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Group Name: MOAT GOATS

1. **Restate**

Rapunzel is locked away in a tower that has a moat surrounding it. The square-shaped moat surrounding the tower is 25 feet in width (from bank to castle). Flynn Rider needs to find a way to use two beams of wood that are 24 feet in length and 9 inches in width without any tools to get across the moat and save Rapunzel.

2. **Attempts**

First, we came up with a variety of hypothetical situations to better understand the problem. We questioned if Flynn could rip off pieces of his shirt and tie them together before tying the two wooden pieces together. This didn't work because the pieces would have still had a weak point in them, making them unstable. Also, with no tools, there would be no way to safely tie the pieces together (no holes can be made). In another attempt, we hypothesized that if Flynn could use the Japanese technique of kigumi or kumiki, the process of interlocking wood without nails or fasteners. This did not work as Flynn did not have any tools to cut the wood with, making him not able to cut the wood in a way that would have them interlock and support each other. Next, we questioned if Flynn could make small cuts in the wood to make them similar to velcro, as they would "lock together". This did not work as Flynn had no tools to do this with, and this would have given Flynn splinters, preventing him from saving Rapunzel. Then, we thought that if Flynn could just take one plank and set it over the moat, leaving the last inch to jump over. This did not work as the plank would have nothing keeping it secured to the ground,

and along with Flynn's weight, he would have fallen into the moat before he could jump the moat. Finally, we reflected that if Flynn could throw over a plank and, with another person, position the planks similar to a simple card tower and walk across. This did not work as first, Flynn would have no help on the other side of the moat and second, any weight on either plank would have caused the planks to fall over, so Flynn would not be able to pass.

3. **Solution**

Rapunzel's recommendation is to position the first 24-foot plank diagonally across a corner of the moat, then to use the second 24-foot plank as a perpendicular bisector from the first's midpoint, overlapping it by 2 inches. This creates a span of 35 $\frac{5}{8}$ feet, which is long enough to cross the $25\sqrt{2}$ foot diagonal distance of the moat.

Plank Placement

1. First, we position the initial plank to create a right isosceles triangle (45º - 45º - 90º) with the corner of the moat (specifically, the corner opposite the land inside the moat). The hypotenuse of this triangle, which is the full length of the plank, is 24 feet long.
2. From the midpoint of this first plank, we can then place the second plank. This second plank is a perpendicular bisector of the first. When we position this plank, we push it forward so that 2 inches of its length rest on the previous plank, with the remainder extending onto the grass beyond the moat.

Moat Span

How can we ensure that this configuration will reach the other end of the moat?

3. Since it is a rectangular moat, we know that any corner forms a 90º angle. Additionally, by drawing construction lines from the corner of the ground so that they are perpendicular to the edges of the moat, we create a square with a side length of 25 feet. From this, we can determine the distance between the corner of the ground and the corner of the moat. Because these lines are perpendicular and define three points, they form a right triangle. With each side length being 25 feet, we can apply the Pythagorean Theorem to show that the diagonal length is $25\sqrt{2}$ feet.

Our primary objective then becomes to demonstrate that our plank configuration covers a span greater than this $25\sqrt{2}$ ft.

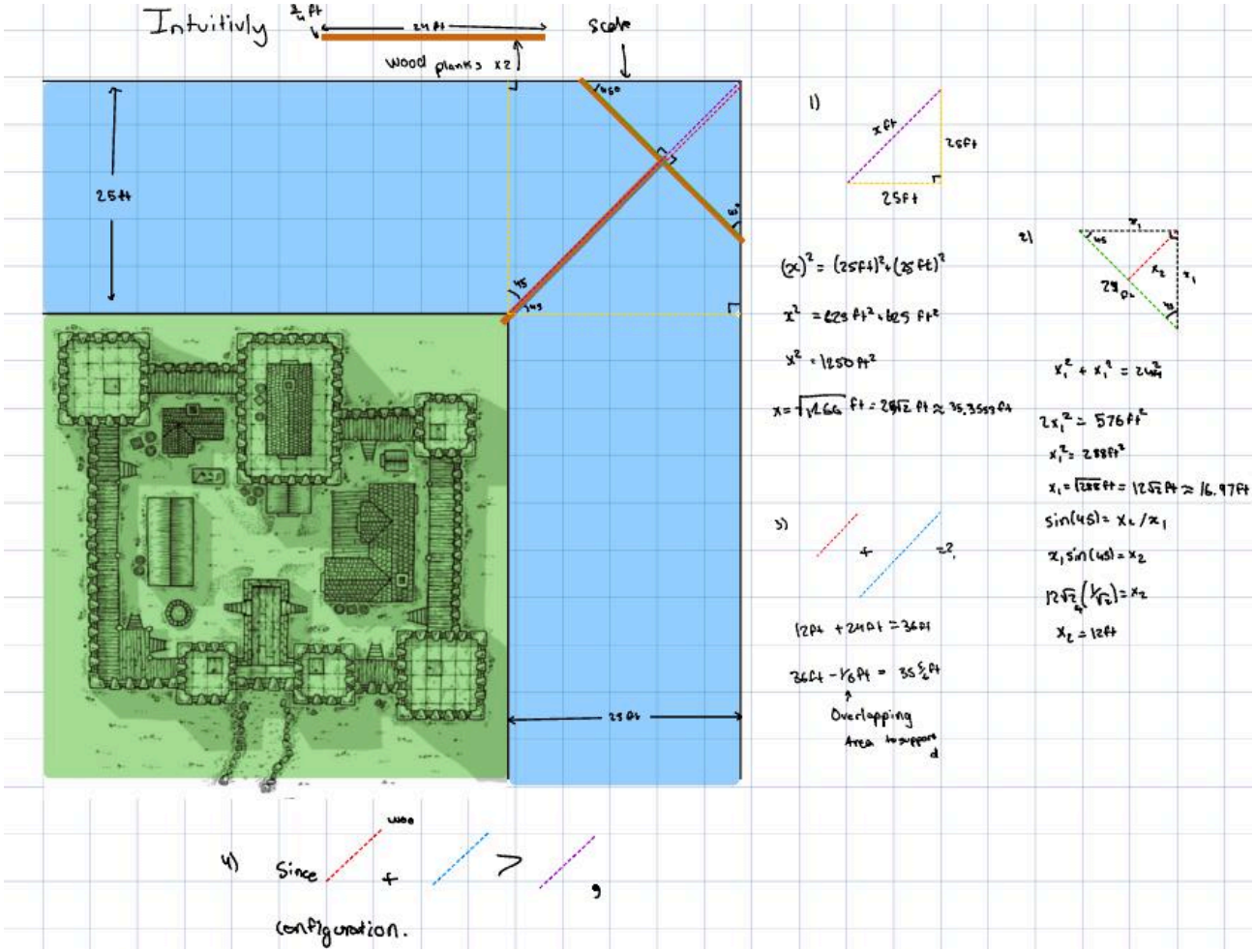
<u>Total Span</u>

4. The first step in calculating the total span is to analyze the length from the midpoint of the first plank to the corner. Since the corner is 90º and we placed the first plank as the hypotenuse of an isosceles right triangle with a length of 24 feet, the lengths of the legs of this triangle (from the corner to each end of the plank) were $24/\sqrt{2}$ feet, which simplifies to $12\sqrt{2}$ feet. From here, if we draw an altitude from the corner that forms a 90° angle with the first plank, this altitude divides the initial triangle into two smaller 45º - 45º - 90º triangles. As a result, using the ratio between side lengths and hypotenuse in an isosceles right triangle, we can determine the distance from the corner to the midpoint of the first plank, which is 12 feet.

5. Next, we calculate the total span. If we now add the length of the second 24-foot plank and subtract the 2 inches (1/6 foot) that rest on the previous plank (allowing for overlap and stability), the calculation becomes: 12 + 24 - ⅙ = 35 ⅚ feet.

Conclusion

Since 35 ⅚ feet (approximately 35.83 feet) is greater than $25\sqrt{2}$ feet (which is approximately 35.36 feet), this plank configuration would be effective enough to successfully cross the moat.



4. Extension

There are many directions that this problem can be expanded towards, changing the shape of the wood, terrain, or by altering a key mechanic of the problem. One of these variations could be seeing the effects if we elevated the land inside the moat, thus involving extending the problem to three dimensions and adding another potential aspect of improvement. Another way to warp the terrain is to change the shape of the moat from the square into a taurus and see the impact on the strategies that could happen. Changing the shape of the wood would also be an interesting idea because if they were trapezoids, they might have different geometries with how they interact with corners. A more farfetched idea could be to try and make this a minimization problem where Flynn has to take as much wood as he can into the castle to power a fire or something like that. This would add a unique twist to the problem and might cause more innovative solutions to form.