Question: Does the relationship between the acceleration of a cart on an inclined plane (a modified Atwood's Machine) and the hanging mass follow Newton's Second Law?

Hypothesis: The relationship between mass of the hanging weight and the acceleration down the incline will be linear, where the slope is g(1+sin(θ)/m_T and the y-intercept is -gsin(θ). (m_T = total mass).

Strategy

- The total mass was kept constant by adding washers to each mass. Each trial, a washer would be swapped between them, keeping the mass constant.
- The cart is either pulled up the ramp or released from the top of the ramp (based on whether acceleration is up or down the ramp) and the slope of the velocity vs. time (acceleration) is observed using a Vernier motion detector within the cart.
- The acceleration and m2 are graphed to confirm that the slope is equal to the gravity in the parallel direction divided by the total mass of the system.

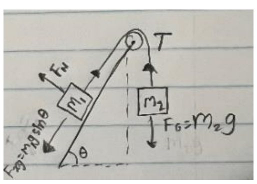

Figure 1. Modified Atwood's Machine & Free Body Diagram, shows the forces involved in experiment (excludes friction).

Derivation of expected slope

Friction is negligible, so it is not included in the following equations, which are derived from the free body diagram and Newton's Second Law (F = ma).

$$F_{net} = a * (m_1 + m_2)$$

$$m_2 g - sin(\theta) m_1 g = a(m_1 + m_2)$$

$$a = \frac{m_2 g}{m_1 + m_2} - \frac{m_1 g \sin(\theta)}{m_1 + m_2}$$

$$a = \frac{m_2 g}{m_1 + m_2} - \frac{g(m_1 + m_2 - m_2)\sin(\theta)}{m_1 + m_2}$$

$$a = \frac{m_2 g - m_T g \sin(\theta) + m_2 g \sin(\theta)}{m_1 + m_2}$$

$$a = \frac{m_2 g(1 + \sin(\theta))}{m_1 + m_2} - g\sin(\theta)$$

Because the total mass is constant, when we compare acceleration and m_2 as y and x respectively, then the slope we would expect (as per slope-intercept form) is:

$$\frac{g(1 + \sin(\theta))}{m_T}$$

where g = 9.8, theta does not change during testing (13 degrees), and m_T is constant, leaving a linear function.

Data

Theta (θ) = 13° |*m_2* = Hanging Mass | *m_1* = Cart Mass | *a* = Acceleration

a (m/s^2)	m_2 (kg)	m_1 (kg)	m_1 + m_2
3.225	0.25	0.3	0.55
2.376	0.21	0.34	0.55
1.457	0.17	0.38	0.55
0.5994	0.13	0.42	0.55
-0.1816	0.09	0.46	0.55
-1.067	0.05	0.5	0.55

Negative a (acceleration) means the cart is falling down the ramp.

Analysis

When a and m_2 are graphed as y and x respectively, the relationship is indeed linear as was determined during the derivation.

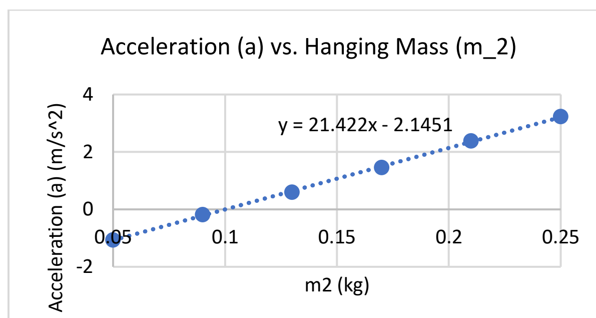

Figure 2. Measured Acceleration vs. Hanging Mass

As indicated through a linear regression of the relationship, the slope is 21.422 while the expected slope as calculated in the derivation is 21.864, a 1.85% error. The y-intercept can also be compared. The expected value of the y-intercept is -2.205 (g*sin(13)) while the experimental value is -2.1451, a 2.70% error. The lower slope indicates that there was less acceleration within the experiment than expected. The most likely source of error is friction, which was considered negligible in this experiment but, if included, would lower the acceleration as m_2 increases. Additionally, minor, uncontrollable pushes as the cart was released may have altered the measured velocity and, therefore, the acceleration.