Happy Birthday!

Group Name: NDHC

Problem Statement:

Using the calendar for the current month, the list of famous birthdays, and the basic calendar information given, create a way for people to determine what day of the week they were born. This only applies to birthdays from 1900-2099. Develop a step-by-step process so that others can follow to find the day of the week they were born.

Requirements for formula:

Takes in a date (Day, Month, Year) and outputs the day of the week the user was born on and finds the day of the week that day was on.

Process:

We started by defining the givens:

- 7 days in a week
- 365 days in a year, except leap years (every 4 years) which have 366 days
- 1900 was **NOT** a leap year
- September, April, June, November have 30 days
    - All others have 31 days, except February which has 28 (29 on leap years)

Want to break the problem down into two major steps:

- Solve the relation between years
- Solve the relation between each month/day WITHIN each year

The General Idea

Objective: Find a baseline

- What makes a good baseline?
  - A good baseline will work for every year input possible (1900-2099).
  - A good baseline would be the first possible day (not today) meaning it would be 1-1-1900
  - Looking at the differences between day shifts and years:]

Calendar for January 19XX (Just a baseline):

Sun	Mon	Tues	Wed	Thurs	Fri	Sat
	1	2	3	4	5	6

Goes to December 31th of 19XX

Sun	Mon	(NY) Tues	Wed	Thurs	Fri	Sat
	31	1	2	3	4	5

Calendar for January 19XX (+1):

Sun	Mon	Tues	Wed	Thurs	Fri	Sat
		1	2	3	4	5

January 1st, 19XX is a Monday, however, we are using 1-1-1900 as a baseline for now to find a general pattern. We found that as years progress one by one, the day of the week January 1st occurs on moves by 1 to the right each time.

We know that January 1st, 19XX is a Monday only if it is a leap year. Exactly 364 days, or 52 weeks, from January 1st, 19XX is December 30th. After that, you would need to add 2 more days to go to January 1st of the next year, which is +2 days and lands on a Wednesday.

January 1st 19XX

Sun	Mon	Tues	Wed	Thurs	Fri	Sat

	1	2	3	4	5	6

Goes to December 30th of 19XX

Sun	Mon	Tues	(NY) Wed	Thurs	Fri	Sat
	30	31	1	2	3	4

January 1st 19X(X+1) --> 1 year after 19XX

Sun	Mon	Tues	(NY) Wed	Thurs	Fri	Sat
	30	31	1	2	3	4

Using this same methodology, we extrapolated the amount of days moved form 10-6-25, or October 6th, 2025.

October 2025

Sun	Mon	Tues	Wed	Thurs	Fri	Sat
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

After counting the number of weeks until January 1st of next year, we landed on December 29th, 2025, which lands on a Monday. After 3 days, you would land on January 1st, 2026. This means that January 1st 2026 is a Thursday. This would be our single baseline date based on what we know about how the Gregorian calendar works.

Now, we calculated the numbers of years between 1-1-1900 and 1-1-2026, which is 126 years. Therefore, the numbers of days shifted from 2026 to 1900 would be 126 days backward because every year, the day of the week shifts 1 spot backward. However, we would also need to account for leap years, which is calculated by using the floor operation when dividing 126 by 4.

The following calculations are detailed down below:

# of years = 2026 – 1900 = 126 years

126 years = 126 days moved

126 days moved + 31 days moved (126/4 = 31.5 ==> 31 leap years) --> truncate the decimal

= 157 total days moved

Take mod 7 of total days moved to find the day of the week

157 mod 7 = **3 days moved in the calendar week**

3 days from Thursday January 1st, 2026, is Monday, which is when January 1st, 1900, lands on

Therefore, we can conclude that January 1st, 1900, lands on a Monday

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Another conclusion we came to was that since there is 1 leap year for every 4 years, that means that for every 4 years, there would be a shift of 4 days in the calendar week with an added day to account for the leap year.

Therefore, we determined that **for every 4 years, there would be a 5-day shift in the calendar week**

Building the Formula

To recap, we came to the following conclusions:

- January 1st, 1900, occurs on a Monday
- For every 4-year change, the day of the week shifts back by 5 days in the calendar week

We are also assumed that we are given the birth date in the format (Month, Day, Year)

Using these findings, we can determine a general formula, but here is the analysis of what we are given to create the formula:

- Since every 4 years, there is a 5-day shift, that means that after finding the difference in the number of years from 1900 and the birth year of the user, the days shifted will be equal to 5/4 * (birth_year – 1900)
- Since the decimal value doesn't matter, we can apply the floor operation to chop off the decimal
- Since we only care the day number from (0-6) with 0 referring to Monday, we will take the mod 7 of the result of the previous equation

However, we encountered one problem:

Formula doesn't account for leap years

We found out this problem when we found that if Jan 1, 1900, was a Monday, Jan 1, 1904 should be a Friday, not a Saturday.

Due to this repeated problem with more test cases, we noticed a 1-day difference in the model's output and the actual date.

This is where we decided to subtract 1 day from the total amount of days shifted to get the final number of days shifted.

Solution:

Y = year of birth

N = (if date in question is a leap year (year is divisible by 4 and not the year 1900) and in the month of January/February) 1, (all other situations) 0

X = Days moved past January 1st 1900

B = Day of the week of January 1st where Monday = 0, Tuesday = 1, etc.

D = Day of the month of birth

I = Summation of days in previous months, BEFORE the birth month (use table below)

W = Day of the week of birth (in numbers where 0 = Y)

Birthday: (M, D, YYYY)

Month (M)	1	2	3	4	5	6	7	8	9	10	11	12
I =	0	31	59	90	120	151	181	212	243	273	304	334

Explanation:

Step 1:

To develop our solution we decided to use January 1st as our baseline for the day of the week. Each year January 1st moves up one day, and during a leap year it moves up by two. So, when we subtract 1900 from somebody's year of birth, we get the number of days January 1st has moved since 1900 without leap years. Leap years make up roughly ¼ of every year passed, so that number then must be multiplied by 5/4 to account for them. In the case of somebody having a birthday during a leap year before the month of January or February, N must equal one because the model then over accounts for the solution. Last, the entire solution must be floored (when a number is rounded down no matter the remainder) because we cannot have half of a day of the week in this scenario.

Then the number of days past January 1st 1900 (X) must be put into mod 7 (which divides the number by 7, and leaves behind the numerator of the remainder as a number) to get the days past Monday that January 1st of the given year will be on as shown in the chart below.

Birthday: (M, D, YYYY)

$$\lfloor (YYYY-1900) * 5/4 - N \rfloor = X$$

$$X \bmod 7 = B$$

If YYYY = leap year --> N = 1, or else N = 0

B = # of days shifted

Step 2:

Once you have the day of the week of January 1st of a birth year, you can find the day of the week any day is on. To do this take the day of birth (D) and subtract 1 to account for the 1st of the year then you will need to add I. I is the summation (all numbers added up) of how many days were in the months before the birth month. Since every month has a different number of days, this will vary for every month. See the chart above with the variable definitions for a quicker addition. Then B, which we found in Step 1, must be added because the first day of every year is different. Then to get the final answer, all of that must be in mod 7. Finally, W will be the amount of days after Monday. Think of Monday as 0, and whatever number you have is that many days after it. The chart below can also be used to quickly find out what the day was.

$$W = (D - 1 + I + B) \bmod 7$$

W =	0	1	2	3	4	5	6
Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday

Example:

Birthday: 3-31-1978

First formula:

$$\lfloor (YYYY-1900) * 5/4 - N \rfloor = X$$

YYYY=1978

N = 0 (It is not a leap year and it is not January or February)

$$\lfloor (1978-1900) * 5/4 - 0 \rfloor = X$$

$$\lfloor (78) * 5/4 - 0 \rfloor = X$$

$$\lfloor 97.5 \rfloor = X$$

The floored symbol ⌊ ⌋ says that the decimal should just be removed, not rounded.

$$X = 97$$

Next, we plug in X to the following equation to get B

$$X \bmod 7 = B$$

$$97 \bmod 7 = B$$

To use the mod, we need to divide 97 by 7 to get the remainder

$$97 / 7 = 13\ (6/7)$$

$$\text{Remainder: } 6/7 \rightarrow 6$$

$$B = 6$$

Y=6 means that in the end, we will count 6 days from Monday to get the year to line up.
Monday → Tuesday → Wednesday… → Sunday

Sunday means that 1-1-1978 was a Sunday

Now that we have found the change in the years, we need to find to the days and months.

$$D = 31 \text{ (Day of month)}$$

$$I = 59 \text{ (Using chart above)}$$

$$B = 6 \text{ (solved for before)}$$

Then use the formula:

$$W = (D - 1 + I + B) \bmod 7$$

$$W = (31 - 1 + 59 + 6) \bmod 7$$

$$W = (95) \bmod 7$$

$$95/7 = 13\ (4/7)$$

$$\text{Remainder: } 4/7 \rightarrow 4$$

$$W = 4$$

W = 4

Use the chart above to convert W into a day

Day = Friday

Congratulations! You just found out the day of a week of a random date.

Extensions:

Every year that is a multiple of 100 but not 400 does NOT have a leap day. Is there a way to edit the formula to account for any date, after or even before, year 1?

The Gregorian calendar is not the only type of calendar used across the world. Is there a way to adapt the formula or even change it completely to work for other calendars, such as the Chinese Calendar, Hebrew Calendar, or even a Hindu Calendar?

Given a birth date, birth time, and birth location: can you determine the weekday and time of birth in UTC and local time? Consider births around midnight, along with how daylight savings time affects days and times?

Given a class of *n* students with known birth dates: 1. How many pairs share the same weekday of birth? 2. The probability that a randomly chosen pair shares the same weekday. 3. What is the sample size *n* students needed so there is a >50% (or 75%) chance that two students have a birthday on the same weekday? 4. Is there a faster way to calculate which student was born on which day if we know that they were born between just 2 different years (ex. All born 2008-2009), is there a simpler formula?