WILCOXON SIGNED RANK TEST AND MANN WHITNEY TEST

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WHAT DOES THE TEST DO?

- The Wilcoxon Signed Rank test demonstrates whether or not after two measures have been performed, their average difference is 0. Because it uses rankings of differences, data does not need to be evenly distributed (Statistics Solutions 2019).
- Therefore, the test is useful to show that there is a statistically significant difference in the distributions.

WHEN DO YOU USE THIS TEST?

- Use this test when you would like to use a T-test, but your data points are not evenly distributed (McDonald 2014).
  - The T test determines whether or not the means, or averages, of two groups are significantly different, and therefore whether or not a hypothesis stands (Kenton 2019)
  - When there are two groups separated into individual categories (that do not have a range of values), you can use this test (McDonald 2014).
  - You must also have one measurement group, or value with a range of numbers (McDonald 2014).
- This process differs from a T-test in that in the T-test, it is assumed that the mean difference between different groups is 0, whereas the median difference is assumed to be 0 with the Wilcoxon test (McDonald 2014).

HOW DOES THE TEST WORK?

- With your individual ordered measured data points, rank the differences between points progressively from smallest to largest. The points that are the closest together will have a rank of 1, the next closest points will have a rank of 2, and so on. If two values are the same distance apart, give them an average rank. (McDonald 2014).
- These original ranks were based solely on absolute value, but now, based on whether the number is increasing or decreasing, add up all the absolute values of each group to get two W values: both positive, but one representing positive change and one representing negative change (Statistics Solutions 2019)
- Whichever of the two sums is smaller is called W, and is the statistic that will be used to prove or disprove the hypothesis. (McDonald 2014). Under the null hypothesis, it is more likely W will be larger.

HOW DOES THIS TEST WORK? (PART 2)

- For the research hypothesis to be true, the positive representation of W should be greater than the negative representation of W. If the null hypothesis is true, the research is not supported, and the magnitudes of W will be similar (LaMorte 2017).

EXAMPLE FIGURE

Case	X	Y	X-Y	Absolute Y-X	Rank	With tied ranks	Signed ranks
14	6	6	0	0	-	-	-
11	5	6	-1	1	1	1.0	-1.0
2	5	7	-2	2	2	2.0	-2.0
5	5	8	-3	3	3	4.5	-4.5
6	0	3	-3	3	4	4.5	-4.5
7	5	2	3	3	5	4.5	4.5
8	5	2	3	3	6	4.5	4.5
4	5	9	-4	4	7	7.5	-7.5
13	6	10	-4	4	8	7.5	-7.5
16	0	5	-5	5	9	9.0	-9.0
1	1	7	-6	6	10	10.5	-10.5
3	4	10	-6	6	11	10.5	-10.5
18	1	8	-7	7	12	12.5	-12.5
19	2	9	-7	7	13	12.5	-12.5
20	2	10	-8	8	14	14.0	-14.0
10	0	10	-10	10	15	17.0	-17.0
15	0	10	-10	10	16	17.0	-17.0
9	0	10	-10	10	17	17.0	-17.0
12	0	10	-10	10	18	17.0	-17.0
17	0	10	-10	10	19	17.0	-17.0

Here is an example figure.

Children were tested for literacy before and after a new teaching method.

Before and after the method, their literacy was rated on a scale of 0 to 10 (the x and y groups).

(Statistics Solutions 2019).

$$\mu_w = \frac{n(n+1)}{4}, \ \sigma_w = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

- Must reduce the variance by $\frac{t^3-t}{48}$

$$z = \frac{MAX(\mathrm{W}+,\mathrm{W}-) - \frac{n(n+1)}{4}}{\sqrt{(\frac{n(n+1)(2n+1)}{4*6}) - (\frac{4^3-4}{48} + 3\left(\frac{2^3-2}{48}\right) + \frac{5^3-5}{48})}}$$

$$z = \frac{181 - \frac{19(20)}{4}}{\sqrt{(\frac{19(19+1)(2(19)+1)}{4*6}) - (\frac{4^3-4}{48} + 3\left(\frac{2^3-2}{48}\right) + \frac{5^3-5}{48})}}$$

$$z = \frac{181 - 95}{\sqrt{617.5} - 4.125}$$

$$z = 3.47$$

(Statistics Solutions 2019).

MANN WHITNEY U TEST

- In the Mann Whitney U Test, ranks are assigned in a similar way, but are summed for two separate groups where two independent variables are being tested (an example might be a placebo drug tested against an effective drug). However, the same dependent variable is being tested, so the ranks are summed overall. (LaMorte 2017).
- The test statistic in this case is U (LaMorte 2017).
- Theoretically, in the test group, there should be a lower U value since the population will not be as greatly distributed (LaMorte 2017). Having a lower U value for the test group suggests the research hypothesis is correct.

OUR EXAMPLE AND OUR DATA

- Our group took weather data at Buffumville Lake in Charlton, MA from the years 1968 and 2018, and used the Mann Whitney U Test to measure if the max temperatures of each day in each year had the same distribution
- Since the p-value is much larger than 0.05, there is weak evidence to show that the alternative hypothesis is correct and disprove the null hypothesis
- Therefore, we cannot draw a definite conclusion by using the Mann Whitney test

```
stats.mannwhitneyu(tmax68,tmax18)

MannwhitneyuResult(statistic=64761.0, pvalue=0.2578966368172395)
```

INFOGRAPHIC

- https://create.piktochart.com/output/42557624-wilcoxon-signed-rank-test

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Wilcoxon Signed-Rank Test

Use this test when you would like to use a t-test, but your data points are not evenly distributed

Demonstrates whether or not after two measures have been performed, their average difference is 0

In the t-test, it is assumed that the mean difference between different groups is 0, but the median difference is assumed to be 0 with the Wilcoxon test

How to Conduct the Test

- Rank the differences between points from smallest to largest. The points that are the closest together will have a rank of 1, the next closest points will have a rank of 2, and so on.
- Add up all the absolute values of each group to get two W values: both positive, but one representing positive change and one representing negative change
- Whichever of the two sums is smaller is called W, and is the statistic that will be used to prove or disprove the hypothesis.

Case	X	Y	X-Y	Absolute Y-X	Rank	With tied ranks	Signed ranks
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17	0	10	-10	10	19	17.0	-17.0

- For the research hypothesis to be true, the positive representation of W should be greater than the negative representation of W. If the null hypothesis is true, the research is not supported, and the magnitudes of W will be similar

$W+$ = 4.5 + 4.5 = 9

$W-$ = 1+ 2 + 4.5 + 4.5 + 7.5 + 7.5 + 9 + 10.5 + 10.5 + 12.5 + 12.5 + 14 + 17+ 17 + 17 + 17 + 17 = 181

How to Conduct the Wilcoxon Sign Test. (2019). Retrieved from https://www.statisticssolutions.com/how-to-conduct-the-wilcox-sign-test/.

Mann Whitney U Test

Ranks are assigned in a similar way to the Wilcoxon test, but are summed for two separate groups where two independent variables are being tested

- In the test group, there should be a lower U value since the population will not be as greatly distributed
- Having a lower U value for the test group suggests the research hypothesis is correct.

SOURCES

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