Uber Problem: Leaping Larry's Luge Launcher

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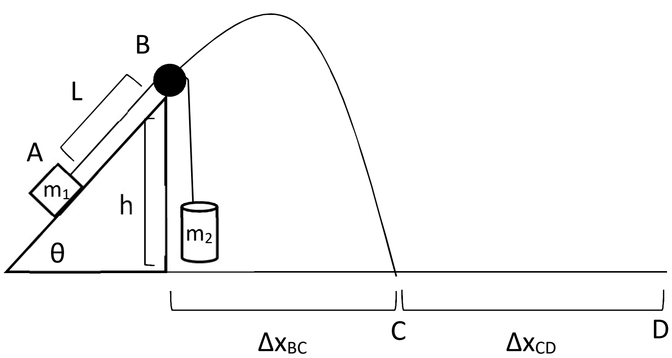
Description:

Leaping Larry decided to make a laborious launcher for his luxury luge using a pulley and ramp system (see diagram). His method was to attach one end of a massless stretchless rope to a barrel of rocks and to hold the other end of the rope. He placed the rope over a massless frictionless pulley, and then walked down the ramp far as down possible to point A (where L = h). When he sat in the luge he accelerated up the ramp to point B and then launched off the top at the same angle as the ramp (all while releasing the rope and avoiding the pulley). He flew through the air as a projectile to point C, transitioning all of his speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any height differences between luge height, barrel height, and size of the pulley, and the diagram is not drawn to scale.

Diagram:

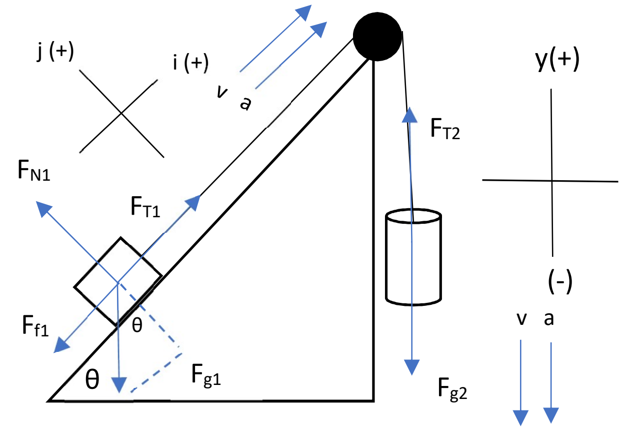


Givens:

Launch Angle θ = 28° m_1 = 37 kg m_2 = 48 kg μ_R = 0.23 Δx_{BD} = 68 m	L = h h = 6.3 m L = 6.3 m Assume: $F_{T1} = F_{T2}$ $a_i = -a_y$

Steps:

I. The Ramp:



Find the sum of forces in the y, j, and i directions, leaving them in variables. Set the equations of the FT1 and FT2 forces equal to one another to find the acceleration, then use kinematics equations to find the velocity of the luge at B.

$$\Sigma F_j: F_{N1} + F_{g1}cos\theta = m_1 a_j$$

$$\underline{F_{N1} = m_1 g cos\theta}$$

$$\Sigma F_i: -F_{f1} + F_{T1} - F_{g1}sin\theta = m_1 a_i$$

$$-\mu F_{N1} + F_{T1} - mgsin\theta = m_1 a_i$$

$$\underline{F_{T1} = \mu(m_1 g cos\theta) + m_1 g sin\theta + m_1 a_i}$$

$$\Sigma F_y: F_{T2} - F_{g2} = m_2 a_y$$

$$\underline{F_{T2} = m_2 g + m_2 a_y}$$

$$\mu(m_1 g cos\theta) + m_1 g sin\theta + m_1 a_i = m_2 g - m_2 a_i$$

$$0.23(37 * 9.8 * cos28) + (37 * 9.8 * sin28) + 37 * a_i = 48 * 9.8 - 48 * a_i$$

$$73.6361 + 170.23 + 37 * a_i = 470.4 - 48 * a_i$$

$$85 * a_i = 226.534$$

$$\underline{a_i = 2.66511\, m/s^2}$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

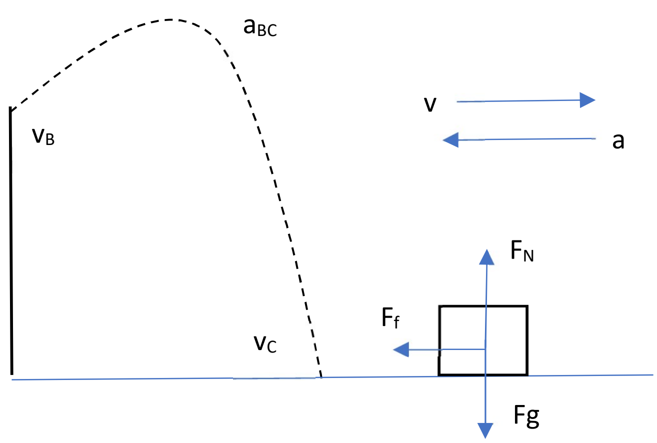
$$v_f^2 = 2 * 2.66511 * 6.3$$

$$v_f^2 = 33.5804$$

$$\underline{v_B = 5.79486\, m/s}$$

II. The Ground

Find the x displacement from B to C , then from C to D. Then, find the time from B until C, find the total velocity right before the luge hits the ground at point C, and convert it all into the x-direction. Find the acceleration necessary for the luge to stop in the distance between C and D using kinematics equations, then use the acceleration to calculate the coefficient of friction from C to D by calculating the sum of forces in the x-direction.



$$y_C = \frac{1}{2}a\Delta t^2 + v_{yB}\Delta t + y_B$$

$$0 = \frac{1}{2}(-9.8)\Delta t^2 + 5.794(sin28)\Delta t + 6.3$$

$$0 = -4.9\Delta t^2 + 5.794(sin28)\Delta t + 6.3$$

$$\underline{\Delta t_{BC} = 1.44499\ s}$$

$$\Delta x_{BC} = \frac{1}{2}a\Delta t^2 + v_i\Delta t$$

$$\Delta x_{BC} = 5.794(cos28)(1.44499)$$

$$\underline{\Delta x_{BC} = 7.39337\ m}$$

$$\Delta x_{CD} = \Delta x_{BD} - \Delta x_{BC}$$

$$\Delta x_{CD} = 68 - 7.39$$

$$\underline{\Delta x_{CD} = 60.6066\ m}$$

$$v_{xC} = v_B cos\theta$$

$$v_{xC} = 5.794cos28$$

$$\underline{v_{xC} = 5.11656\ m/s}$$

$$v_{yC} = a\Delta t + v_B sin\theta$$

$$v_{yC} = 9.8 * 1.44 + 5.794 * sin28$$

$$\underline{v_{yC} = -11.4404\ m/s}$$

$$v_C = \sqrt{v_{xC}^2 + v_{yC}^2}$$

$$v_C = \sqrt{5.116^2 + -11.44^2}$$

$$\underline{v_C = 12.5324\ m/s}$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$0 = 12.53^2 + 2a\Delta * 60.61$$

$$\underline{a = -1.29574\ m/s^2}$$

$$\Sigma F_y: -F_g + F_N = ma_y$$

$$\underline{F_N = mg}$$

$$\Sigma F_x: -F_f = ma$$

$$-\mu(mg) = ma$$

$$-\mu(37 * 9.8) = 37(-1.29)$$

$$-\mu(362.6) = 35.704$$

$$\boxed{\mu = 0.132}$$