Advanced Physics POW #1: Yellow Light Problem

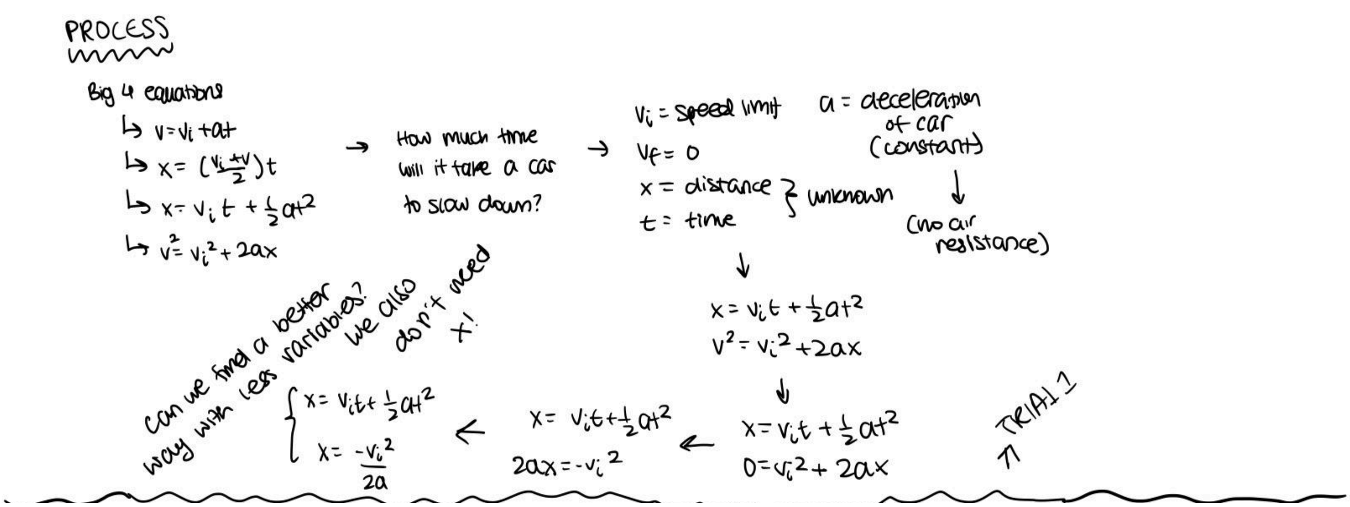
Problem Statement: The problem asks us to determine the duration a traffic light should remain yellow to allow an individual to make an informed decision — whether to proceed through the intersection (GO ZONE), or to stop before the intersection (STOP ZONE). We also must determine any factors that would affect this duration. We are given that the car must be going the speed limit, and we determined that it must maintain a constant acceleration/deceleration. Finally, we are told that the car must pass through the intersection COMPLETELY before the light turns red, and it also should be able to stop before the intersection (the line) before the light turns RED if the driver decides so.

Process:

Trial 1:

- To begin with, we tried calculating how much time it takes for a car to slow down and how much distance it would cover.
  - We considered the variables of the speed limit, the distance to travel, and the deceleration of a car (which is constant without air resistance)

- We set up a system of 2 equations to get x as a function of time, deceleration, and speed limit.
  - Soon, we realized that this approach is much too complex, and a simpler approach considering fewer unknowns would be better
  - Additionally, we realized that we do not need to isolate x just yet, as we only need the stopping time for the initial bound for the yellow light time



Trial 2:

- We started with a simpler equation: $v = v_i + at$, and from this, we derived that t (our stopping time) will be $-v_i / a$
    - This describes the absolute minimum time that our traffic light should be yellow. We can define a variable t_r for reaction time, and add it
    - Therefore, we have MINIMUM LIGHT TIME = $(-v_i / a) + t_r$

- Now, let's calculate the STOP ZONE. This will be crucial in our final calculation. We did this by using the equation $x = ((v_i + v) / 2) * t$
    - We plugged in $t = (-v_i / a)$
    - Therefore, our STOP ZONE is any value GREATER than $(-v_i^2 / 2a)$
        - ***Note that this value will end up being positive, as 2a is a negative value (car is decelerating)

$v = v_i + at \rightarrow 0 = v_i + at \rightarrow t = \frac{-v_i}{a}$

TRIAL 2

Now we know that the yellow light time should be more than $\frac{-v_i}{a}$

Let's find "STOP" zone

$x = \left(\frac{v_i + v}{2}\right)t$

$t = \frac{-v_i}{a}$ $\rightarrow$ $x = \left(\frac{v_i + 0}{2}\right)\left(-\frac{v_i}{a}\right) = \frac{-v_i^2}{2a}$

$v = 0$

"STOP" zone is anything greater than $\frac{-v_i^2}{2a}$, therefore, the "GO" zone is anything less than $\frac{-v_i^2}{2a}$

Let's try an example to verify:

$v_i = 30 m/s$

$a = -3 m/s^2$

car goes 30m/s

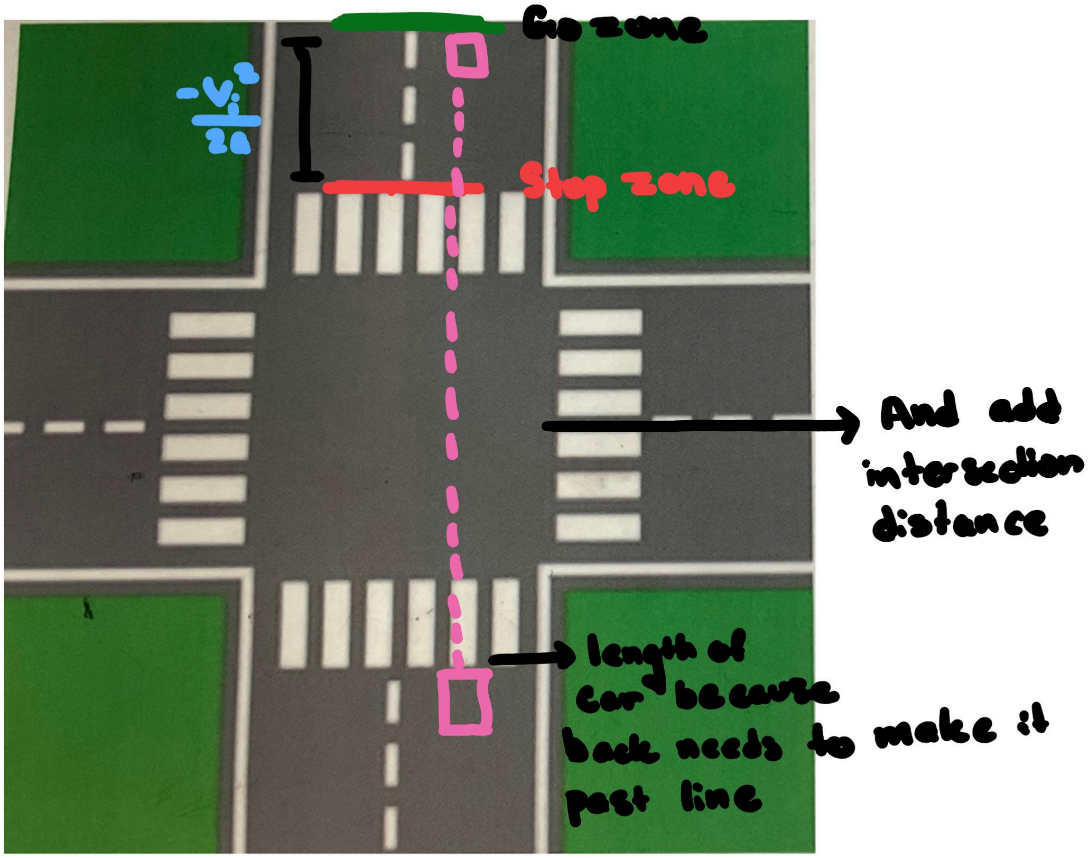
↳ stopping time = 10 s

↳ stopping distance = $\frac{900}{20} = 45m$

from this, we have → based on stop zone → minimum time of light should be stopping time + reaction time = $\frac{-v_i}{a} + t_r$

(t_r: reaction time)

- However, we cannot just stop here. We need to account for the GO ZONE distance, which consists of the length of the car, the length of the intersection, and finally, the MINIMUM STOPPING DISTANCE, which we found to be the STOP ZONE.
- See the figure below



- Therefore, we have identified some new factors. The variables in our final equation are as follows
  - t = yellow light time
  - c = length of the car
  - i = length of the intersection
  - v_i = speed limit (since we assume that the car is driving the speed limit)
  - a = deceleration of the car (will be a negative number)
  - t_r = reaction time

- Finally, we can simply utilize the total distance over speed to get time.
  - You might be wondering, *if there is speed in the denominator, doesn't that mean that the time will decrease as speed increases?* Not necessarily. Please keep reading.

Solution:

What about time it takes to safely go?

↳ distance to go = length of car + length of intersection + minimum distance from intersection

length of car → c; length of intersection → i; minimum distance from intersection → $\left(-\frac{v_i^2}{2a}\right)$ → from prev. part

we have: $\text{time} = \frac{\text{distance}}{\text{speed}} = \boxed{\frac{\left(c+i-\frac{v_i^2}{2a}\right)}{v_i} + t_r}$

↳ this is ok because as v_i increases, stopping distance also increases

- We have, $((c + i + (-v_i^2 / 2a)) / v_i) + t_r$
    - Total distance = car length + intersection length + minimum stopping distance
    - Speed = speed limit

- This solution is correct, as it considers the STOP ZONE and the GO ZONE, while including reaction time as well as the speed limit. There is no issue with the speed being in the denominator, because it is also in the numerator
    - As the denominator gets bigger, the minimum stopping distance also gets bigger; therefore, they counteract each other and yield an accurate result

Discussion Questions:

a) What must be true about the STOP and GO zone for the intersection to be safe? Describe what happens at the intersection if the zones do not adhere to this rule?

The GO zone should extend further behind the intersection than the STOP zone. This ensures that a car driving at the speed limit into the intersection always has the option of continuing through the intersection during a yellow light, but can also stop before it.

b) How would the following conditions affect the required yellow light time? Which zone would they affect? Via which variable?

1. Speed limit - An increase in v_i would lead to an increase in breaking distance, as the breaking distance directly increases as v_i does. This would increase the time a car takes to cross the intersection.
2. Bald tires - Acceleration would increase, as the car would not be able to decelerate as efficiently. This would increase the stopping distance, in turn increasing the total time the light needs to be on.
3. For long vehicles such as a semi-truck, the distance required for a truck to cross would be increased to ensure the back end of the truck completely crosses the intersection.
4. Distracted driver - increase t_r due to reaction time increasing

c) Would it be a good idea to mark the stop-and-go zone on the road before the intersection? Why or why not?

Yes. It would be a good visual indicator of whether a car should cross or not. It could also help with finding fault in accidents; if a car is in the stop zone during a yellow light, crosses the intersection, and gets into an accident, we can confirm that they are at fault for the accident.

d) In the 1960s, traffic engineers piloted a system that would display a countdown timer to show how much time was left before the light would turn red. Why do you think the engineers decided against the idea?

Engineers decided against this process because individuals who saw a time limit would most likely concentrate on the timer instead of their driving. They could be motivated to speed up to try to make the time, putting others in danger. Additionally, people may underestimate the amount of time it takes for them to cross the intersection and cause an accident.