# Happy Birthday POW

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**Problem Statement:**

Our problem was to find out the day of the week that someone was born on using their birthdate. The only resources we could use was a calendar of October 2025 (the current month), some basic information about calendars, and a four-function calculator (+, -, *, /).

**Process:**

We started off the process by understanding what layers there are to this problem. We realized there are a few parts to this problem that make it a bit more complicated. There is the fact that the day of the week of a date from year to year differs, along with the fact that the day of the week from month to month differs.

We also wanted to make note of any given data that would be helpful in creating our final formula. As a part of the assignment, we were given a calendar of October and the birthdays of famous people. Another piece of data that will prove central to our procedure was that June 1st, 2025 was a Sunday, which a group member happened to know.

We decided we would first figure out how the day of the week changed from one year to the next before considering months in the equation.

Realizing that the same days of the week per month are just 7 more or less than each other, we figured out that adding a week to a day must maintain the day of the week. So for instance, if the day of the week on June 1st, 2025 is Sunday, we know that every day that is simply 7 days, a week, before or after it should be a Sunday, like June 8th. Knowing this, we can take the amount of days in a year and mod it by 7. This gives us 1, meaning from year to year, the day of the week for any given date differs by one. Meaning, that on any date, next year it should be the next day, like if June 1st, 2025 is a Sunday, June 1st, 2026 should be a Monday.

Now, we haven't actually found our full solution to how days change from year to year yet. This is because every 4th year is a leap year, meaning February has one extra day (compensating for subtracting 0.25 days from each year till then). This is actually a very easy thing to consider, as we just follow the same process, taking mod 7 of 366 (the amount of days in a leap year), giving us 2. This means that any date will shift 2 days of the week after a leap year rather than just 1. Now there are more rules for leap years, specifically that every 100 years is not a leap year unless it is a multiple of 400, but we don't actually need to worry about this. This is because our formula only needs to work up until 1900, making the only multiple of 100 within range 2000, which is also a multiple of 400, thus being a leap year and not important to consider when using our formula.

We decided to abstract away this part of the formula for ease in integrating into a formula. We did this by seeing how many days the date moved forward for a given interval. We chose to make the interval over 4 years to include a single leap year. Since a normal year shifts the date 1 day and a leap year 2 days, these 4 year intervals will shift the date by a total of 5 years. So for every 4 years, the day of the week will shift by 5 days. So if October 8th, 2025 is a Wednesday, October 8th, 2029 will be a Monday.

With our method to find how a date changed depending on change in solely the year, we now need to find a method for switching months. As of this point, we know how the day of the week for a date changes from year to year. This means that if we have the day of the week for any given date, we can very easily find what the day of the week will be in any year. For switching years, we simply take mod 7 of the days we are going forward, which is either 365 or 366 for a year. But since our method for changing years is just finding the day after a given amount of weeks and a remainder of some days, we realized that we could also employ this system for the months.

If we count the number of days from one date to any other date, we can undergo the same procedure we did for calculating the years portion to find the day of the week after any given amount of months, so long as we have the day of the week for a date. That process is simply to find the number of days in between the dates and then mod it by 7. So if we were going from June 1st, 2025, which is a Sunday, to October 9th, 2025, we can count the days and take mod 7 to see how many days forward October 9th will be from Sunday. Putting this into practice, we figure there are 130 days between June 1st and October 9th: 29 in the remainder of June, 31 in July, 31 in August, 30 in September, and another 9 in October. Taking mod 7 gives us 4, and 4 days after Monday is Thursday, and checking the calendar of October we were given, October 9th is indeed a Thursday.

We came to realize our system for dates could be a bit cumbersome, so we decided to make them fully numerical, which will also later help when finding a formula for our discoveries. The key we assigned was:

Sunday = 0
Monday = 1
Tuesday = 2
Wednesday = 3
Thursday = 4
Friday = 5
Saturday = 6

Now that we have found how to go from date to date regardless of month and year, we can now combine the parts and test it to come to a formula. At this moment, we have how to find the day of the week switching date in terms of month and day, along with separately having how to switch years. Our year process depends on the fact that the dates are the same between years, so we knew our month process should come first. This tells us our full process should simply be to find the day of the week of the date in 2025, before changing the year. We decided to use Billie Eilish's birthdate as a starting place (12/18/2001).

Going off the basis that 6/1/2025 was on a Sunday, we can easily find the day Billie's birthday will be in 2025. We follow the same process as we did earlier, finding the amount of days in each month to the given date. So we get a total of 200 days: 29 for the remainder of June, 31 for July, 31 for August, 30 for September, 31 for October, 30 for November, and 18 more of December to get us to Billie's birthday. Modding this number by 7, we get 4, meaning Billie Eilish's birthday in 2025 will be 4 days after Sunday, or a Thursday, seen in our key. Now we find the number of years between Billie's birth year and 2025, 2001 - 2025, which gives us -24. Now multiplying this number by our 5 days / 4 years cycle, we get the number of days that the date will move forward. Then, mod by 7 to get a difference of 2 days of the week. In this case we actually got pretty lucky since -24 is evenly divisible by 4, but not all numbers are, which will lead to decimals. Having decimal days for our discrete calendar doesn't make sense, so in the case we do get decimals, we will round the number down. Since we were finding a birthday in the past, we need to subtract this number from the 2025 equivalent, Thursday, or 4, to get 4 - 2, or 2. This means that Billie Eilish was born on a Tuesday, which indeed, is the correct answer.

We had a few disputes over creating our full process. We tried and tested 2 other procedures before finally deciding to use this one due to conciseness and simplicity. Each method attempted to address a problem that may be exploited in the current one. One such method we were developing attempted to replace the 5 day / 4 year cycle with a per year addition of days. This method did work, however we chose not to use it because we thought this method would be more concise when it came to our solution. The second method we were developing attempted to get rid of the rounding in the current one. We thought that the rounding may lead to edge cases where our algorithms wouldn't work, but the added complexity of the alternative solution deterred us from finishing it. Later we found an exception to our solution and found a way to make those values work. We decided to define that exception as a special case which we will address more specifically in the solution.

Now from this process, we can flesh out a formula. The steps that we have been following are as listed, in order:

1. Find n days between the date and June 1st.
2. Finding the day of the week the day rests on.
3. Find m years between the date and 2025.

4. Find how many days should be taken away from to the day of the week in 2025 and apply mod 7 to find the amount of days (less than 1 week)
5. Round
6. Find the sum of the original day of the week and the offset number of days.

**Solutions**:

This concludes our procedures and leads into our final, written and proved solution. Our formula is based on the fact that in 365 days, the day of the week will move forward by one day every year and two days if it were a leap year. In every 4 years, the day of the week would move forward 5 days. By calculating the day of the week that the date is on in 2025, we can move forward by the amount of years to calculate the day of the week that the date was on in a certain year.

These steps clearly lay out our final formula which we can now begin to construct step by step:

1. Let days between date mm/dd and 06/01 (known Sunday) = d
2. $y = d \bmod 7$
3. Let end year = c
    a. 2025 - c
4. $(c - 2025) * 5 / 4$
5. $(c - 2025) * 5 / 4) \bmod 7$
6. $x = \lfloor((c - 2025) * 5 / 4) \bmod 7 - 0.1\rfloor$
7. $f = (x + y) \bmod 7$

where f is the final day of the week, according to our key listed earlier in the paper.

From this we get our 3 main steps/formulas:

1. $y = d \bmod 7$
2. $x = \lfloor((c - 2025) * 5 / 4) \bmod 7\rceil$
3. $f = (x + y) \bmod 7$

Following the formulas in this order will give the user the day of the week of any date, so long as they find d, the number of days between the picked date and July 1st.

While this formula can be turned into one giant equation, we elected to keep it in 3 steps just to enhance usability of it.

Example 1: Thursday, 8/26/1982, the birthday of John Mulaney

1. Number of days between 6/1 and 8/26: 29 + 31 + 26 = 86
2. y = 86 % 7 = 2
3. x = $\lfloor((1982 - 2025) * 5 / 4) \bmod 7\rfloor = 2$
4. (x + y) mod 7 = (2 + 2) mod 7 = 4, which corresponds with Thursday.

**EXCEPTION**

There is an exception to this formula for dates after 2/29 in every leap year (every year number that is a multiple of 4). Because we used the general trend that the day of the week moves 5 days every 4 years, we also need to account for the extra day. Basically, to find the value for those dates, we just need to add 1 day to the last part to count in the leap year date.

Example 2 (Exception Case): Wednesday, 4/4/1928, the birthday of Maya Angelou

1. The number of days between 6/1 and 4/4 (given that they're in the same year) is -31 - 27 = -58 (- (#days between 5/1 and 6/1 + #days between 4/4 and 5/1). The reason why this number is negative is because we move backwards in the calendar).
2. y = -58 % 7 = 5
3. Her birth year is 1928, so c = 1928. x = $\lfloor((1928 - 2025) * 5 / 4) \bmod 7\rfloor = 4$
4. (4 + 5) mod 7 = 2, and 2 + 1 = 3, which corresponds with Wednesday.

**Extensions:**

1. Will our formula work for years in the future?
    a. Considering we made our formula generalized by leap years, it would make logical sense for the formula to also apply in the future years. The process is very similar because the new idea

2. Can we make our formula a generalized formula instead of using a set date?
    a. Right now, our formula depends on June 1, 2025 being a Sunday. This allowed us to set it as a zero. You technically can use the same idea for any date and it would work. We will go through every step and see what changes. Our first step does not change because it is not dependent on the day of the week. The second step would become
    (day of the week initial + (# days between mod 7)) mod 7.
    b. The idea we used used a Sunday, which was set as 0, so the initial day of the week was left out. Step 3 would be a different initial year than 2025 because it would be whatever year that is chosen for the given date. The last step depends on two values that we already changed, so it doesn't need a change.
3. Factor in years that are multiples of 100 but not 400, such as 2100, 2200, etc
    a. We have factors in the 100 and 400 in our equation that can apply a modulus of 100 and 400 to the years, respectively.
4. Certain exceptions to the calendar such as the Gregorian calendar reform
    a. In the 1500s, there was a calendar reform when 10 days were taken out. We have to account for the special calendar reforms including the Gregorian calendar by subtracting 10 days.