Question: How does adding an incline affect the relationship between the force, mass, and acceleration of m_1 and m_2 in a modified Atwood's machine?

Hypothesis: The relationship between the sine of the incline and the acceleration will be linear. The slope of the incline vs. acceleration graph will be equal to the negative weight of m_1 divided by the mass of the whole system.

Strategy:

- The experimental setup was the Vernier track laid over a stack of textbooks of height h calculated to have an incline of sin(θ). m_1, beginning at rest from the top of the ramp, was connected through a pulley to the hanging mass m_2, held down by an external human force until the start of each experiment. Refer to Figure 1 (right).

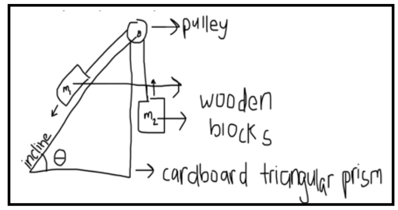


Figure 1: Modified Atwood's Machine with Incline - Diagram of Setup

- The independent variable was theta, the angle of the incline of the ramp. Three trials were completed for each of the three incline angles: 20º, 25º, and 30º. The resulting acceleration of m_1 upon the release of m_2 was recorded using a Vernier motion detector.
- The average acceleration of the three trials was graphed vs. sine of theta to verify that the slope was equal to the negative weight of the cart (m_1) divided by the combined mass of the cart (m_1) and the hanging mass (m_2).

Data:

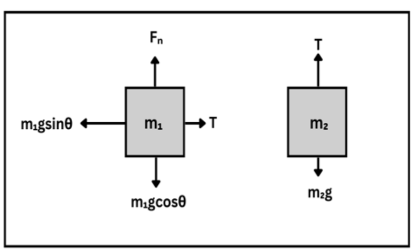
- The mass of the cart (m_1 with a mass of 0.2834 kg) and the hanging mass (m_2 with a mass of 0.05 kg) in the modified Atwood's machine remained constant.

Angle of incline (θ) (degrees)	Independent variable: sinθ	Dependent variable: acceleration (m/s^2)
20	0.342	-1.312
25	0.423	-1.917
30	0.5	-2.232

The acceleration is an average of three trials.

Analysis:

The free body diagrams in Figure 2 show the forces on the masses in the modified Atwood's machine with the two components of gravity.



Friction between the cart and the track is somewhat negligible because the cart's wheels are relatively smooth and glide easily down the ramp. The following equations are based on the free body diagrams.

$$T - mg\sin\theta = m_1 a \qquad mg - T = m_2 a$$
$$T = m_1 a + mg\sin\theta \qquad T = mg - m_2 a$$

Fig 3: Equations derived from individual FBDs

T is equivalent across both systems, so the equations were combined. Then, a was isolated and the equation was rearranged into the linear equation format.

$$m_1 a + mg\sin\theta = mg - m_2 a$$
$$m_1 a + m_2 a = mg - mg\sin\theta$$
$$a(m_1 + m_2) = mg - mg\sin\theta$$
$$a = \frac{mg - mg\sin\theta}{m_1 + m_2}$$
$$a = \frac{m_2 g}{m_1 + m_2} - \frac{m_1 g}{m_1 + m_2}\sin\theta$$

y-intercept   slope

Fig 4: Derivation of equation of acceleration in terms of theta

This equation indicates that there is a linear relationship between sine theta of the incline and the acceleration. The slope of this line, the negative weight of the cart (m_1) divided by the combined mass of the system, is the coefficient of sin(θ).

The linear relationship is confirmed by the graph of the relationship between the sine of theta vs. the acceleration. The slope is -7.6372. To create the best fit line, the point (0, 1.470) was added. 1.470 is the calculated, theoretical acceleration of the cart when sin(θ) = 0.

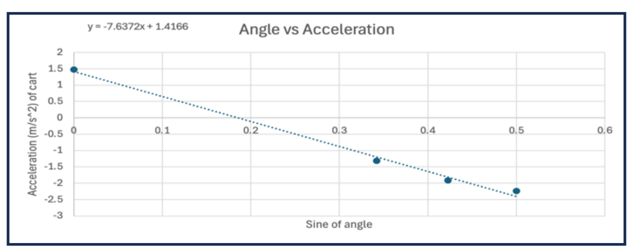


Fig 5: Sine of angle of incline vs. acceleration of cart

The theoretical weight of the cart (m_1) over the combined mass of the system calculated using Figure 3 equations is -8.33. This yields a percent error of 8.32%. The experimental value was less than the expected value, primarily due to the initial assumption that friction is negligible. The wheels of the cart or any minor debris on the tracks, such as sand or dust particles, could contribute to the discrepancies in the cart's acceleration.