y_E = 0m$
$v_{Dy} = 11\frac{m}{s}$ S	$x_A = 0m$
$v_{Dx} = 15\frac{m}{s}\ W$	$a_g = 9.8\frac{m}{s^2}\ S$
$v_{4x} = v_{Dx}$	$a_{2x} = 0\frac{m}{s^2}$

Overall strategy: Track the horizontal and vertical displacement of the rocket from sections A to B, B to C, C to D, and D to E. The final horizontal displacement equals the sum of the horizontal displacements of the 4 sections.

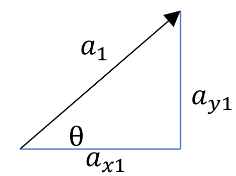
Important Info:

- Stage AB = Stage 1
- Stage BC = Stage 2
- Stage CD = Stage 3
- Stage DE = Stage 4
- Strike through means term is equal to 0
- Underlining an equation underlines the whole thing, so sub answers are also bolded for emphasis.
- South and West are negative

Stage AB(1)

Strategy for part 1.

a. Find the constant acceleration in x and y directions
b. Find final velocities in x and y directions
c. Use velocities to find displacements



x-dir	y-dir
a. $a_{x1} = \cos(57) * 5.8$	$a_{y1} = \sin(57) * 5.8$
b. $v_{Bx} = a_{x1} * \Delta t_1 + \cancel{v_{Ax}}$	$v_{By} = a_{y1} * \Delta t_1 + \cancel{v_{Ay}}$
$v_{Bx} = \cos(57) * 5.8 * 8.9$ $\underline{v_{Bx} = \mathbf{28.1143}\frac{\mathbf{m}}{\mathbf{s}}}$	$v_{By} = \sin(57) * 5.8 * 8.9$ $\underline{v_{By} = \mathbf{43.2922}\frac{\mathbf{m}}{\mathbf{s}}}$
c. $\Delta x_1 = \frac{1}{2}(v_{Bx} + \cancel{v_{Ax}}) * \Delta t_1$	$\Delta y_1 = \frac{1}{2}(v_{By} + \cancel{v_{Ay}}) * \Delta t_1$
$\Delta x_1 = \frac{1}{2}(\mathbf{28.1143}) * 8.9$	$\Delta y_1 = \frac{1}{2}(\mathbf{43.2922}) * 8.9$
$\underline{\Delta x_1 = \mathbf{125.108}\boldsymbol{m\ E}}$	$\underline{\Delta y_1 = \mathbf{192.650}\boldsymbol{m\ N}}$

Stage BC(2)

a. Use the fact that velocity in the y-dir at the apex(C) is 0 and other known values to find displacement in y-dir
b. Use the displacement in the y-dir to find time
c. Substitute in the value for time to find displacement in x-dir

Part a (y-dir)

$$\cancel{v_{Cy}^2} = v_{By}^2 + 2a_g \Delta y_2$$
$$0 = \mathbf{43.2922^2} + 2(-9.8)\Delta y_2$$
$$-1874.21 = -19.6\Delta y_2$$
$$\underline{\Delta y_2 = \mathbf{95.6232\,m\,N}}$$

Part b (y-dir)

$$\Delta y_2 = \frac{1}{2}(\cancel{v_{Cy}} + v_{By})\Delta t_2$$
$$\mathbf{95.6232} = \frac{1}{2}(\mathbf{43.2922})\Delta t_2$$
$$\Delta t_2 = 2 * \frac{\mathbf{95.6232}}{\mathbf{43.2922}}$$
$$\underline{\Delta t_2 = \mathbf{4.41757s}}$$

Part c (x-dir)

$$\Delta x_2 = \cancel{\frac{1}{2} a_{2x} t_2^2} + v_{Bx} * \Delta t_2$$
$$\Delta x_2 = \mathbf{28.1143} * \mathbf{4.41757}$$
$$\underline{\Delta x_2 = \mathbf{124.197\,m\,E}}$$

Stage CD(3)

a. Use the fact that velocity in the y-dir at the apex(C) is 0 and other known values to find velocity in y-dir at D
b. Use the previously found velocity to calculate time for part 3
c. Use the fact that no air resistance means that velocity in x-dir does not change:

$$\underline{v_{Bx} = v_{Cx} = \mathbf{28.1143\,\frac{m}{s}}}$$

d. Use time to find horizontal displacement

Part a (y-dir)

$$v_{Dy}^2 = \cancel{v_{Cy}^2} + 2a_g \Delta y_3$$
$$v_{Dy}^2 = 2(-9.8)(-71)$$
$$v_{Dy}^2 = 1391.6$$
$$v_{Dy} = \pm 37.3042\,\frac{m}{s}\text{, value is negative, as the rocket is falling}$$
$$\underline{v_{Dy} = \mathbf{37.3042\,\frac{m}{s}\ S}}$$

Part b (y-dir)

$$\Delta y_3 = \frac{1}{2}(v_{Dy} + \cancel{v_{Cy}})\Delta t_3$$
$$-71 = \frac{1}{2}(\mathbf{-37.3042})\Delta t_3$$
$$\frac{-142}{(\mathbf{-37.3042})} = \Delta t_3$$
$$\underline{\Delta t_3 = \mathbf{3.80654s}}$$

Part c/d (x-dir)

$$x_D = \cancel{\frac{1}{2} a_{3x} t_3^2} + v_{Cx} * \Delta t_2 + x_C$$
$$\Delta x_3 = v_{Cx} * \Delta t_2$$
$$\Delta x_3 = \mathbf{28.1143} * \mathbf{3.80654}$$
$$\underline{\Delta x_3 = \mathbf{107.018\,m\,E}}$$

Stage DE(4)

a. Use the fact that the rocket is at y = 0 at point E and the previously found displacements in y-dir to find y at point D
b. Use given velocity to find time for stage 4
c. Use time to find horizontal displacement

Part a (y-dir)

$$y_D = \cancel{y_A} + \Delta y_1 + \Delta y_2 + \Delta y_3$$
$$y_D = \mathbf{192.650} + \mathbf{95.6232} - 71$$
$$\underline{y_D = \mathbf{217.273m}}$$

Part b (y-dir)

$$v_{4y} = v_{Dy}$$
$$\underline{v_{4y} = \mathbf{-11\,\frac{m}{s}}}$$
$$v_{4y} = \frac{\cancel{y_E} - y_D}{\Delta t}$$
$$-11 = \frac{\mathbf{-217.273}}{\Delta t}$$
$$\underline{\Delta t = \mathbf{19.7521s}}$$

Part c (x-dir)

$$\underline{v_{4x} = \mathbf{-15\,\frac{m}{s}}}$$
$$v_{4x} = \frac{\Delta x_4}{\Delta t}$$
$$-15 = \frac{\Delta x}{\mathbf{19.7521}}$$
$$\underline{\Delta x_4 = \mathbf{296.282\,m\,W}}$$

Final Answer

Sum together all the previously found horizontal displacments

$$\Delta x_{tot} = \cancel{x_A} + \Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4$$
$$\Delta x_{tot} = \mathbf{125.108 + 124.197 + 107.018 - 296.282}$$
$$\boxed{\Delta x_{tot} = 60.04m\ E\ of\ x_A}$$