**Using Math Modeling to Objectively Identify and Correct Gerrymandering**

**Grant Proposal**

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**Author Note**

This STEM project was conducted under the supervision and support of Dr. C.

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**Executive Summary**

Several legislatures have been established to prevent gerrymandering by making compactness mandatory and outlawing discriminatory redistricting plans in Section 2 of the Voting Rights Act. However, these laws are ultimately deemed ineffective because gerrymandering cannot be objectively measured or solved (Kirschenbaum & Li, 2021). Current models aimed at outputting optimized maps fail to account for all factors. Chatterjee et al., Liu et al., and Guest et al.'s models are accurate but solely consider geographical size, compactness, and preserving old district cores. Existing models leave out one crucial factor that is one of the main causes of the continued usage of the archaic electoral college system: just representation for minorities. Thus, the project goal is to develop a mathematical model to objectively and accurately identify and correct gerrymandering, specifically of minorities. This research could significantly change the way we redistrict and provide objective legal evidence to prosecute gerrymandering to make every vote count the same.

*Keywords:* gerrymandering, math model, electoral voting, just representation for minorities

**Using Math Modeling to Objectively Identify and Correct Gerrymandering**

Several Americans have identified and agreed that gerrymandering is a major issue that threatens democracy and unfairly allows politicians to maintain power through manipulation (Kirschenbaum & Li, 2021). In a recent case taken to the Wisconsin Supreme Court, voters declared that current electoral district maps were gerrymandered and suggested replacing them with fairer ones (Associated Press, 2024). The case was difficult to fight for because there was no objective evidence to support the gerrymandering. If evidence could be obtained, then gerrymandering could be prosecuted and the way politicians redistrict would be impacted greatly.

**Commented [A2]:** This initial section is NOT identified as the Introduction (Hoffman, 2000, 2007) but can include a broad overview of introductory information. Thereafter, each section would be formatted according to APA heading level guideline - https://owl.purdue.edu/owl/research_and_citation/apa_style/apa_formatting_and_style_guide/apa_headings_and_seriation.html (Davidov et al., 2013; Knafo et al., 2008). Section Headers would be the same size and font as the text (Warneken & Tomasello, 2006). As a general guide, a grant proposal should start with general relevance and move towards identifying knowledge gaps in the field and potential researchable questions/needs.

**Background Information**

Gerrymandering, visually represented in Appendix A (Ingraham, 2015), is a major political issue that describes the manipulation of electoral borders to work in favor of, or against, a specific party (Diller, 2018). It is a deeply undemocratic process, but with no objective evidence, it is impossible to act against it (Kirschenbaum & Li, 2021). There are two variations of gerrymandering: packing and cracking. Packing describes grouping several voters of the same party in one district to ensure that one district wins by a tremendous margin, but the surrounding districts are less competitive, giving an unfair advantage to the party of the politicians that are developing the boundary map. Cracking also makes districts less competitive, but it does this by splitting a party's voters across several districts, making them a minority in each one (Jones, 2018).

There are several constitutional requirements for redistricting to prevent gerrymandering. These include compactness or the degree to which districts are tightly packed together with relatively smooth district borders and relatively normally shaped electoral districts. Additionally, Section 2 of the Voting Rights Act states that redistricting to intentionally pack or crack minorities is strictly prohibited (Section 2 of the Voting Rights Act, 2015). Despite these regulations, gerrymandering still occurs because the power of politicians goes unchecked. The nature of redistricting is so subjective and complex, that attempts to prosecute gerrymandering are often fruitless (*Gill v. Whitford*, 2018).

## Existing Models

### *Chatterjee et al.'s Model*

Past gerrymandering research involved developing math models to identify the optimal map. Many of these mathematical models exist, but each contains its flaws. It is impossible to achieve a model with complete accuracy. One existing model that comes close to full accuracy utilizes the efficiency gap, a measure first developed by McGhee and Stephanopoulos in 2015 (Stephanopoulos & McGhee, 2015). The efficiency gap is deemed a mathematically accurate measure of the quantity of "votes that did not count" (Cover, 2017). Chatterjee et al. began by formalizing the method and then creating several proofs and studying the method's mathematical properties. They were able to find that the efficiency gap measure attains only a finite discrete set of rational values which significantly reduced the range of possible district maps. Chatterjee et al.'s solution was a rapid randomized algorithm that cycled through several potential district maps before selecting the least gerrymandered one (Chatterjee et al., 2018).

Despite the accuracy of this model, certain fundamental flaws remain. The model is simple and does not consider other factors that are typically accounted for when redistricting. Other models that do consider factors, such as compactness or preserving the cores of previous districts, are typically less accurate or too slow (Pandit et al., 2023). Ultimately, a math model that accounts for several factors and can accurately output a non-gerrymandered map in a short period does not exist but would have a major impact on the political world.

***Liu et al.'s Model***

Liu et al. also recognized the gap in Chatterjee et al.'s model and in several other common non-gerrymandered redistricting models: they outputted maps that only followed non-political constitutional laws regarding redistricting, such as compactness, preserving old district cores, and geographical symmetry. However, Liu et al. argued that solely focusing on those factors fails to consider essential political influences on redistricting. Thus, Liu et. al formulated two mathematical optimization models to implement two new criteria: fairness and competitiveness (Liu et al., 2020). Fairness ensures that seats are fairly allocated to political parties based on voter decisions and competitiveness aims to maximize the number of competitive districts to prevent districting solutions that favor one political party over another. This model provides a more thoughtful, appropriate approach, but it also fails to consider one of the most important nonpolitical constitutional factors: just representation for minorities (Liu et al., 2020).

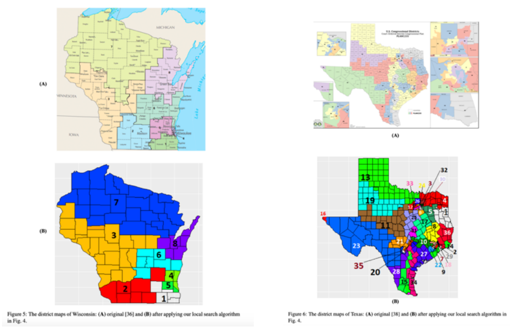

**Figure 1:** The original (top) and non-gerrymandered (bottom) district maps of Wisconsin (left) and Texas (right).

### *Guest et al.'s Model*

Guest, Kanayet, and Love identified partisan gerrymandering as a dangerous threat to democracy but considered redistricting fairly to be a task that may exceed human capacities. The researchers used computational models to automate redistricting by optimizing criteria. The criteria included compactness and reducing the pairwise distance between voters in a district. The outputted maps, refer to Figure 2, were accurate, but the researchers identified that an ideal model should also consider municipal boundaries, historic communities, and relevant legislation in addition to compactness (Guest et al., 2019).

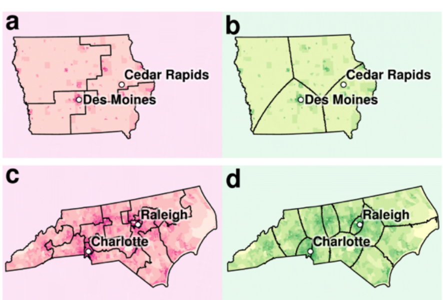


**Figure 2:** The actual (a, c) and computed (b, d) district maps for Iowa (a, b) and North Carolina (c, d). Darker areas represented a densely populated location.

**Problem Statement**

Gerrymandering is a deeply undemocratic process that is currently unchecked with no objective method of measuring it (Kirschenbaum & Li, 2021). This project seeks to maintain the accuracy and efficiency of Chatterjee et. al, Liu et. al, and Guest et al.'s models while considering a crucial factor that defines the purpose of the electoral college system: just representation for minorities. This law is represented in Section 2 of the Voting Rights Act (Section 2 of the Voting Rights Act, 2015).

**Methodology**

This project will be completed according to the typical mathematical modeling process: define the problem, make assumptions, define variables, get a solution, analyze the solution, iterate, and communicate (*What Is Mathematical Modeling?*, 2023). Each factor will be added one at a time, beginning from the simplest, even distribution of the population, and working towards the most complex one, and the final goal: just representation for minorities. Each factor will be added to the model one at a time and several trials will be completed for each one using existing voter datasets. The data will be analyzed by simulating an election and comparing the predicted election result of the outputted map to the popular vote.

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**Results**

The solution and result of this project will be a mathematical model that can effectively output a score that describes the level of current gerrymandering and provides a corrected electoral district map with as minimal gerrymandering as possible when the coordinates of a current gerrymandered map are inputted.

**Section II: Specific Aims**

This proposal's objective is to describe the purpose, methodology, and impact of the project "Using Mathematics to Objectively Identify and Correct Gerrymandering".

Our long-term goal is to provide an objective measure of gerrymandering and a method to produce electoral district maps without human bias where the central goal of this proposal is to develop a mathematical model that can output non-gerrymandered maps that provide just representation to minorities. The rationale is that the current process of redistricting is vulnerable to human bias and malicious manipulation, but a mathematical model could serve as an effective, unbiased map developer. The work we propose here will change the redistricting process and provide a check on gerrymandering and the power of incumbent politicians. The specific goals are as follows below.

**Specific Aim 1:** Develop an effective mathematical model that outputs non-gerrymandered maps with simple factors, such as compactness and geographic symmetry.

**Specific Aim 2:** Iterate the second model to add another factor: just representation for minorities.

**Specific Aim 3:** Develop a mathematical model to objectively identify the degree of gerrymandering present in a current electoral district map.

**Section III: Project Goals and Methodology**

**Relevance/Significance**

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[1] Adam Smith (1790/1976) claimed that empathy (or sympathy, as he called it; see Jahoda, 2005, for the shared history of the terms) with negative emotions leads to pleasure. The philosopher David Hume quipped in response that "if all Empathy were agreeable . . . a hospital would be a more entertaining place than a ball" (Hume, as cited in Frazer, 2010, p. 100).

Gerrymandering and unfair redistricting have devastating consequences on minority communities. In 2021, Louisiana state officials passed a congressional map where Black people were only the majority in one of six districts, despite making up 33% of the population. Further, Black voters can only influence the electoral outcome in around 17% of Louisiana's districts and white voters determine the outcome of 83% of them (Wiley, 2022). One of the primary purposes of retaining the archaic electoral system is to counteract the majority rule and provide just representation to minorities. However, this blatant disparity in Louisiana represents just one of several states that use redistricting as a maliciously subtle method of taking away voting power from minorities (Wiley, 2022). State officials can pass these redistricting plans because of the lack of objective evidence and the inability to develop an ideal non-gerrymandered map. Thus, a mathematical model to identify and correct gerrymandering without human bias could revolutionize the redistricting process and balance the power of state map developers. Ultimately, this research aims to protect the rights of minorities and ensure the fundamental right to vote is equally counted for all people.

**Innovation**

As described in the Existing Studies section, several math models exist that attempt to correct gerrymandering, but none of the current models account for racial inclusivity considerations. Most models focus on the basic constitutional requirements of redistricting: geometrical symmetry, even population distribution, and compactness. Thus, a model that could effectively correct gerrymandering and prove just representation to minorities would be unique and fill a knowledge gap in the field of applied mathematics, specifically in politics.

**Methodology**

***Specific Aim #1:***

The objective is to develop an effective mathematical model that outputs non-gerrymandered maps with simple factors, such as compactness and geographic symmetry. Our approach (methodology) is to use the popular vote instead of the efficiency gap and use computational models to randomly generate thousands of maps to identify the optimal one. Our rationale for this approach is that gerrymandering is a combinatorial optimization problem with a finite discrete set of rational values (Chatterjee et al., 2018) so a rapid randomized algorithm is necessary to identify the optimal map.

| | Vote share | | Number of Seats | | Normalized efficiency gap (current) |
|---|---|---|---|---|---|
| | Democrats $\frac{\text{PartyA}(\mathcal{P})}{\text{Pop}(\mathcal{P})}$ | GOP $\frac{\text{PartyB}(\mathcal{P})}{\text{Pop}(\mathcal{P})}$ | Democrats | GOP | $\text{Effgap}_{\kappa}(\mathcal{P},\ldots\ldots)/\text{Pop}(\mathcal{P})$ |
| Wisconsin | 50.75% | 49.25% | 3 | 5 | 14.76% |
| Texas | 43.65% | 56.35% | 12 | 24 | 4.09% |
| Virginia | 51.96% | 48.04% | 4 | 7 | 22.25% |
| Pennsylvania | 50.65% | 49.35% | 5 | 13 | 23.80% |

**Figure 3:** The results of Chatterjee et al.'s model. The calculated efficiency gaps closely match those of other published models and comparisons to the popular vote.

**Justification and Feasibility.** These methods are relevant because Chatterjee et al.'s team was able to achieve significant accuracy through their usage of a rapid randomized algorithm and by using the efficiency gap (Chatterjee et al, 2018). Thus, by adopting a similar model to Chatterjee's and accounting for new factors, it is possible to maintain the accuracy of Chatterjee's model while considering additional redistricting factors.

**Summary of Preliminary Data.** The preliminary data shown in Figures 4 and 5 support Specific Aim #3 because they support the claim that both iterations, especially Version B, were able to accurately identify the extent of gerrymandering in a provided map.

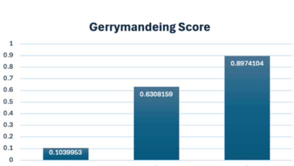



**Figure 4:** Bar graph displaying the outputted gerrymandering scores of Identification Model Version A when given Map 1 (low gerrymandering), Map 2 (medium gerrymandering), and Map 3 (high gerrymandering).


**Figure 5:** Bar graph displaying the outputted gerrymandering scores of Identification Model Version B when given Map 1 (low gerrymandering), Map 2 (medium gerrymandering), and Map 3 (high gerrymandering).

**Expected Outcomes.** The overall outcome of this aim is to develop a model that outputs non-gerrymandered maps that consider simple factors. This model will serve as a base for the project goal, adding the factor of just representation for minorities.

**Potential Pitfalls and Alternative Strategies.** We expect some fallbacks to occur, primarily due to knowledge gaps or resource deficits. The latter will be solved by seeking help from our project mentors. Any knowledge gaps that arise will be resolved by researching or consulting our mentor or another field expert.

***Specific Aim #2:***

The objective is to develop a second version of the model developed in Specific Aim #1 to account for another factor: the representation of minorities. Our approach (methodology) is to add one factor at a time because this will allow the model to retain its prior accuracy while adjusting to consider another factor.

***Specific Aim #3:***

The objective is to develop a mathematical model to objectively identify the degree of gerrymandering present in a current electoral district map. Our approach (methodology) is to compare the calculated "gerrymandering score" to the electoral district demographics vs. the popular vote. Our rationale is that this will allow us to confirm the calculated measure with known data that represents the percentage of a minority in a city vs. the number of districts in which this minority is a majority.

## Section III: Resources/Equipment

The nature of this project does not require extensive physical resources to complete the research. It solely requires access to background information, datasets, and computer programming software. Essentially, the only equipment for this project is a computer, which is already obtained.

## Section V: Ethical Considerations

Electoral voting itself has been a long-debated, controversial issue. However, it does allow for a counteract to the majority rule: it, theoretically, provides a voice to people from minority groups. Thus, there are significant positive ethical considerations of this research, but no considerable negative ones.

## Section VI: Timeline

Phase 1 (Aug-Sep): Brainstorming/choosing project idea

Phase 2 (Sep-Nov): Research existing information and fill personal knowledge gaps

Phase 3 (Nov-Dec): Develop methodology/project development plan and complete zfairs paperwork

Phase 4 (Nov-Dec): Develop a measure to identify the degree of gerrymandering present in a current map

Phase 5 (Dec): Collect preliminary data and analyze it

Phase 6 (Dec): Present preliminary findings to the audience at the December Fair

Phase 7 (Dec-Feb): Further refine the model and develop a model that can output a non-gerrymandered map that does not pack or crack minorities

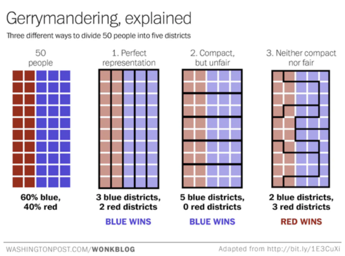
Phase 8 (Jan-Feb): Analyze data

Phase 9 (Feb): Present project results at Feb Fair

Phase 10 (March): Make the refined model accessible to the public

## Section VII: Appendix

*Appendix A: Visual representations of ideal (second from right) and gerrymandered (fourth from right) electoral districts (Ingraham, 2015).*

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