Radioactive Contamination

In an article taken from the Journal of Environmental Health, May-June 1965, Volume 27, Number 6, pages 883-897, author Robert Fadely explains that the Atomic Energy Plant in Hanford, Washington has been a plutonium production facility since the Second World War. Some of the waste have been stored underground in the same area. Radioactive waste has been seeping into the Columbia River, and eight Oregon counties and the city of Portland have been exposed to radioactive contamination. The table below lists the number of cancer deaths per 100,000 residents for Portland and these counties. The table also includes an index of exposure that measures the proximity of the residents to the contamination. The index is based on the assumption that city or county exposure is directly proportional to river frontage and inversely proportional both to the distance from Hanford, WA site and to the square of the county's or city's average distance from the river.

Data Table

In[168]:= **index = {2.5, 2.6, 3.4, 1.3, 1.6, 3.8, 11.6, 6.4, 8.3};**

In[169]:= **deaths = {147, 130, 130, 114, 138, 162, 208, 178, 210};**

In[170]:= **lista = Transpose[{index, deaths}];**

In[171]:= **Text[Grid[Prepend[lista, {"index", "deaths"}],**
**   Alignment → Center, Dividers → {2 → True, 2 → True}, Spacings → {1, 1}]]**

Out[171]=

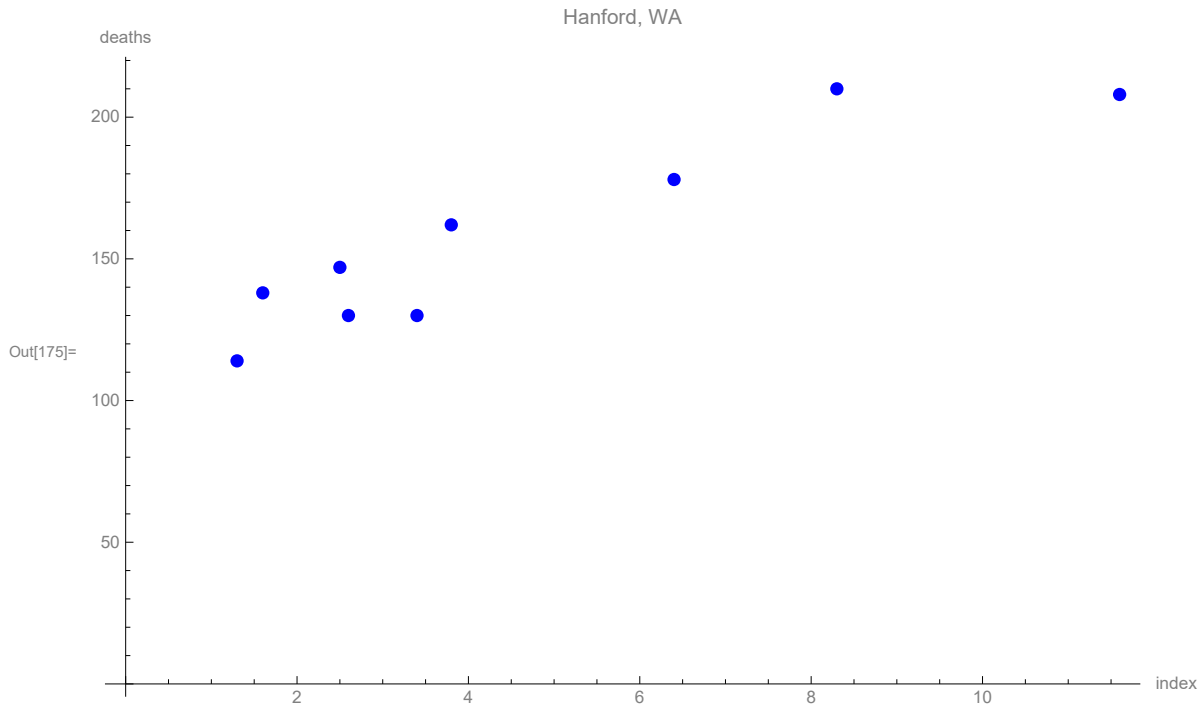| index	deaths
2.5	147
2.6	130
3.4	130
1.3	114
1.6	138
3.8	162
11.6	208
6.4	178
8.3	210

In[172]:= **points = Prepend[{deaths}, index];**

In[173]:= **points1 = Transpose[points];**

In[174]:= **points2 = Sort[points1]**

Out[174]= {{1.3, 114}, {1.6, 138}, {2.5, 147}, {2.6, 130},
    {3.4, 130}, {3.8, 162}, {6.4, 178}, {8.3, 210}, {11.6, 208}}

Data Graph

In[175]:= `plota = ListPlot[lista, AxesLabel → {"index", "deaths"}, ImageSize → Large,`
     `PlotStyle → {Blue, Point}, AxesOrigin → {0, 0}, PlotLabel → "Hanford, WA", LabelStyle → Gray]`

Out[175]=

Two Point Line

In[176]:= $\texttt{tpLine[x\_] :=} \dfrac{\texttt{lista[[9]][[2]] - lista[[4]][[2]]}}{\texttt{lista[[9]][[1]] - lista[[4]][[1]]}} \left(\texttt{x - lista[[9]][[1]]}\right) + \texttt{lista[[9]][[2]]}$
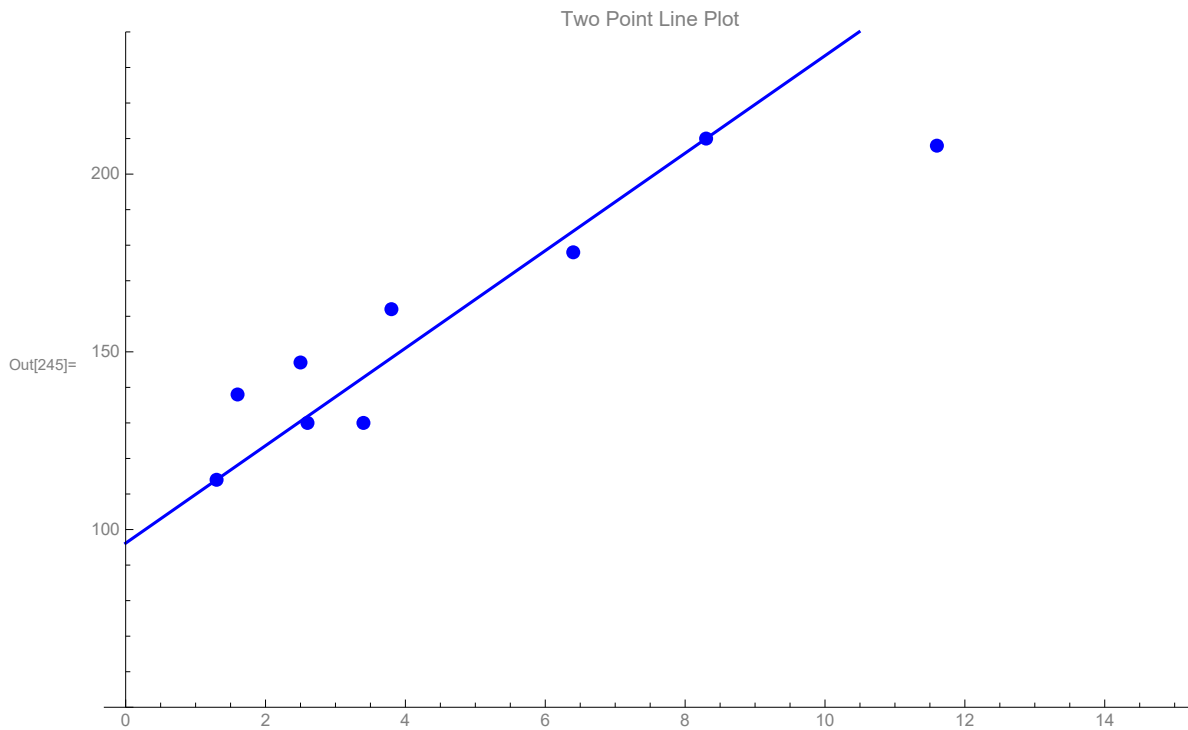
In[177]:= `tpLine[x]`

Out[177]= $210 + 13.7143 \left(-8.3 + x\right)$

In[244]:= `tpPlot = Plot[tpLine[x], {x, 0, 15}, PlotStyle → Blue, PlotRange → {50, 240},`
     `PlotLabel → "Two Point Line Plot", LabelStyle → Gray, ImageSize → Large];`

In[245]:= **Show[tpPlot, plota]**

Two Point Line Plot
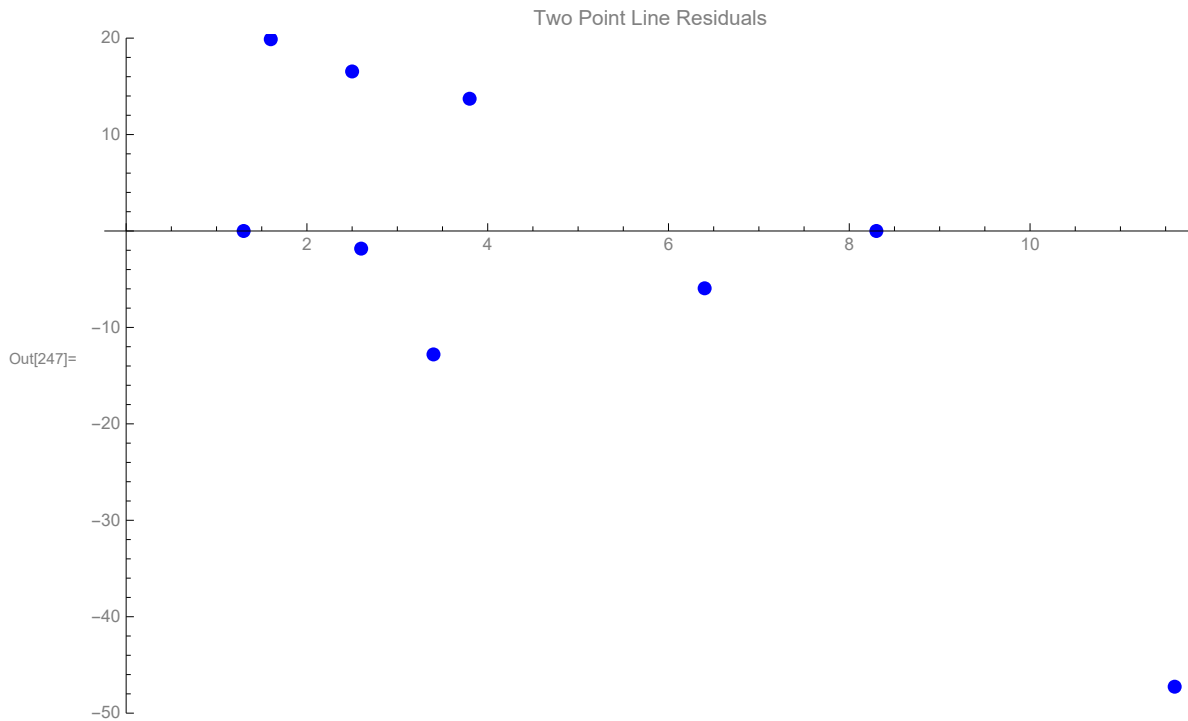
Out[245]=

Two Point Line Residuals Plot

In[180]:= **oneRes = {points2[[1]][[2]] - tpLine[points2[[1]][[1]]],**
 points2[[2]][[2]] - tpLine[points2[[2]][[1]]],
 points2[[3]][[2]] - tpLine[points2[[3]][[1]]],
 points2[[4]][[2]] - tpLine[points2[[4]][[1]]],
 points2[[5]][[2]] - tpLine[points2[[5]][[1]]],
 points2[[6]][[2]] - tpLine[points2[[6]][[1]]],
 points2[[7]][[2]] - tpLine[points2[[7]][[1]]],
 points2[[8]][[2]] - tpLine[points2[[8]][[1]]],
 points2[[9]][[2]] - tpLine[points2[[9]][[1]]]}

Out[180]= {0., 19.8857, 16.5429, -1.82857, -12.8, 13.7143, -5.94286, 0., -47.2571}

In[181]:= **index1 = Sort[index];**

In[182]:= **tplist = Transpose[{index1, oneRes}];**

In[247]:= `tpRes = ListPlot[tplist, PlotStyle → Blue, PlotRange → {-50, 20},`
　　　　`PlotLabel → "Two Point Line Residuals", LabelStyle → Gray, ImageSize → Large]`

Out[247]=

In[184]:= `Total[oneRes]`

Out[184]= -17.6857

Median-Median Line

Summary Points

In[185]:= `sect1 = points2[[{1, 2, 3}]];`

In[186]:= `med1 = Median[sect1]`

Out[186]= {1.6, 138}

In[187]:= `sect2 = points2[[{4, 5, 6}]];`

In[188]:= `med2 = Median[sect2]`

Out[188]= {3.4, 130}

In[189]:= `sect3 = points2[[{7, 8, 9}]];`

In[190]:= `med3 = Median[sect3]`

Out[190]= {8.3, 208}

Median-Median Line

Method 1:

In[191]:= **medLine1[x_] := $\frac{\text{med3}[[2]] - \text{med1}[[2]]}{\text{med3}[[1]] - \text{med1}[[1]]}$ * $\left(x - \text{med1}[[1]]\right)$ + med1[[2]];**

In[192]:= **medLine2[x_] := medLine1[x] - $\frac{1}{3}$ * $\left(\text{medLine1}[\text{med2}[[1]]] - \text{med2}[[2]]\right)$**

In[193]:= **medLine2[x]**

Out[193]= $129.065 + 10.4478 \left(-1.6 + x\right)$

In[194]:= **Simplify$\left[129.06467661691542` + 10.44776119402985` \left(-1.6` + x\right)\right]$**

Out[194]= $112.348 + 10.4478 \, x$

Method 2:

In[195]:= **medLine3[x_] := $\frac{\text{med3}[[2]] - \text{med1}[[2]]}{\text{med3}[[1]] - \text{med1}[[1]]}$ * $\left(x - \text{med2}[[1]]\right)$ + med2[[2]];**

In[196]:= **medLine4[x_] := $\frac{\text{med3}[[2]] - \text{med1}[[2]]}{\text{med3}[[1]] - \text{med1}[[1]]}$ * x + $\frac{1}{3}$ * $\left(2 * \text{medLine1}[0] + \text{medLine3}[0]\right)$;**

In[197]:= **medLine4[x]**
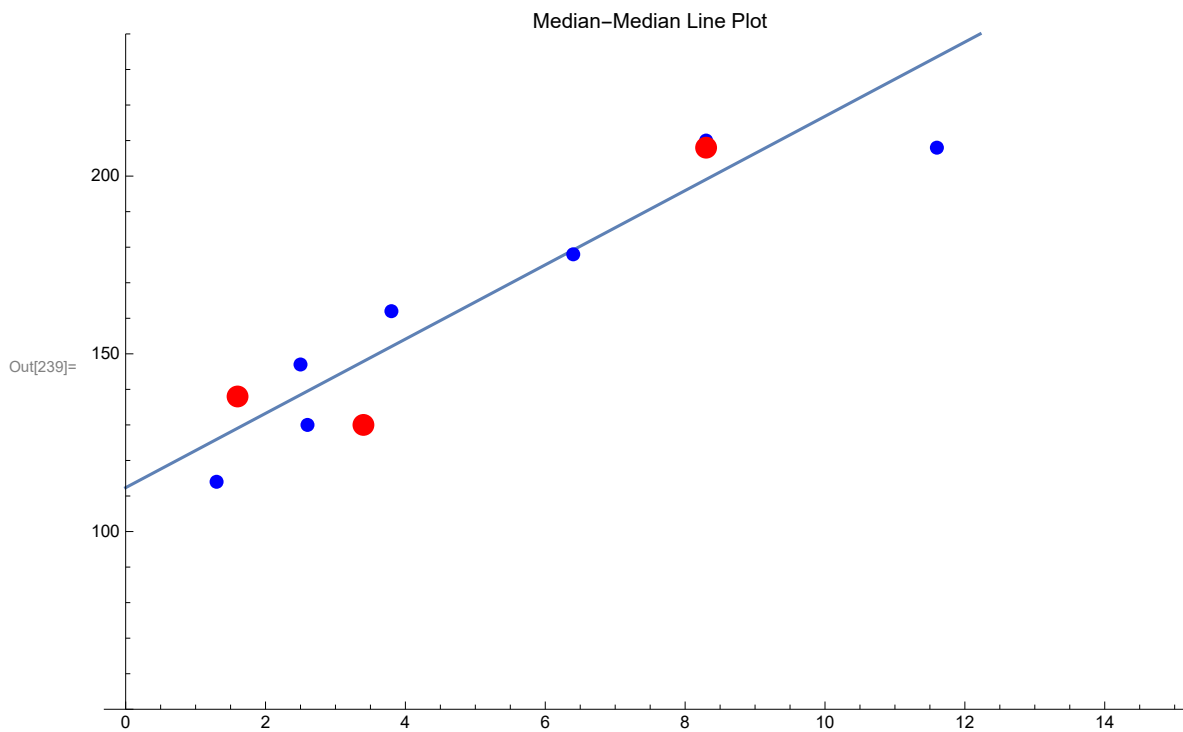
Out[197]= $112.348 + 10.4478 \, x$

Median-Median Graph

```
plotc = ListPlot[{med1, med2, med3}, PlotStyle → {Red, PointSize[0.02]}];
```

In[237]:= **plotd = Plot[112.34825870646766` + 10.44776119402985` x,**
       {x, 0, 15}, PlotRange → {50, 240}, PlotLabel → "Median–Median Line Plot"];

In[239]:= **Show[plotd, plota, plotc]**

Median–Median Line Plot

Out[239]=

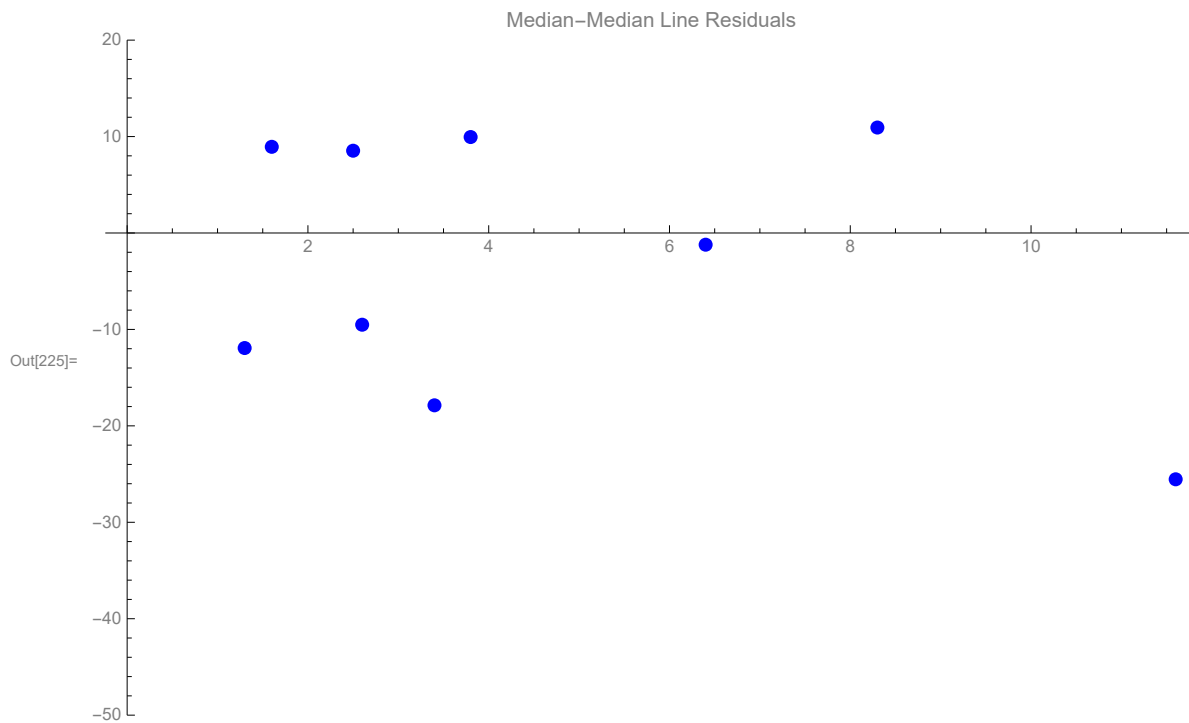## Median-Median Residuals

In[201]:= **twoRes = {points2[[1]][[2]] - medLine2[points2[[1]][[1]]],**
      points2[[2]][[2]] - medLine2[points2[[2]][[1]]],
      points2[[3]][[2]] - medLine2[points2[[3]][[1]]],
      points2[[4]][[2]] - medLine2[points2[[4]][[1]]],
      points2[[5]][[2]] - medLine2[points2[[5]][[1]]],
      points2[[6]][[2]] - medLine2[points2[[6]][[1]]],
      points2[[7]][[2]] - medLine2[points2[[7]][[1]]],
      points2[[8]][[2]] - medLine2[points2[[8]][[1]]],
      points2[[9]][[2]] - medLine2[points2[[9]][[1]]]}

Out[201]= {-11.9303, 8.93532, 8.53234, -9.51244, -17.8706, 9.95025, -1.21393, 10.9353, -25.5423}

In[202]:= **mmlist = Transpose[{index1, twoRes}];**

In[225]:= `mmRes = ListPlot[mmlist, PlotStyle → Blue, PlotRange → {-50, 20},`
     `PlotLabel → "Median-Median Line Residuals", LabelStyle → Gray, ImageSize → Large]`

Out[225]=

In[204]:= `Total[twoRes]`

Out[204]= -27.7164

Sum of Least Squares Line

In[205]:= $b = \sum_{i=1}^{9} index[[i]] * deaths[[i]]$

Out[205]= 7427.1

In[206]:= $c = \sum_{i=1}^{9} deaths[[i]]$

Out[206]= 1417

In[207]:= $d = \sum_{i=1}^{9} index[[i]] * index[[i]]$

Out[207]= 287.67

In[208]:= $e = \sum_{i=1}^{9} index[[i]]$

Out[208]= 41.5

In[209]:= **mls** = $\dfrac{(b * 9) - (c * e)}{(9 * d) - (e * e)}$

Out[209]= 9.27386

In[210]:= **bls** = $\dfrac{(c * d) - (b * e)}{(9 * d) - (e * e)}$

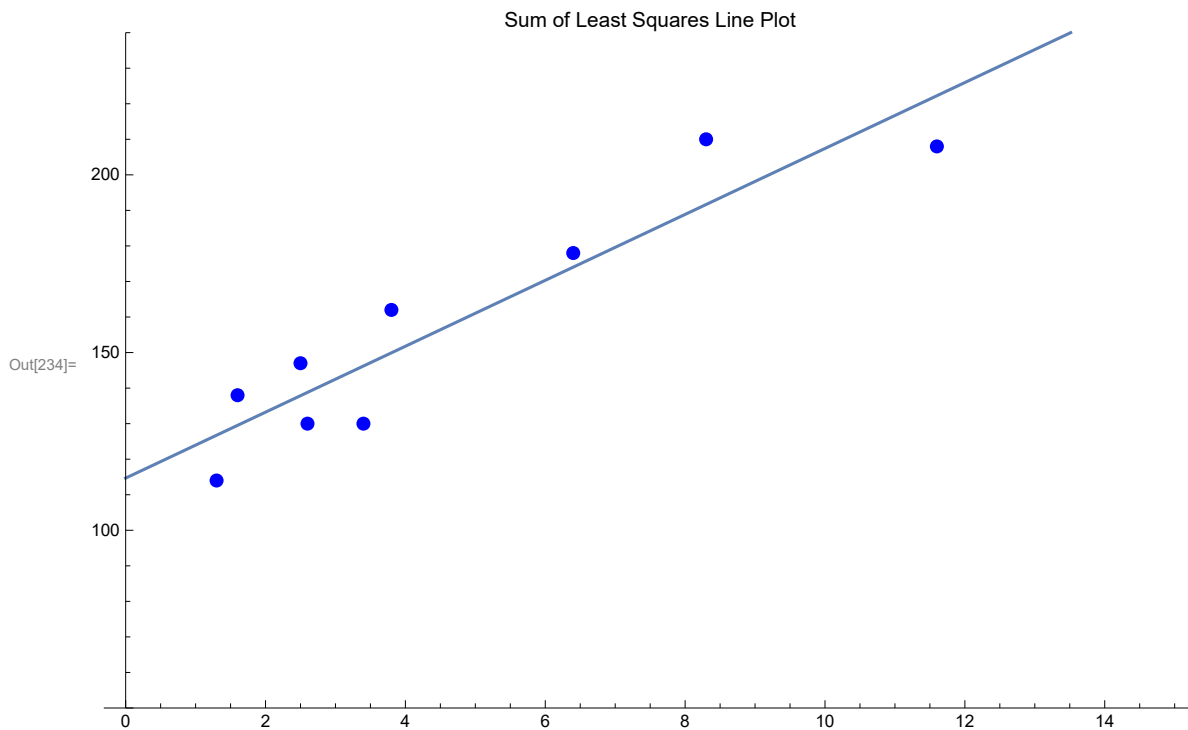Out[210]= 114.682

In[211]:= **lsLine [x_] := mls * x + bls**

In[212]:= **lsLine[x]**

Out[212]= 114.682 + 9.27386 x

Least Squares Plot

In[233]:= **plote = Plot[lsLine[x], {x, 0, 15},**
        PlotRange → {50, 240}, PlotLabel → "Sum of Least Squares Line Plot"];

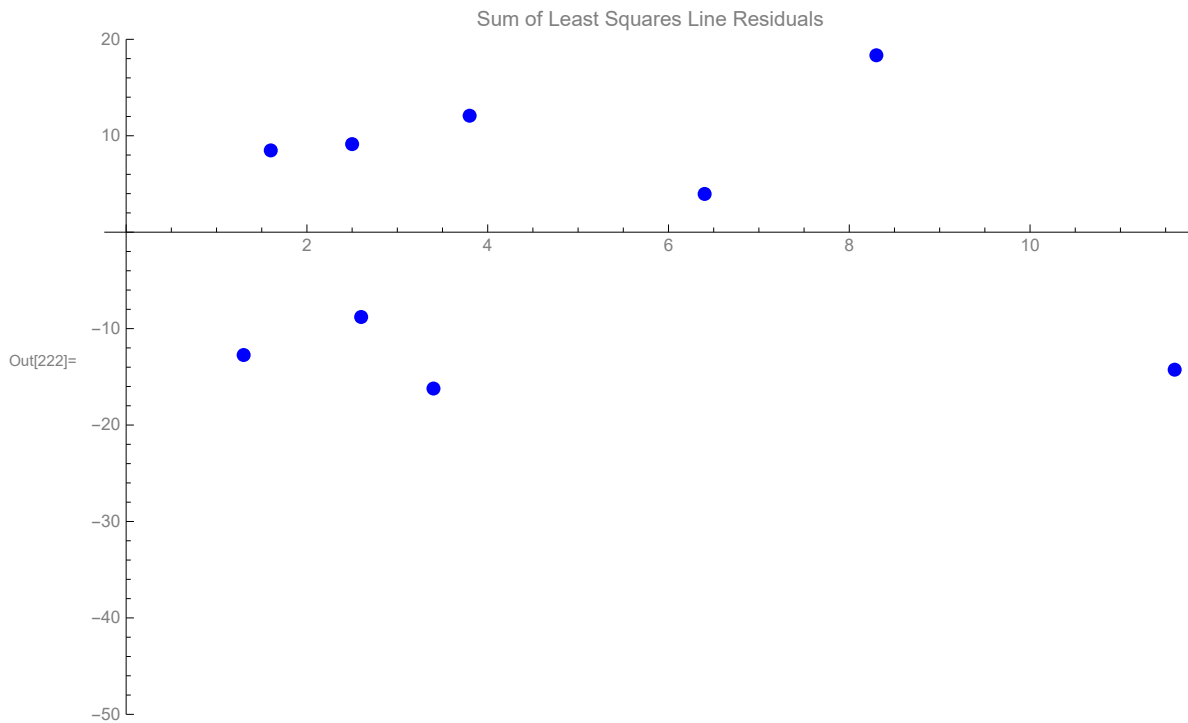In[234]:= **Show[plote, plota]**

Out[234]=

Least Squares Residuals

In[215]:= `threeRes = {points2[[1]][[2]] - lsLine[points2[[1]][[1]]],`
`    points2[[2]][[2]] - lsLine[points2[[2]][[1]]],`
`    points2[[3]][[2]] - lsLine[points2[[3]][[1]]],`
`    points2[[4]][[2]] - lsLine[points2[[4]][[1]]],`
`    points2[[5]][[2]] - lsLine[points2[[5]][[1]]],`
`    points2[[6]][[2]] - lsLine[points2[[6]][[1]]],`
`    points2[[7]][[2]] - lsLine[points2[[7]][[1]]],`
`    points2[[8]][[2]] - lsLine[points2[[8]][[1]]],`
`    points2[[9]][[2]] - lsLine[points2[[9]][[1]]]}`

Out[215]= $\{-12.7376, 8.48019, 9.13371, -8.79367, -16.2128, 12.0777, 3.96564, 18.3453, -14.2585\}$

In[216]:= `lslist = Transpose[{index1, threeRes}];`

In[222]:= `lsRes = ListPlot[lslist, PlotStyle → Blue, PlotRange → {-50, 20},`
`    PlotLabel → "Sum of Least Squares Line Residuals", LabelStyle → Gray, ImageSize → Large]`

Out[222]=

In[218]:= `Total[threeRes]`

Out[218]= 1.84741×10^{-13}

Comparison of the Residuals

In[248]:= `GraphicsRow[{tpRes, mmRes, lsRes}, ImageSize → Full]`

Out[248]=