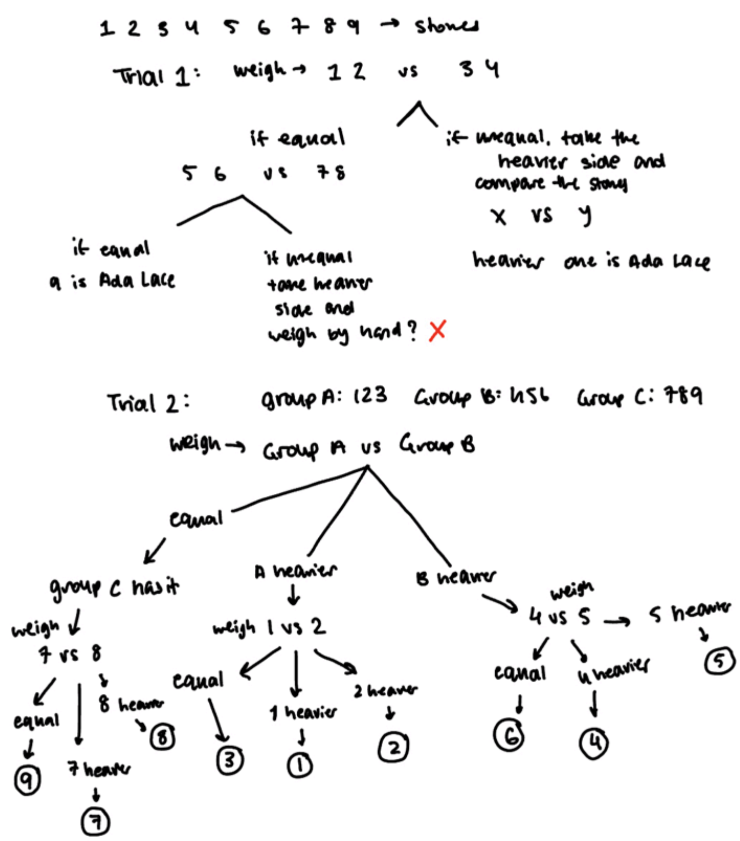
Problem Statement: Given nine Rocks, with one "special" one that is heavier than the others, and the rest being all the same weight, your goal is to determine which one has a different weight. You are allowed to use a balancing scale twice (weigh two sides against each other) to narrow the nine rocks down to the one heavier rock.

Process: We had two trials to find the solution to this problem. They are outlined in the figure below.



Process (Written):

TRIAL 1:

1. Weigh Group A (Rock 1 and Rock 2) against Group B (Rock 3 and Rock 4)
    a. If Group A is heavier…
        i. Weigh Rock 1 vs Rock 2
            1. If Rock 1 is heavier, then the special Rock is Rock 1
            2. If Rock 2 is heavier, then the special Rock is Rock 2
    b. If Group B is heavier
        i. Weigh Rock 3 vs Rock 4
            1. If Rock 3 is heavier, then the special Rock is Rock 3
            2. If Rock 4 is heavier, then the special Rock is Rock 4
    c. If the scale is balanced (they weigh the same)
        i. We know that Rocks 1, 2, 3, and 4 are not the special Rock
2. If the scale is balanced, weigh Group C (Rock 5 and Rock 6) against Group D (Rock 7 and Rock 8)
    a. If Group C is heavier…
        i. This fails because we no longer have a method to weigh the Rocks (ran out of attempts)
    b. If Group D is heavier…
        i. This fails because we no longer have a method to weigh the Rocks (ran out of attempts)
    c. If the groups weigh the same…
        i. The special Rock is Rock 9, as we have noted that all other Rocks are equal in weight

TRIAL 2: (OUR SOLUTION)

1. Divide the 9 Rocks into three separate groups of three, labeling them group A, B, and C, respectively. Assign each Rock a number, and assign A: (1,2,3), B: (4,5,6), and C: (7,8,9)
2. For the usage of the first scale, place group A on one side, and group B on the opposing side. Now, there are 3 possible outcomes:
    a. If both sides of the scale are equal, the heavier Rock must be in Group C (as we know that all the other Rocks are the same weight)
    b. If the side of Group A is heavier, the heavier Rock must be in Group A.
    c. If the side of Group B is heavier, the heavier Rock must be in Group B
3. Separate the group with the heavier Rock from Part 2 into the individual Rocks
4. Now, the usage of the second scale may commence.
    a. If Group A was heavier…
        i. Weigh Rock 1 vs Rock 2
            1. If they are equal, then Rock 3 is the heavier one (as we know that all the other Rocks are the same weight)
            2. If Rock 1 is heavier, the special Rock is Rock 1
            3. If Rock 2 is heavier, the special Rock is Rock 2
    b. If Group B was heavier…
        i. Weigh Rock 4 vs Rock 5
            1. If they are equal, then Rock 3 is the heavier one (as we know that all the other Rocks are the same weight)
            2. If Rock 4 is heavier, the special Rock is Rock 4
            3. If Rock 5 is heavier, the special Rock is Rock 5
    c. If Group A and Group B were equal…
        i. Weigh Rock 7 vs Rock 8
            1. If they are equal, then Rock 9 is the heavier one (as we know that all the other Rocks are the same weight)
            2. If Rock 7 is heavier, the special Rock is Rock 7
            3. If Rock 8 is heavier, the special Rock is Rock 8

This solution is correct as it addresses every possible situation that may occur. Dividing the Rocks into groups of 3, it makes sure 1 outlier will be clear. Moreover, this logic gives the Rock the opportunity to be placed at any location and still able to be identified.

Extensions:

Problem 1: What is the maximum number of stones among which you can guarantee finding the single heavier stone using only 2 weighings on a balance scale?

Solution:

We know that each weighing of the balance scale must have 3 possible outcomes:

- The left side is heavier
- The right side is heavier
- The scale is balanced

Each weighing will provide 3 outcomes, so with 2 weighings we will have:

3*3 = 3^2 = 9 possible scenarios.

With 2 weighings, we can distinguish between 9 possible sequences of outcomes. And each of the 9 possible outcomes from the two weighings can be mapped to one stone being heavy.

So, with 2 weighings, we can identify the heavy stone in a group of up to 9 stones, which is the same as the original problem, meaning the solution we chose above was optimal; there is no way to solve the problem with less than 2 weighings.

Problem 2: You are given n stones, each with different weights. You can use a balance scale, only comparing two stones at a time. What is the minimum number of weighings required to find both the heaviest and lightest stones?

Solution:

Step 1:

Divide the n stones into n/2 pairs. For example, 8 stones would become 4 pairs, with the first and second stone being pairs, the third and fourth being pairs, and so on and so forth.

Comparing each one with the scale tells you which of the two is heavier and which is lighter. Take the heavier stones from each pair and put them in a group. Likewise, do the same with the lighter stones.

There will now be:

n/2 candidates for the heaviest stone

n/2 candidates for the lightest stone

Step 2:
All we have to do now is find the heaviest among the heavy group and lightest among the light group. We will continue to compare each stone with another within these groups, eliminating stones as we go.

In the heavy group,
If you have x stones, after one comparison, the lighter stone is ruled out, and we are left with k - 1 stones. After two, we are left with k-2. This continues for k-1 comparisons, where only one stone remains, which must be the heaviest.

For example:
If k = 4, and we want the heaviest,
Compare stone A and B, where the lighter one is removed
Compare stone C and D, where the lighter one is removed
Compare the 2 winners to find the heaviest overall.
This is 4 - 1 = 3 comparisons.

Therefore, to find the heaviest and lightest stone in their respective groups, we need to take the group size, n/2, and subtract one.

Adding all of the scale comparisons from both steps, we get

$n/2 + (n/2 - 1) + (n/2 - 1) = (3n)/2 - 2$

So to find both the heaviest and lightest stones among n distinct stones using only a 2-stone balance scale, the minimum number of scale uses would be as follows:

$(3n)/2 - 2$

But what if n is odd, and we can’t get even pairs?
To find the minimum number of scale uses while the number of stones is odd, we can repeat step 1. We just have to set the last stone aside for now. So we must have:
x heaviest candidates,
x lightest candidates,
and 1 stone left over.
Once we have our two groups of stones, we can add our extra stone back in. We have not compared it to any other stones yet, so it would need to be weighed in both groups.

This means that both groups would have $x + 1$ stones, with the extra stone being the plus one.

Now we need to find the heaviest or lightest within each group. Similar to the latter half of step 2, we take the number of candidates and subtract one to get the number of comparisons.

For both groups we get $(x+1-1)$.
So for the total:
Step 1 pairing = x scale uses
Finding max and min = $(x+1-1) + (x+1-1)$

The total is $x + (x+1-1) + (x+1-1) = 3x$ comparisons.

Now, all we have to do is write this in terms of n. If we have x pairs, the total number of stones n would equal to 2x+1, where the plus one is the extra stone. This also ensures n is odd, as any number multiplied by 2 is even, and any even number plus one will be odd.

$n = 2x+1$
$x =(x-1)/2$

Plugging x in to the previous expression for the total uses:

$3x = 3((n-1)/2)= (3n-3)/2 = (3n-3)/2 = 3n/2 - 1.5$

General Formula (both odd and even):

If we use $3n/2 - 2$ for all n, odd number inputs return decimals.
However, from our formula $3n/2 - 1.5$, we know that the solution is always 0.5 less than the actual solution. To fix this, we can apply the ceiling function to $3n/2$ in the first formula.

$\lceil 3n/2 \rceil - 2$

This will always return a whole number, which we need, as you cannot have 0.5 scale comparisons.